Square Root Marginalization for Sliding-Window Bundle Adjustment

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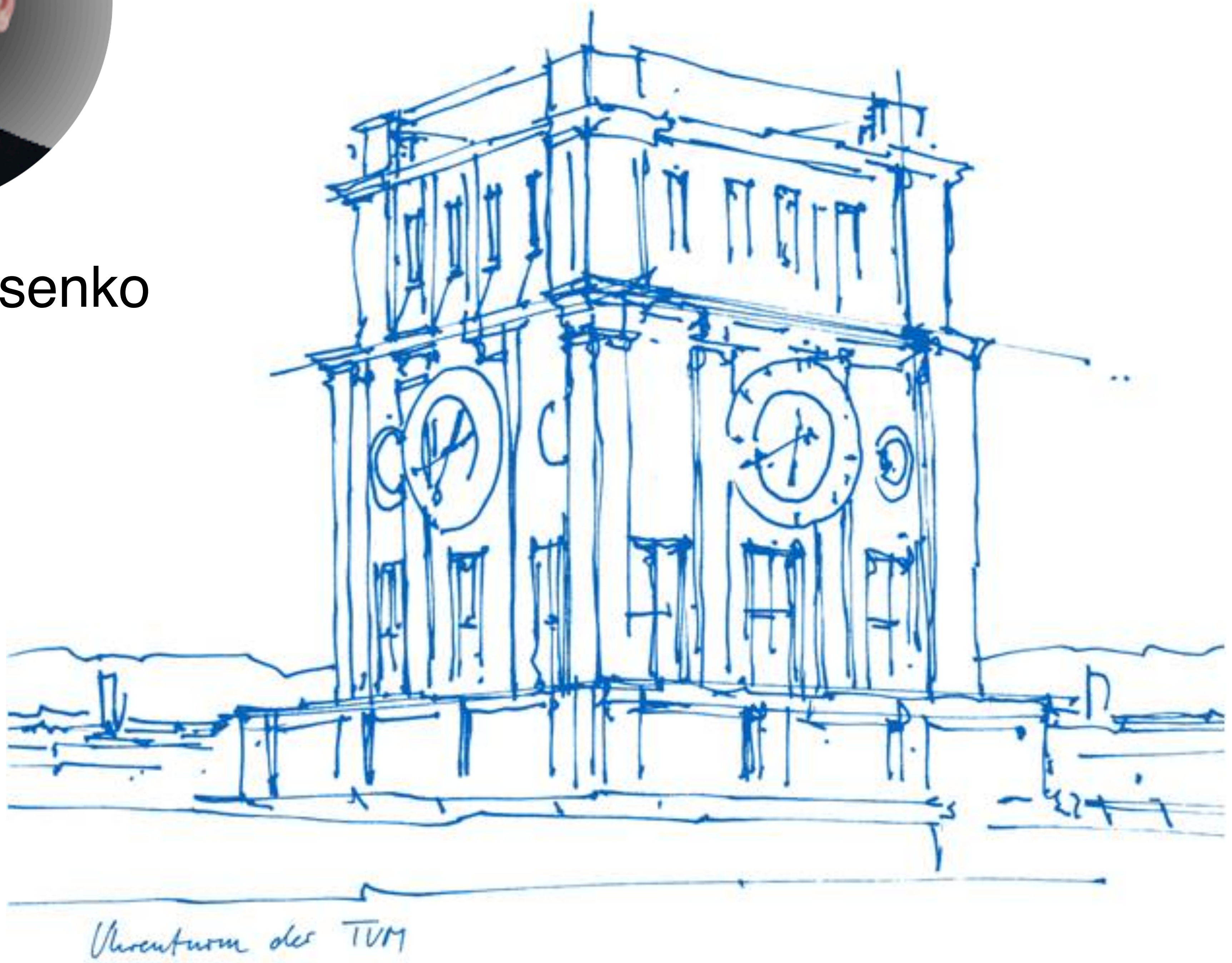
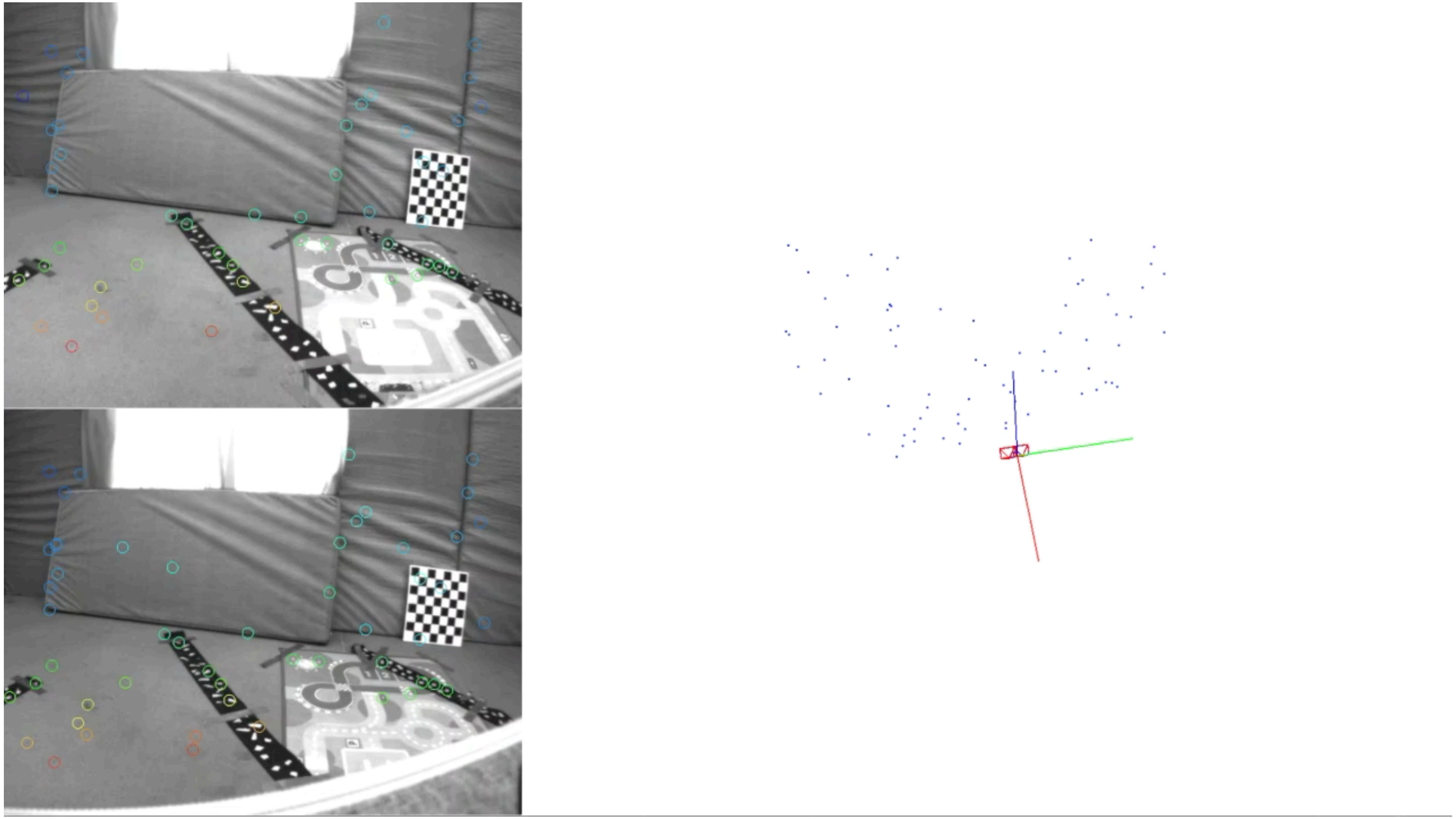


