

Large Scale SLAM

Lina Maria Paz University of Zaragoza Spain

linapaz@unizar.es

Joint work with: Pedro Piniés, José Neira, Juan D. Tardós



Simultaneous Localization and Mapping

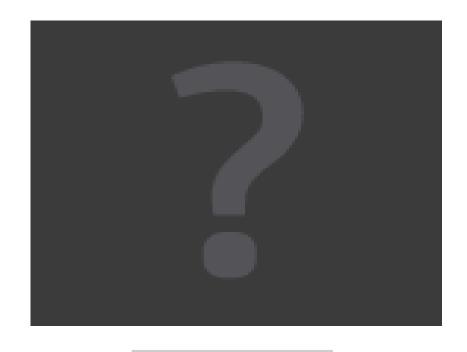
Is it possible to use a vehicle, starting at an

unknown initial location, in an

unknown environment,

to **incrementally** build a map of the environment,

and at the same time use the map to determine the vehicle location?

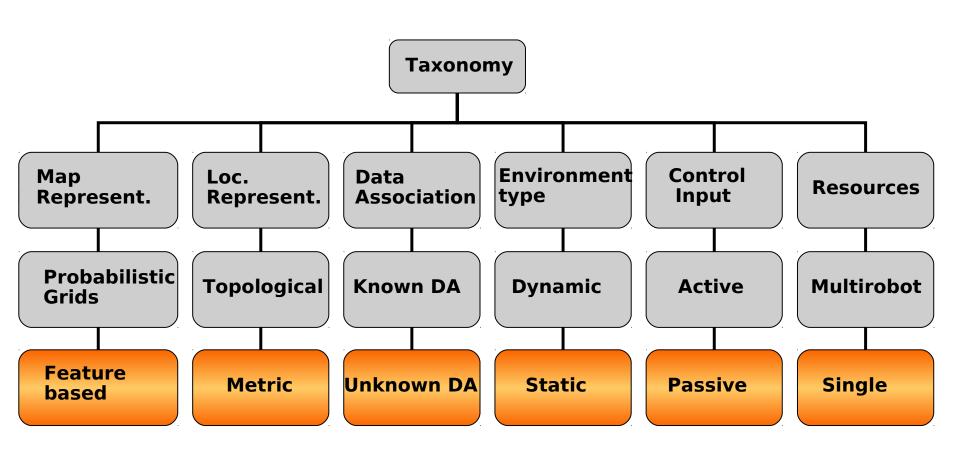


(image: Paul Newman)

Chicken and egg problem?

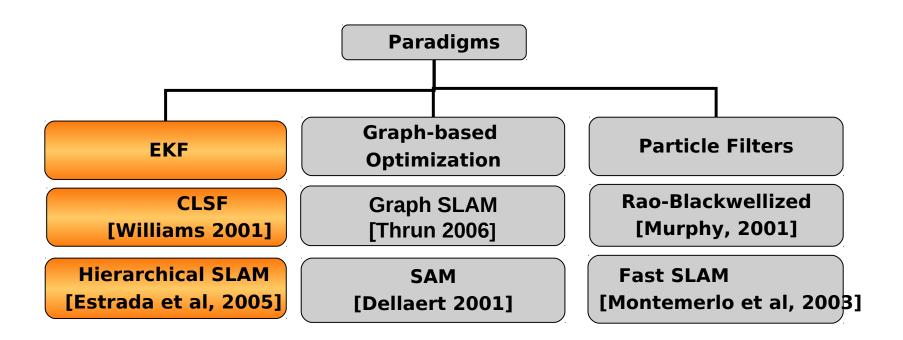


Let's put things in context





Let's put things in context





Outline

- 1. The SLAM scaling problem: Complexity and Consistency
- 2.D&C SLAM: Independent local maps

- 3.CI-Graph SLAM: Conditionally independent maps
- 4. DBA: Decomposable Bundle Adjustment

5. Multirobot SLAM



Outline

- 1.The SLAM scaling problem: Complexity and Consistency
- 2.D&C SLAM: Independent local maps

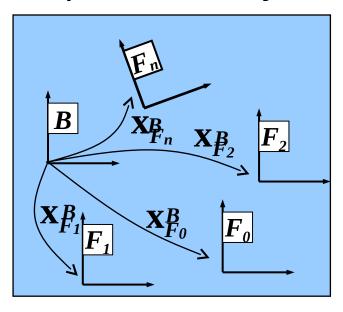
- 3.CI-Graph SLAM: Conditionally independent maps
- 4. DBA: Decomposable Bundle Adjustment

