CVPR #1204

Shape Decomposition: Combining Minima Rule, Short-Cut Rule and Convexity Supplementary Material

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1. Proof of Theorem on Page 3.

Theorem: Given a shape S without holes, for any point pair $(p, q) \in S$, the path R which corresponds to the inner distance between p and q is also the path which can minimize $\max_f \max_{t \in R} g_f(t)$ as defined in Eqn.7 on page 3.

Proof: Let R' be the shortest path with respect to the inner distance for (p, q) and R^*. If R' cannot minimize $\max_f \max_{t \in R} g_f(t)$ as defined in Eqn.7 on page 3, there must be a path R^* which can minimize the concavity for (p, q), i.e.,

$$R^* = \arg\min_R \max_f \max_{t \in R} g_f(t). \tag{1}$$

Therefore $R' \neq R^*$. Then there is the length of R' is shorter than R^*, $L(R') < L(R^*)$. Let Morse function

$$f^* = \arg\max_f \max_{t \in R^*} g_f(t)$$

which means that f^* maximizes the perpendicular distance between projected points of R^* and (p, q) according to this Morse function. Let $Dir(f^*)$ denote its direction. Similarly, let

$$f' = \arg\max_f \max_{t \in R'} g_f(t)$$

which means that f' maximizes the perpendicular distance between projected points of R' and (p, q) according to this Morse function. Let $Dir(f')$ denote its direction. Then we have the following inequality:

$$\max_{t \in R'} g_{f'}(t) > \max_{t \in R^*} g_{f^*}(t) > \max_{t \in R^*} g_{f'}(t). \tag{2}$$

The first sign of inequality holds because R^* is the path which minimizes $\max_f \max_{t \in R} g_f(t)$. The second sign of inequality holds because f^* maximizes $\max_{t \in R^*} g_f(t)$.

Let t' denote the point which maximizes the perpendicular distances between projected R' points and (p, q) on direction $Dir(f')$.

$$t' = \arg\max_{t \in R'} g_{f'}(t). \tag{3}$$

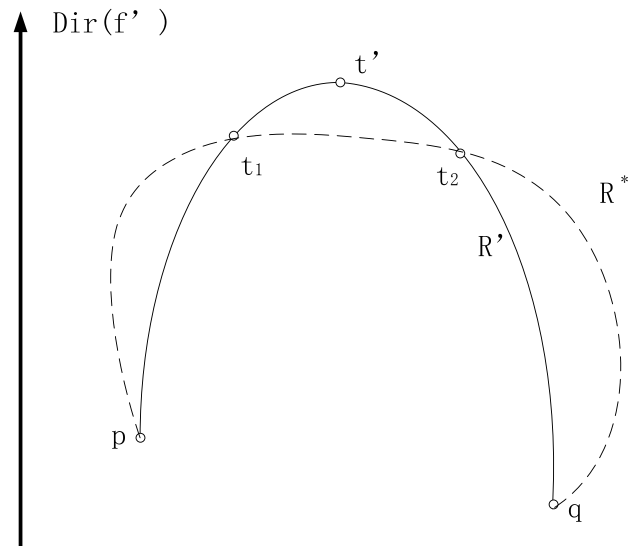

Figure 1. Illustration of relative positions of p, q, t', t_1, t_2, the direction of Morse function f and the two paths R', R^*.

By Eqn. 2 and Eqn. 3, we can infer that $\forall t \in R^*, g_{f'}(t) < g_{f'}(t')$. Without loss of generalization, let $f'(t') > f'(p) > f'(q)$. Figure 1 shows the relationship between the points p, q, t' and the paths R', R^*.

Because there is no hole in S, there must be intersections between R' and R^*. Let t_1 be the closest intersection point to t' between p and t'. Let t_2 be the closest intersection point to t' between q and t'. Since there is no hole in S, t_1 and t_2 are on the same side of $\overline{pq}$ as Figure 1 shows. Then we have $f'(t_1), f'(t_2) < f'(t')$ and any point t on R^* between t_1 and t_2 satisfies: $f'(t) < f'(t')$. Therefore, the subpath between t_1 and t_2 on path R', denoted as $R'(t_1 \rightsquigarrow t' \rightsquigarrow t_2)$ must be outside of the subpath $R^*(t_1 \rightsquigarrow t' \rightsquigarrow t_2)$. So there must be a shorter path connecting t_1 and t_2 than $R'(t_1 \rightsquigarrow t' \rightsquigarrow t_2)$ and we can form a new path R'' which is shorter than R'. This contradicts with the fact that R' is the shortest path.$\sharp$