International Conference on Computer Vision
October 11-17, 2021

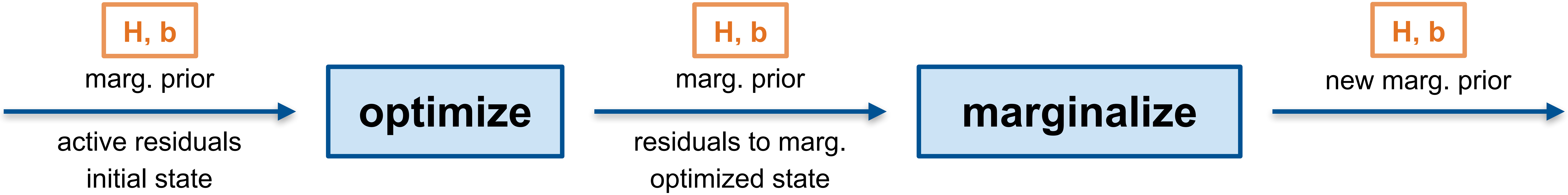
Visual-Inertial Odometry



Odometry based on: Usenko et al., “Visual-Inertial Mapping with Non-Linear Factor Recovery”, RA-L, April 2020

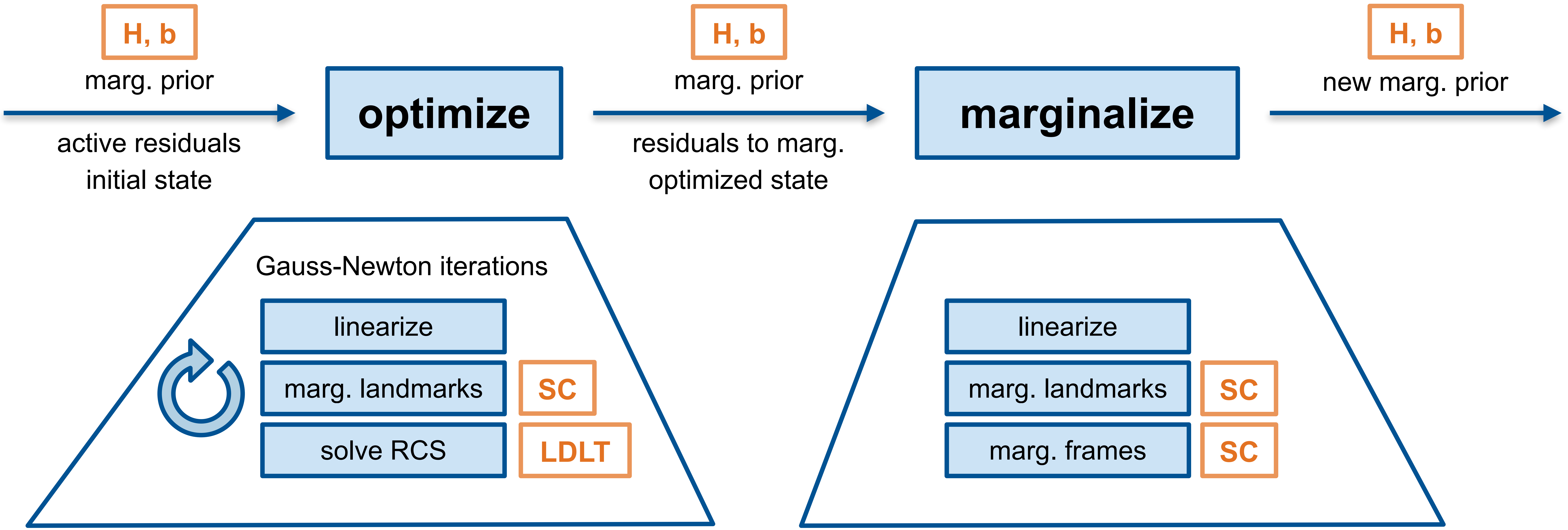
Optimization-based sliding-window estimator