5. Multirobot SLAM



SLAM approach

- Environment information related to a set of elements: $\mathcal{F} = \{B, F_0, F_1, \dots, F_n\}$ $F_0 = \text{Robot}$
- represented by a map: $\mathcal{M}_{\mathcal{F}}^{B} = \left(\hat{\mathbf{x}}_{\mathcal{F}}^{B}, \mathbf{P}_{\mathcal{F}}^{B}\right)$

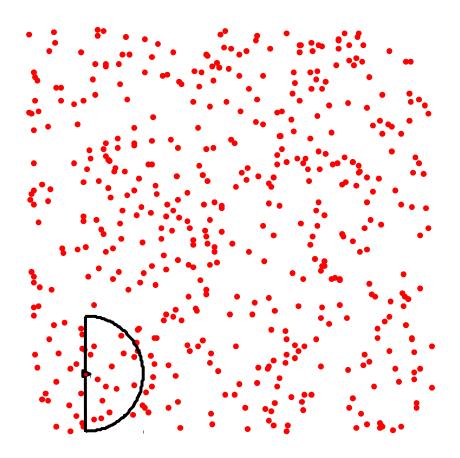


$$\hat{\mathbf{x}}_{\mathcal{F}}^{B} = \begin{bmatrix} \hat{\mathbf{x}}_{F_0}^{B} \\ \vdots \\ \hat{\mathbf{x}}_{F_n}^{B} \end{bmatrix}$$

$$\mathbf{P}_{\mathcal{F}}^{B} = \begin{bmatrix} \mathbf{P}_{F_0F_0}^{B} & \cdots & \mathbf{P}_{F_0F_n}^{B} \\ \vdots & \cdots & \vdots \\ \mathbf{P}_{F_nF_0}^{B} & \cdots & \mathbf{P}_{F_nF_n}^{B} \end{bmatrix}$$