CVPR
#1204

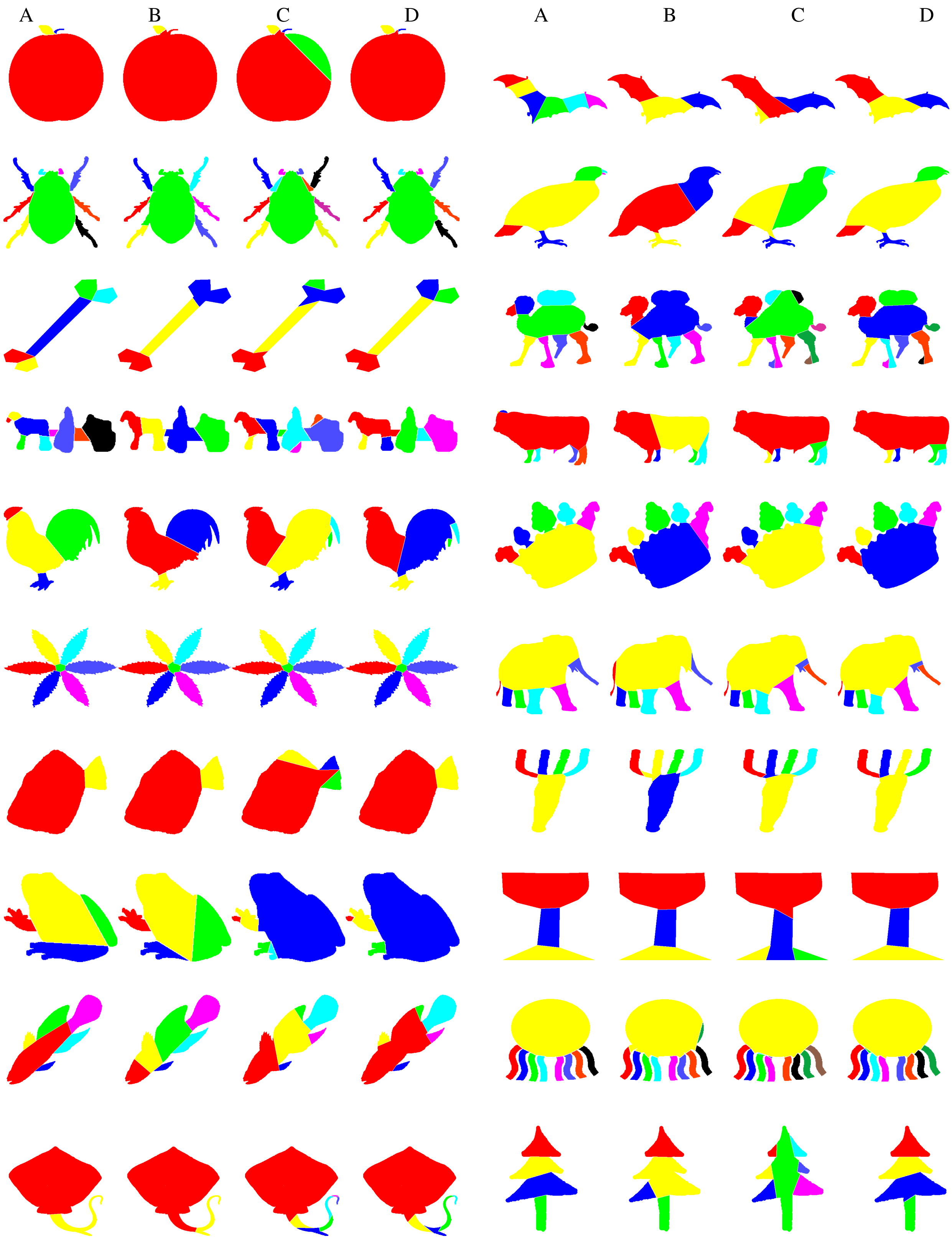

Figure 2. Examples of decomposition results for 20 categories of MPEG-7. There are four results for each shape. From left to right: Column A is the human decomposition result, Column B is Gopalan's result([1]), Column C is Liu's Result([2]) and Column D is our result.

2. More Examples of Decomposition Results

Figure 2 shows more examples of the decomposition results from experiments. For each category of the MPEG-7 dataset, we choose one example shape and display the human decomposition result, Gopalan's result [1], Liu's result [2] and our result. It can seen that for most categories, our decomposition results are closer to human results except chicken, fork and ray (left side of the last row). More comprehensive statistical data of comparison results are reported in the main paper, please see Figure 8 and 9.

References

[1] R. Gopalan, P. Turaga, and R. Chellappa. Articulation-invariant representation of non-planar shapes. In *Proceedings of the 11th European Conference on Computer Vision: Part III*, pages 286–299, 1927030, 2010. Springer-Verlag. 2, 3

[2] H. Liu, W. Liu, and L. L. J. Convex shape decomposition. In *IEEE Conference on Computer Vision and Pattern Recognition*, pages 97–104, 2010. 2, 3