conventional

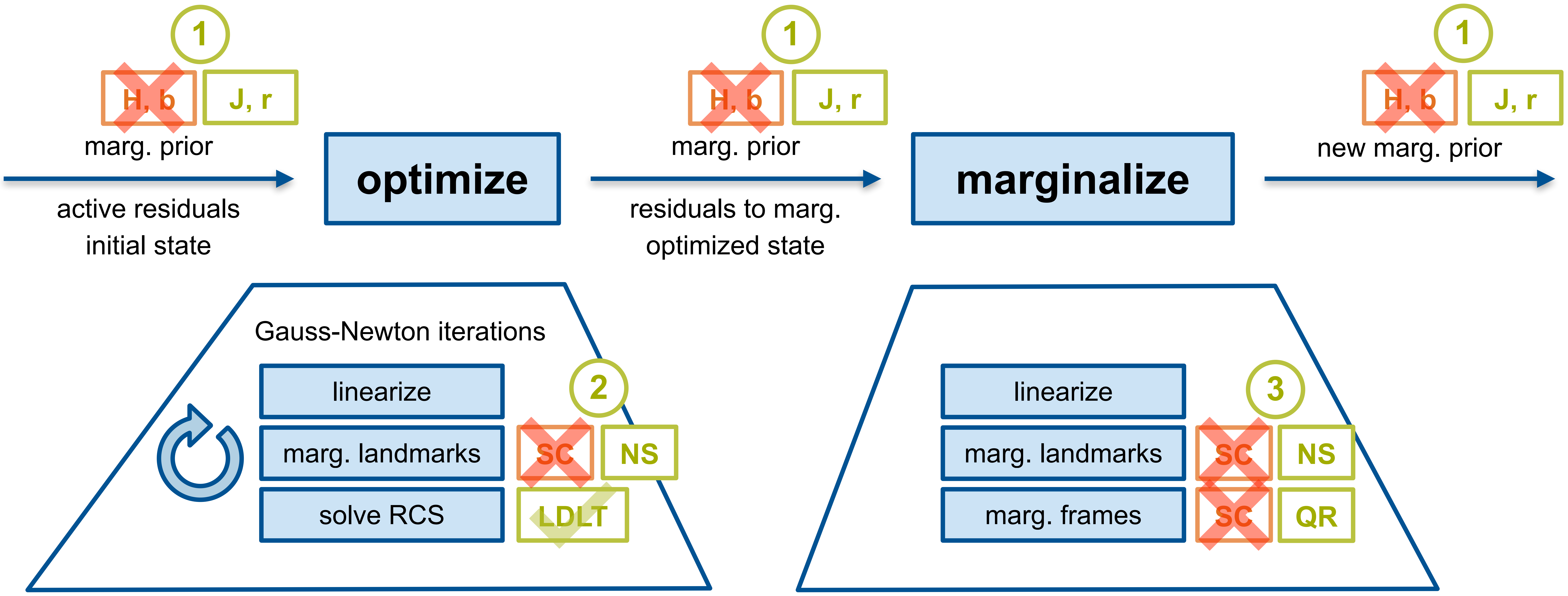


Optimization-based sliding-window estimator

conventional



Optimization-based sliding-window estimator



Sliding-window energy with marg. prior

conventional proposed

sliding-window energy

active residuals (visual, inertial)

$$E_{\text{sw}}(x) = \frac{1}{2}\|r_{\text{a}}(x)\|^2 + E_{\text{m}}(x)$$

marginalization prior

Hessian form to store prior

$$E_{\text{m}}(x) = \frac{1}{2}\Delta x^\top H_{\text{m}} \Delta x + b_{\text{m}}^\top \Delta x + \text{const}$$

H, b

perturbation from current state x

Jacobian form to store prior

$$E_{\text{m}}(x) = \frac{1}{2}\|r'_{\text{m}} + J_{\text{m}}\Delta x\|^2$$

J, r

J_{m} is a square root of H_{m}, i.e., $H_{\text{m}} = J_{\text{m}}^\top J_{\text{m}}$

Demmel et al., “Square Root Marginalization for Sliding-Window Bundle Adjustment”, ICCV21

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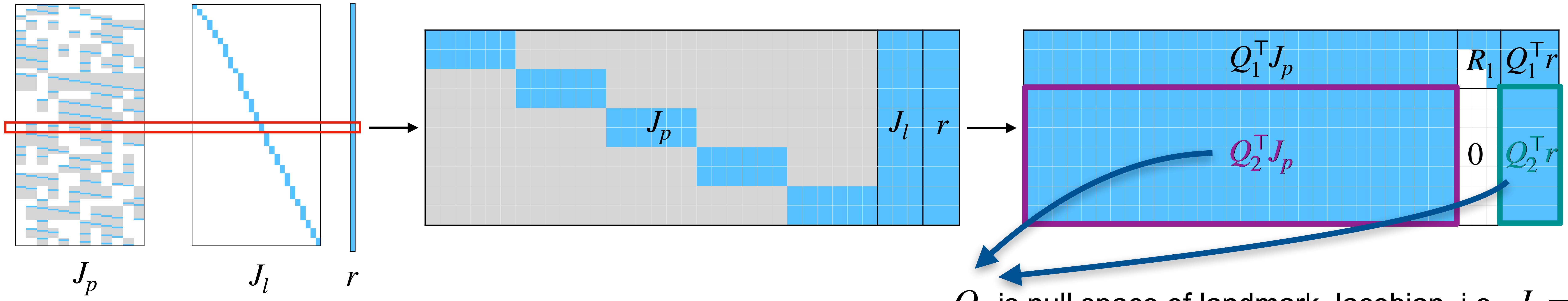
Optimization: nullspace marg. + Cholesky

conventional

proposed

Nullspace marginalization of landmarks based on: Demmel et al., “Square Root Bundle Adjustment for Large-Scale Reconstruction”, CVPR21