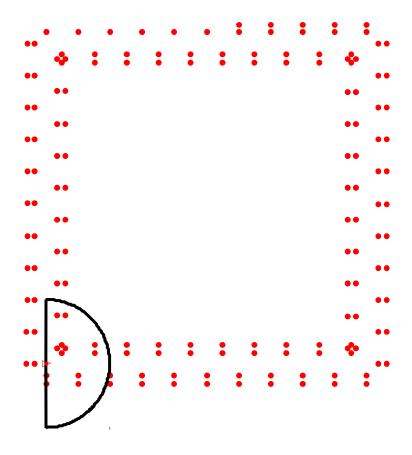


 Environment to be mapped has more or less uniform density of features



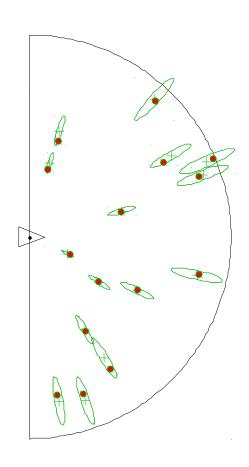


 Environment to be mapped has more or less uniform density of features



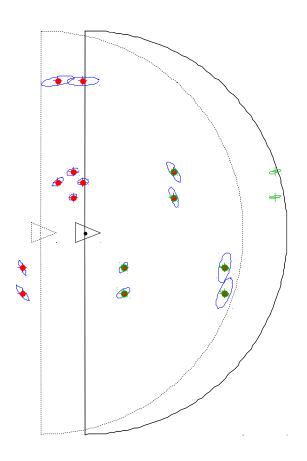


 Onboard range and bearing sensor obtains m measurements



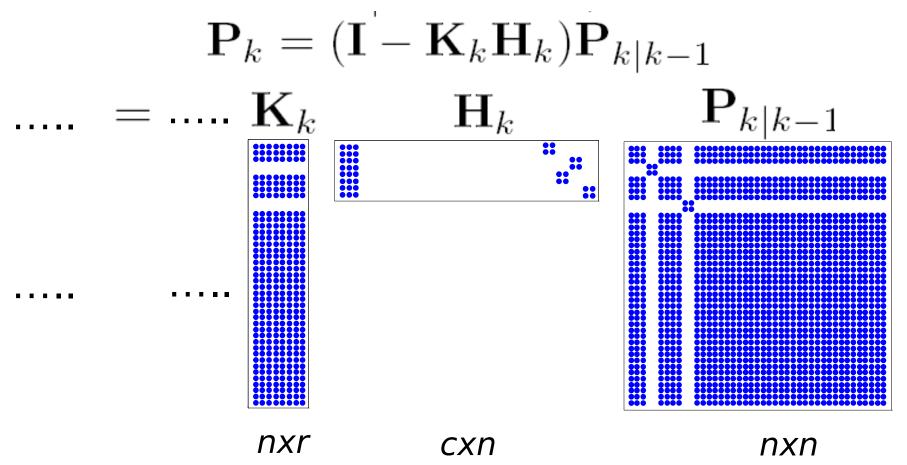


• Vehicle performs an exploratory trajectory, re-observing r features, and seeing s = m - r new features.





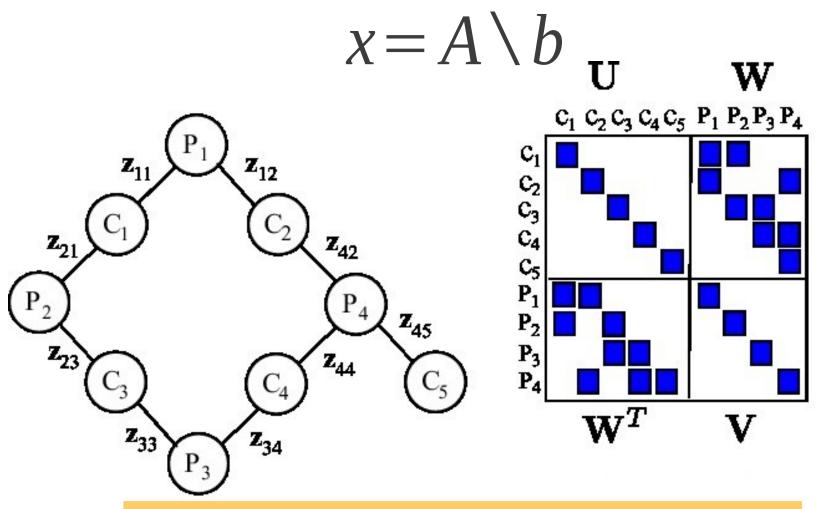
The EKF-covariance matrix update



EKF update step is $O(n^2)$



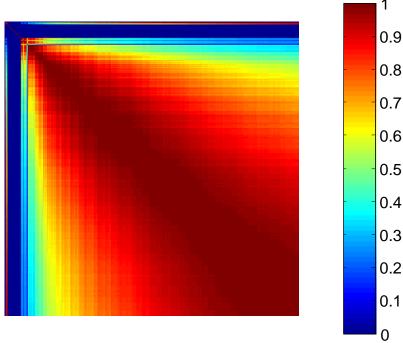
BA Primary structure



BA solution is O(n+p)? $O((n+p)^3)$



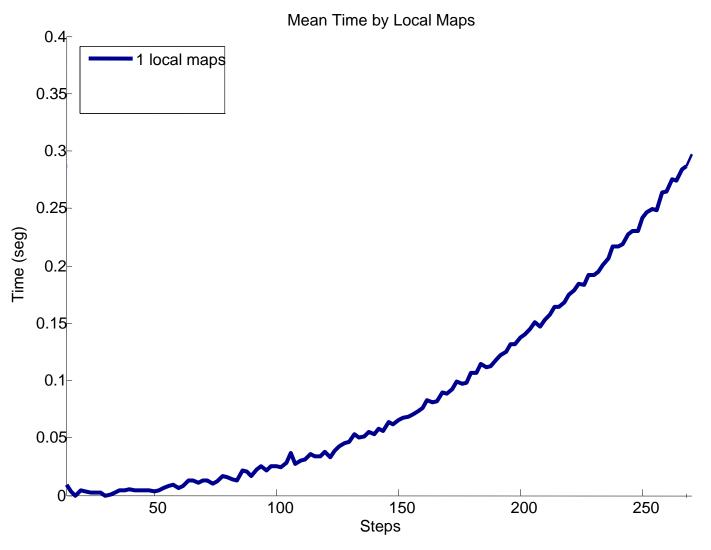
The mixed blessing of Covariance



- Covariance provides data association based on statistical tests
- Covariance-based criteria speeds up convergence
- But the covariance matrix is full (e.g. EKF)