Q_2 is null space of landmark Jacobian, i.e., $J_l = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix} \begin{pmatrix} R_1 \\ 0 \end{pmatrix}$

compute $\tilde{H}$ and $\tilde{b}$ using Schur complement

$$\tilde{H} = H_{pp} - H_{pl} H_{ll}^{-1} H_{lp}$$

$$\tilde{b} = b_p - H_{pl} H_{ll}^{-1} b_l$$

SC

solving the reduced camera system with dense Cholesky decomposition

$$\tilde{H}\,\Delta x = -\,\tilde{b}$$

LDLT **LDLT**

compute $\tilde{H}$ and $\tilde{b}$ after nullspace projection

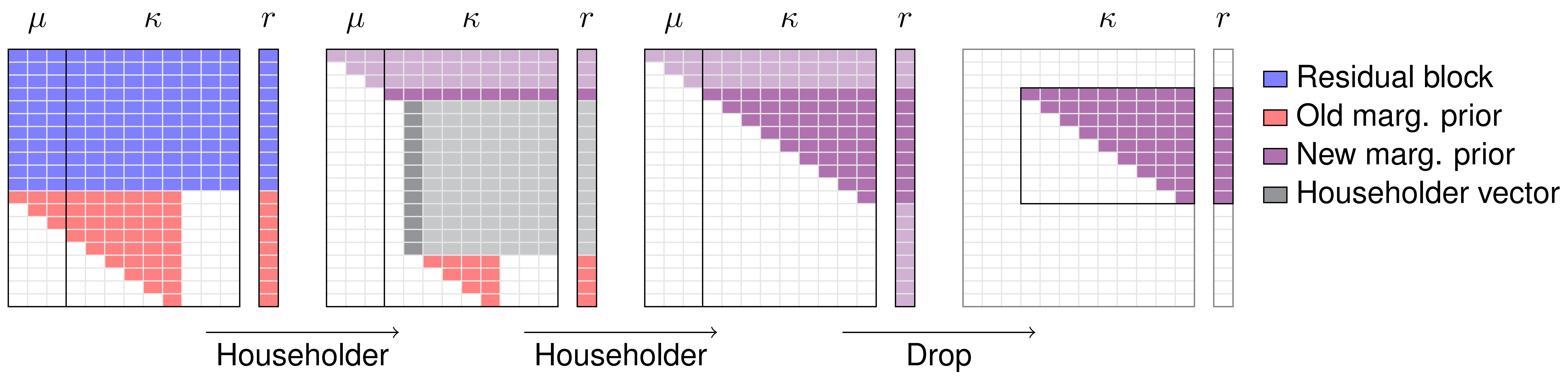
$$\tilde{H} = (Q_2^\top J_p)^\top (Q_2^\top J_p)$$

$$\tilde{b} = (Q_2^\top J_p)^\top (Q_2^\top r)^\top$$

NS

Demmel et al., “Square Root Marginalization for Sliding-Window Bundle Adjustment”, ICCV21

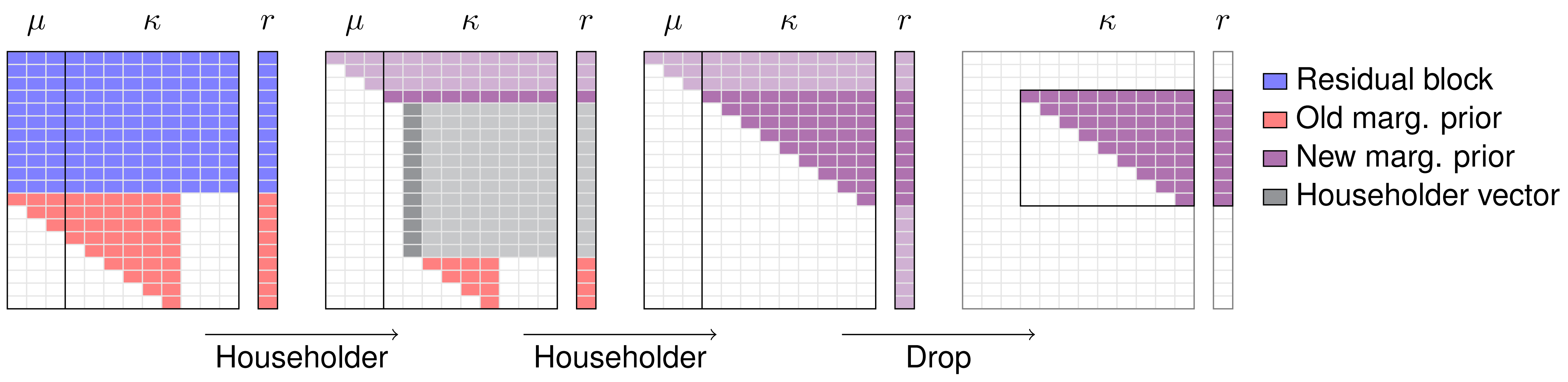
Marginalisation: specialized QR decomposition



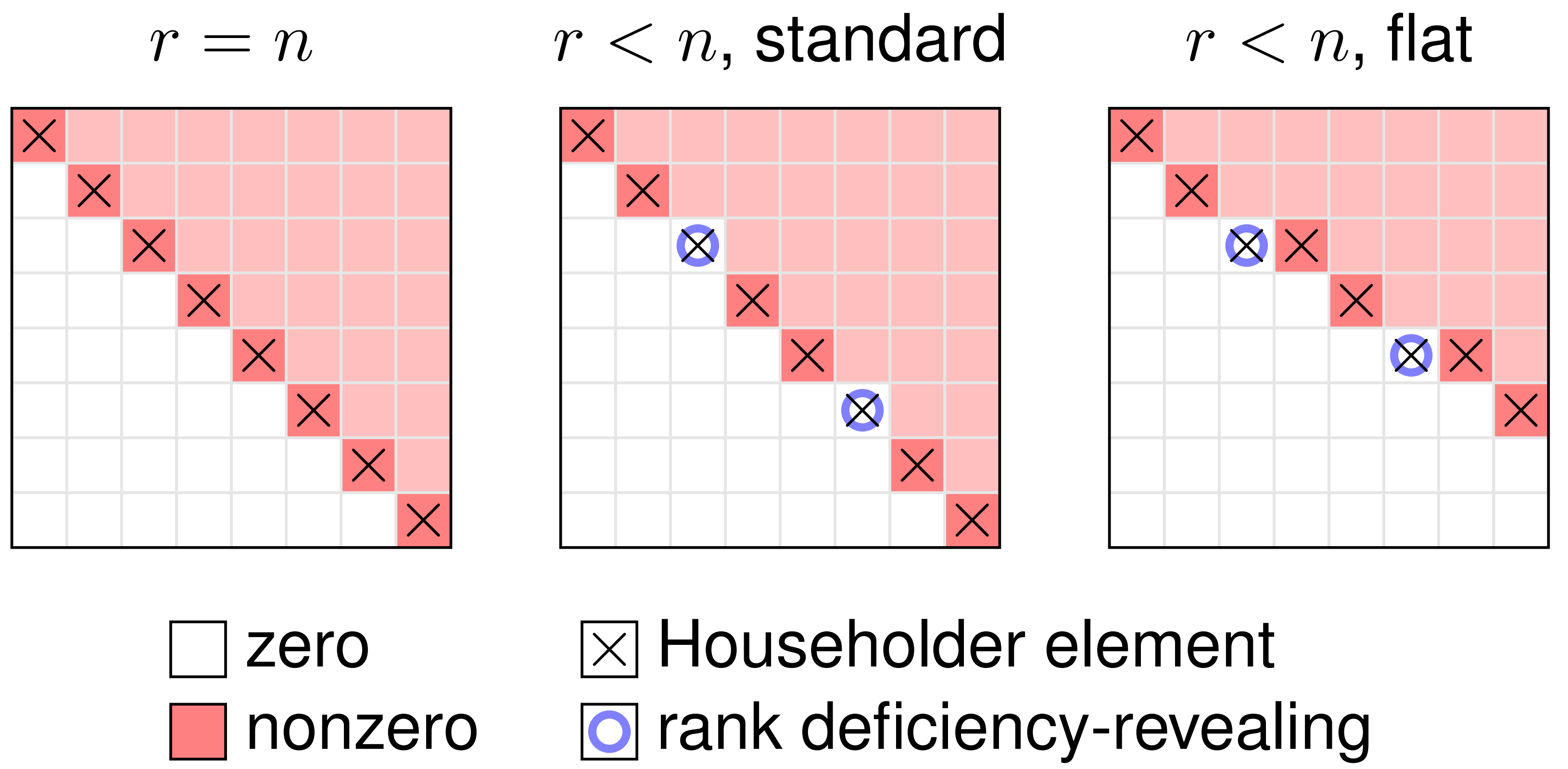
- start with reduced camera system in Jacobian form (including the old marginalization prior and possibly inertial residuals)
- frame states to be marginalized are sorted into the leftmost columns
- Successive in-place Householder transformations result in upper-triangular matrix
- Columns for marginalized states and corresponding top-rows, and zero rows at the bottom are dropped

Demmel et al., “Square Root Marginalization for Sliding-Window Bundle Adjustment”, ICCV21

Marginalisation: specialized QR decomposition



Rank-deficient case



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Results: accuracy and runtime

conventional

proposed

Accuracy

absolute trajectory error in meters

	$\sqrt{VIO}$-64	$\sqrt{VIO}$-32	VIO-64	VIO-32
eurocMH01	0.093	0.093	0.093	0.991
eurocMH02	0.048	0.048	0.048	0.048
eurocMH03	0.051	0.051	0.051	x
eurocMH04	0.109	0.109	0.109	x
eurocMH05	0.137	0.137	0.137	x
eurocV101	0.043	0.043	0.043	0.043
eurocV102	0.048	0.048	0.048	0.048
eurocV103	0.058	0.058	0.058	x
eurocV201	0.037	0.037	0.037	0.037
eurocV202	0.053	0.053	0.053	x
tumvi-corr1	0.300	0.300	0.300	x
tumvi-corr2	0.426	0.426	0.426	x
tumvi-mag1	1.456	1.457	1.456	x
tumvi-mag2	0.908	0.920	0.908	x
tumvi-room1	0.102	0.102	0.102	0.104
tumvi-room2	0.071	0.071	0.071	x
tumvi-slides1	0.310	0.310	0.310	x
tumvi-slides2	0.759	0.759	0.759	x

	$\sqrt{VO}$-64	$\sqrt{VO}$-32	VO-64	VO-32
kitti00	3.92	3.92	3.92	x
kitti02	9.72	9.72	9.72	x
kitti03	1.34	1.34	1.34	1.34
kitti04	1.22	1.22	1.22	1.22
kitti05	2.75	2.75	2.75	x
kitti06	2.61	2.61	2.61	2.61
kitti07	1.52	1.53	1.52	1.44
kitti08	3.85	3.85	3.85	x
kitti09	4.13	4.13	4.13	x
kitti10	1.11	1.11	1.11	26.12

Runtime

total runtime for optimization / marginalization in seconds

	$\sqrt{VIO}$-64	$\sqrt{VIO}$-32	VIO-64	VIO-32
eurocMH01	23.4 / 2.5	18.6 / 2.3	35.9 / 1.8	33.4 / 1.7
eurocMH02	20.0 / 2.1	15.6 / 1.9	31.7 / 1.5	29.0 / 1.4
eurocMH03	17.6 / 1.8	13.9 / 1.6	26.3 / 1.3	x
eurocMH04	13.1 / 1.3	10.3 / 1.2	19.5 / 0.9	x
eurocMH05	15.0 / 1.5	11.6 / 1.3	22.6 / 1.1	x
eurocV101	15.0 / 2.2	12.0 / 2.0	23.6 / 1.5	22.4 / 1.5
eurocV102	8.3 / 1.0	6.8 / 0.9	11.5 / 0.7	10.6 / 0.7
eurocV103	8.3 / 1.0	6.7 / 0.9	11.1 / 0.7	x
eurocV201	12.1 / 1.4	9.5 / 1.4	20.8 / 1.0	19.2 / 1.0
eurocV202	11.4 / 1.3	9.3 / 1.2	15.5 / 0.9	x
tumvi-corr1	24.4 / 3.2	18.7 / 2.6	36.7 / 2.2	x
tumvi-corr2	29.4 / 3.8	22.0 / 3.1	42.2 / 2.6	x
tumvi-mag1	78.1 / 10.5	57.4 / 8.4	112.5 / 7.0	x
tumvi-mag2	59.6 / 7.7	42.2 / 6.3	88.2 / 5.1	x
tumvi-room1	13.2 / 1.7	10.0 / 1.4	21.6 / 1.3	19.6 / 1.3
tumvi-room2	12.2 / 1.8	9.4 / 1.5	20.2 / 1.3	x
tumvi-slides1	28.6 / 3.6	20.9 / 3.0	44.1 / 2.5	x
tumvi-slides2	24.8 / 3.1	18.5 / 2.5	38.8 / 2.1	x