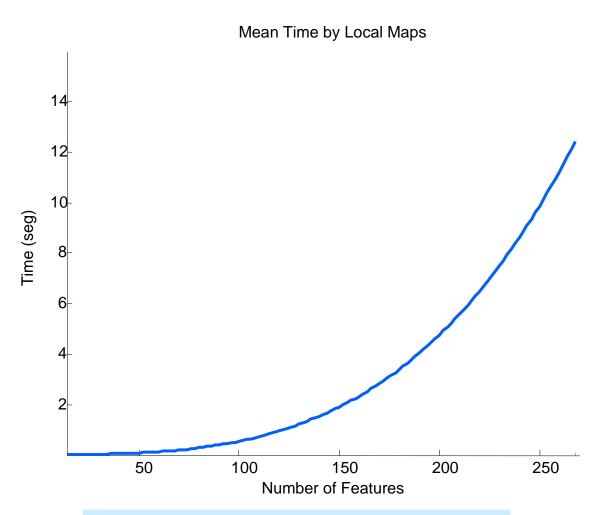


EKF-SLAM updates are $O(n^2)$





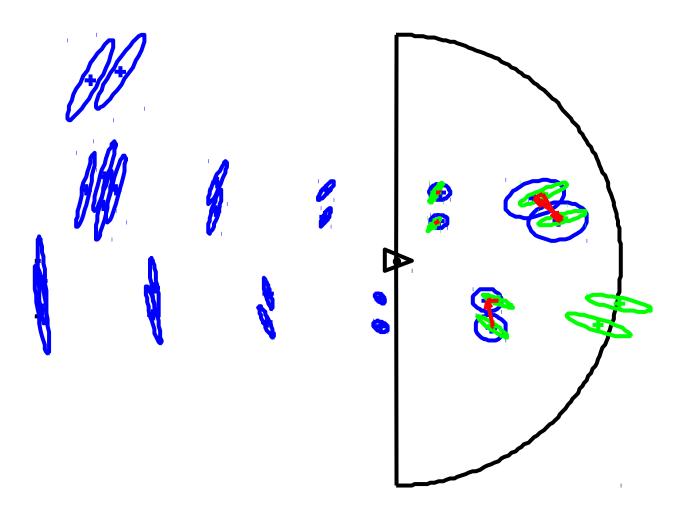
Total cost of EKF-SLAM



Total SLAM is $O(n^3)$



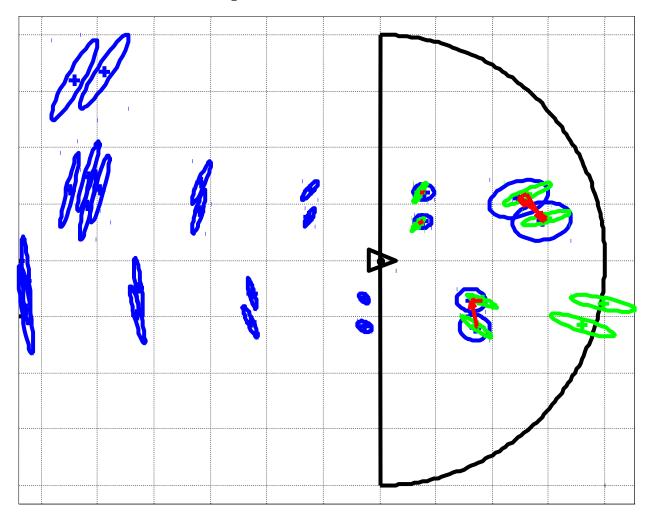
Continuous data association



Individual compatibility is O(nm) = O(n)



Map tessellation



Individual compatibility can be O(1)



Efforts to reduce complexity

- Decoupled Stochastic Mapping (Leonard and Feder, 2000) (Jensfelt 2001) O(1)
- Suboptimal SLAM (Guivant and Nebot 2001)
 O(n)
- Sparse Weight Filter (Julier 2001) O(n)
- Sparse Information Filter (Thrun et al 2004) O(1) amort.