	$\sqrt{VO}$-64	$\sqrt{VO}$-32	VO-64	VO-32
kitti00	29.5 / 2.7	23.6 / 2.2	50.2 / 2.3	x
kitti02	32.0 / 3.0	25.0 / 2.3	53.2 / 2.4	x
kitti03	5.2 / 0.6	4.3 / 0.5	9.4 / 0.5	9.0 / 0.5
kitti04	1.5 / 0.2	1.2 / 0.1	2.6 / 0.1	2.5 / 0.1
kitti05	18.0 / 1.7	15.0 / 1.4	31.1 / 1.5	x
kitti06	5.8 / 0.6	4.8 / 0.5	9.8 / 0.6	9.3 / 0.6
kitti07	6.3 / 0.7	5.3 / 0.6	11.2 / 0.6	10.7 / 0.6
kitti08	26.3 / 2.5	21.2 / 2.0	44.2 / 2.1	x
kitti09	10.1 / 1.0	8.0 / 0.8	16.7 / 0.8	x
kitti10	6.9 / 0.7	5.5 / 0.6	11.6 / 0.6	9.7 / 0.6

Ablation study

different algorithmic choices for optimization and marginalization for VIO on EuRoC

	proposed		ablation study					
opt.	NS+LDLT		SC+LDLT		NS+LDLT		SC+LDLT	
marg.	NS+QR		NS+QR		SC+SC		SC+SC	
precision	64	32	64	32	64	32	64	32
ATE [m]	0.068	0.068	0.068	0.068	0.068	0.232	0.068	0.211
real-time	6.9x	8.2x	5.0x	5.6x	7.1x	7.9x	5.2x	5.5x
t total [s]	17.9	14.9	24.4	21.8	17.4	15.5	23.7	22.2
t opt [s]	14.4	11.4	22.2	20.3	14.4	11.5	22.1	20.4
t marg [s]	1.6	1.5	1.6	1.3	1.4	1.4	1.3	1.2

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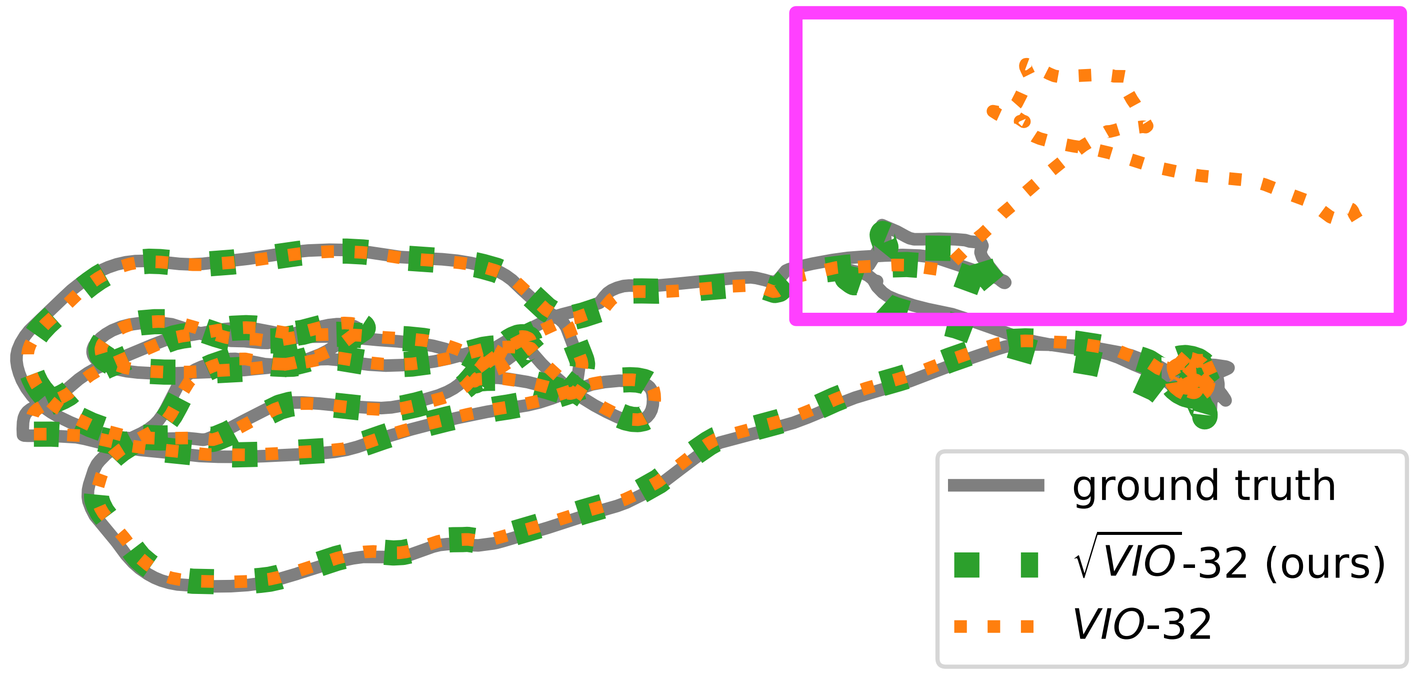
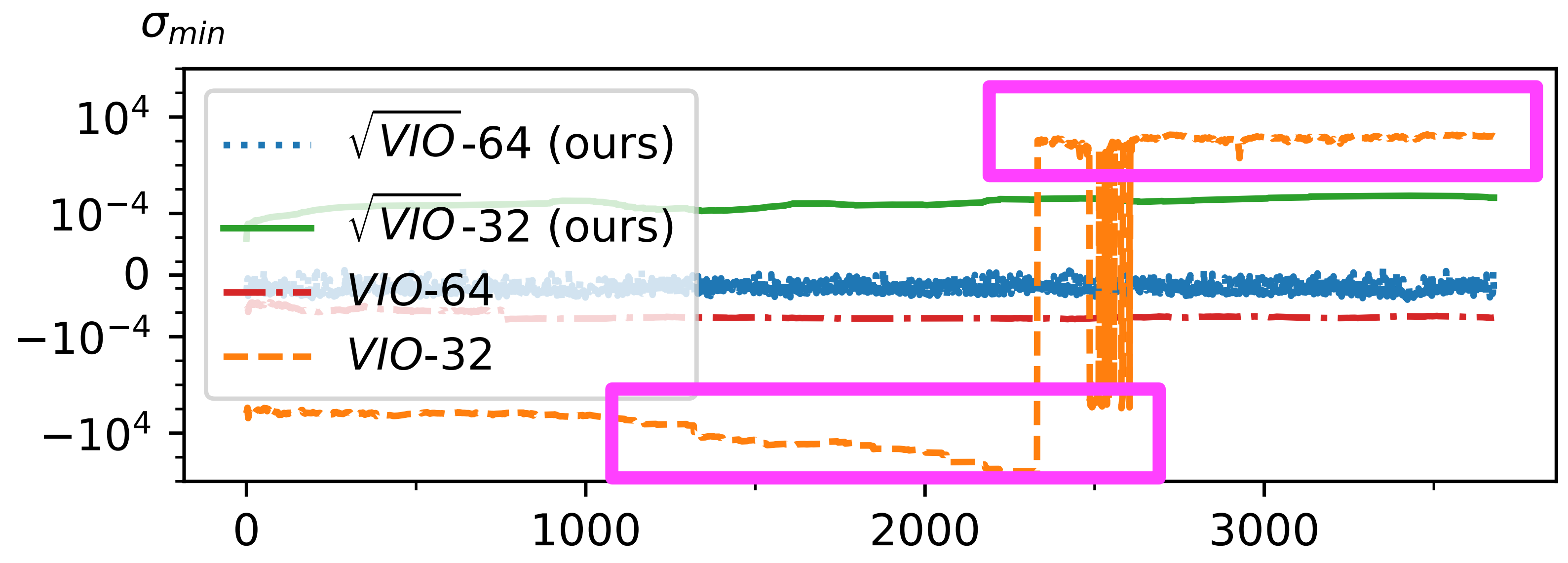
Results: numerical stability of marginalization prior

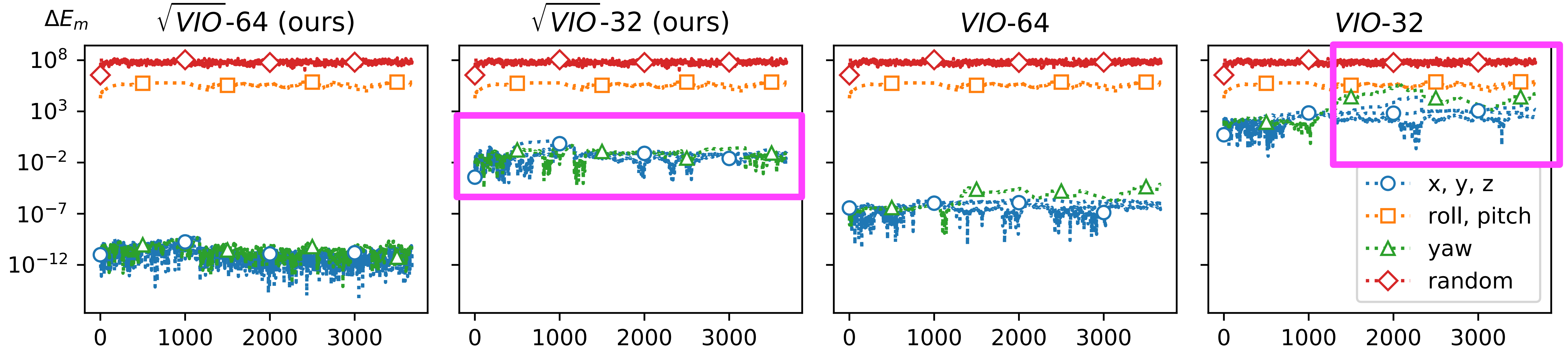
Trajectory



Minimum Eigenvalue of Hessian



Nullspace of Hessian



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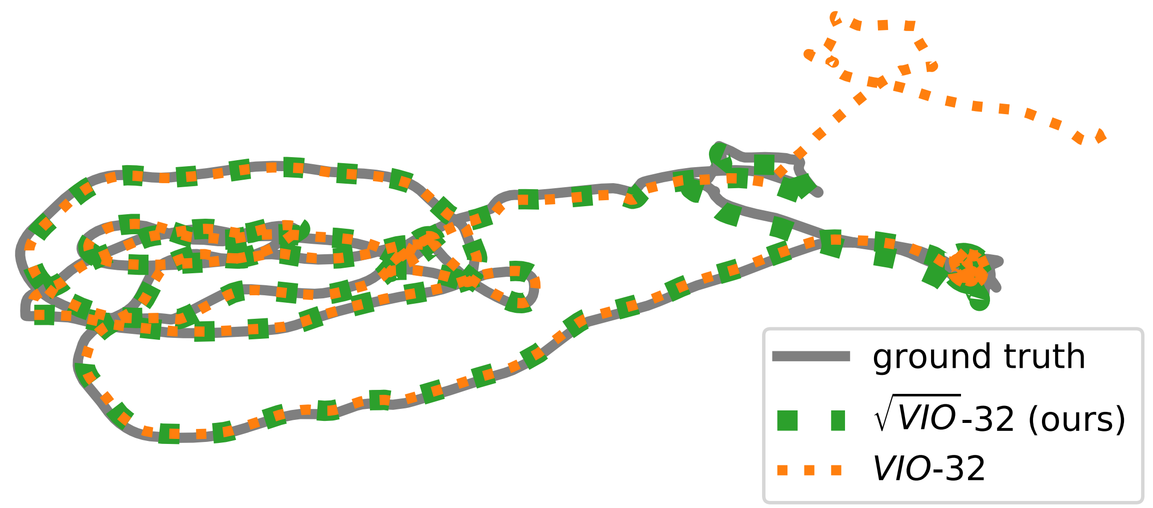
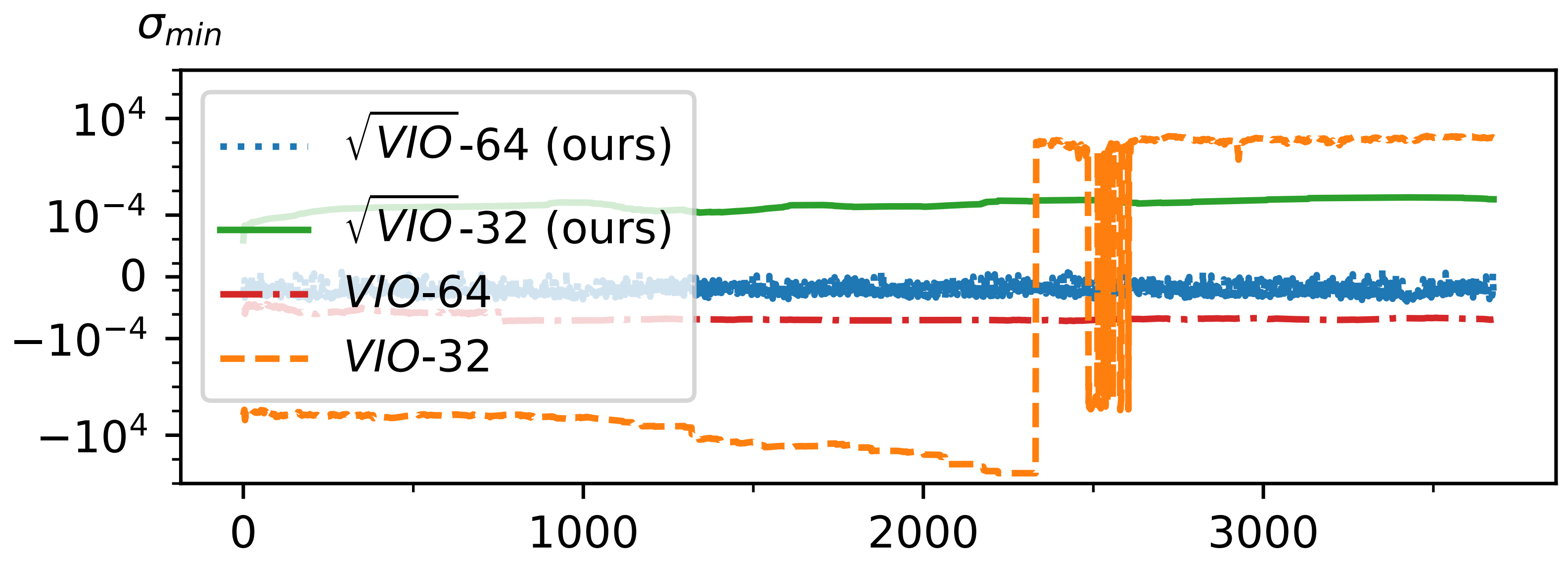
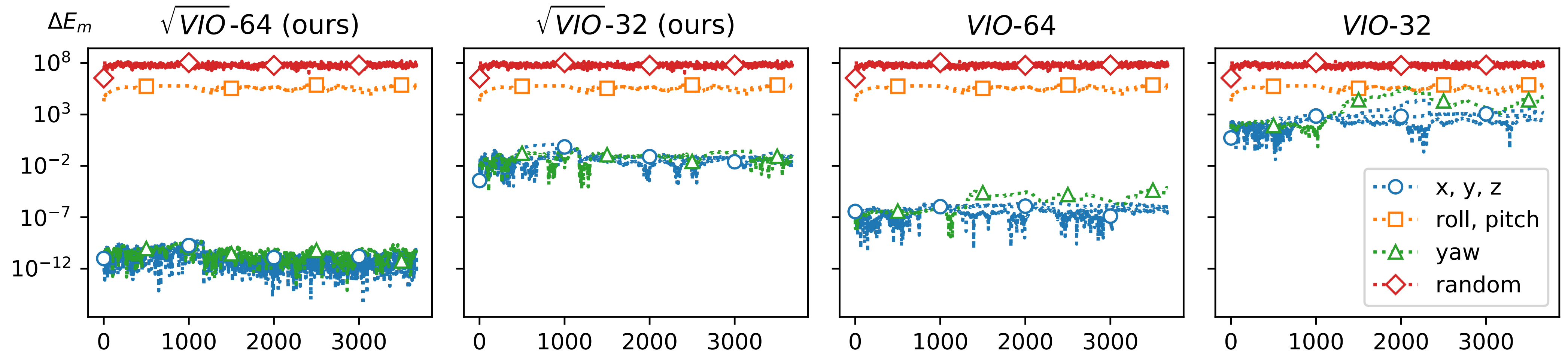
Conclusion

- We propose a novel **square root formulation** for optimization-based sliding-window estimators.
- We prove that the proposed **specialized QR-decomposition** for frame state marginalization is **equivalent** to the conventionally used Schur complement and naturally **deals with rank deficiencies**.
- The resulting odometry estimator runs in **single precision without loss of accuracy** and is **36% faster** than the conventional baseline approach.







Open Source Implementation:

https://go.vision.in.tum.de/rootvo

Acknowledgements: This work was supported by the ERC Advanced Grant *SIMULACRON* and by the German Science Foundation Grant CR 250/20-1 *Splitting Methods for 3D Reconstruction and SLAM*