- Postponement (Knight, Davidson and Reed 2001)
- Compressed Filter (Guivant and Nebot 2001)
- Constrained Local Submap Filter (Williams 2001)
- Map Joining (Tardós et al, 2002)

Aproximate, or pessimistic solutions

Exact solutions that delay global map updating, and strongly reduce cost.

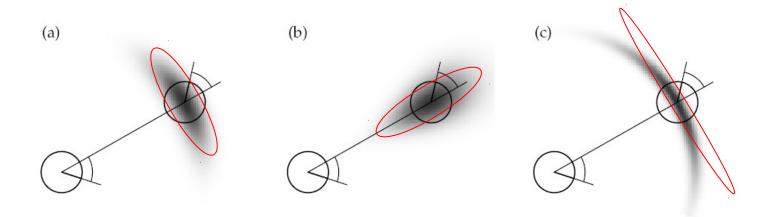
But still O(n²)



Consistency of EKF-SLAM

$$\mathbf{x}_{R_k}^{R_{k-1}} = (0, 0, \phi_1)^t \oplus (x_1, 0, 0)^t \oplus (0, 0, \phi_2)^t$$

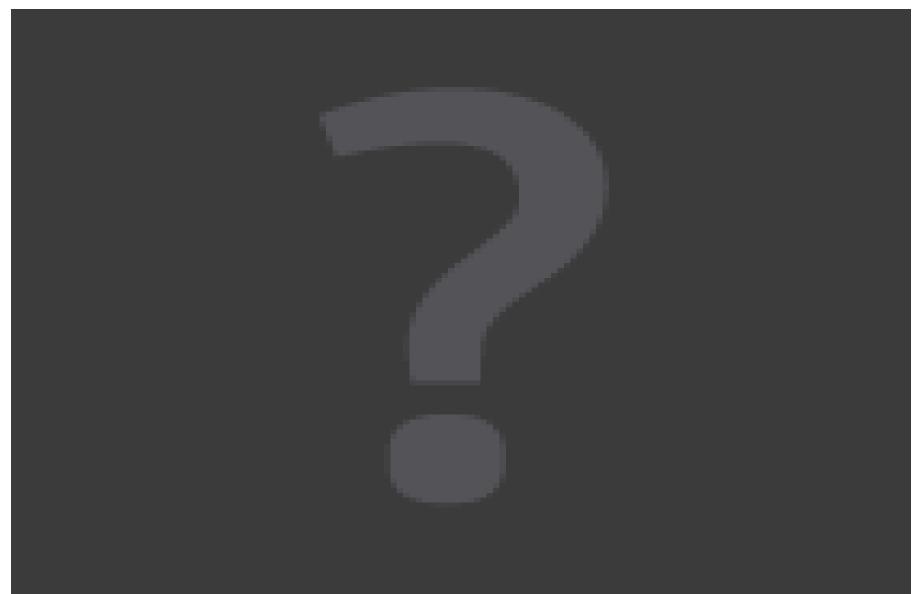
We are linearizing errors!



(image: Thrun, Burgard, Fox)

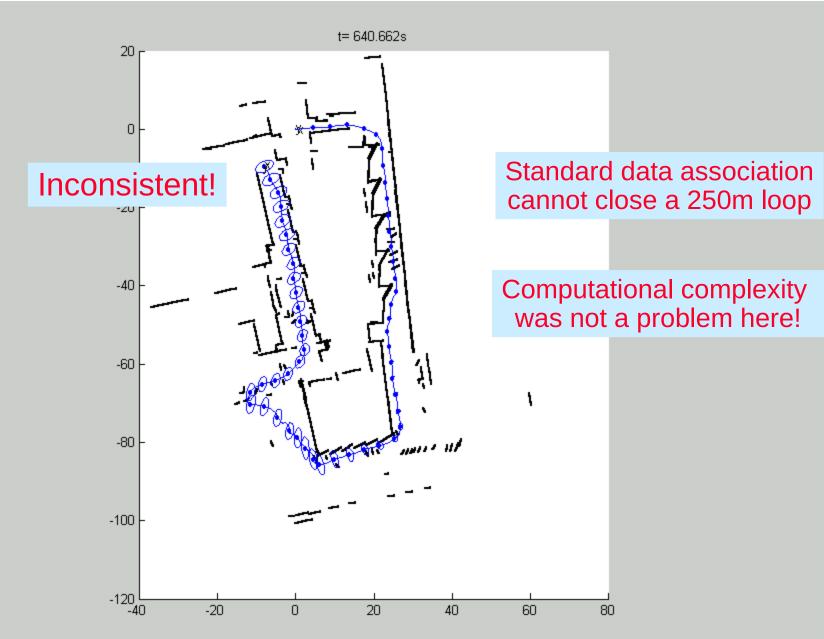


EKF-SLAM: Real Example



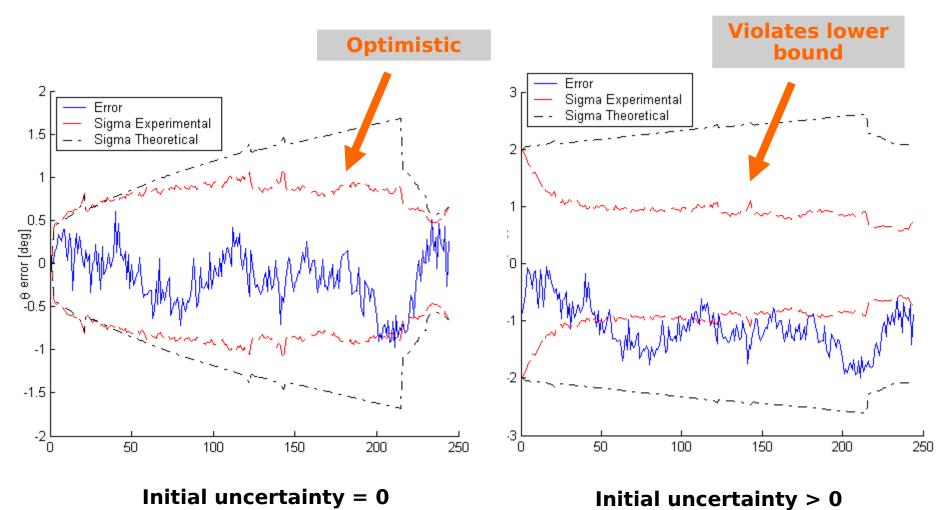


EKF-SLAM: Real Example





EKF-SLAM: Covariance



J.A. Castellanos, J. Neira, J.D. Tardós, **Limits to the Consistency of EKF-based SLAM**, 5th IFAC Symposium on Intelligent Autonomous Vehicles, Lisbon, July 2004



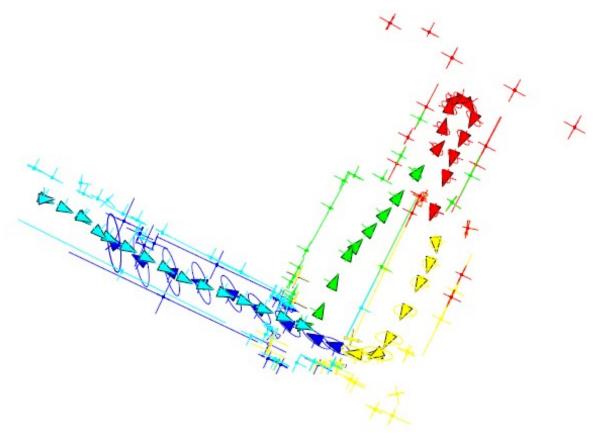
Outline

- 1. The SLAM scaling problem: Complexity and Consistency
- 2.D&C SLAM: Independent local maps

- 3.CI-Graph SLAM: Conditionally independent maps
- 4. DBA: Decomposable Bundle Adjustment

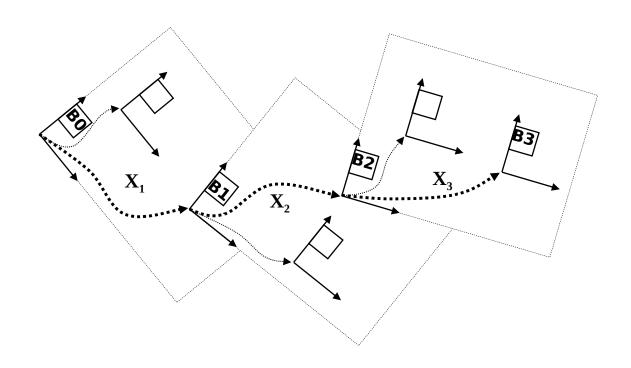
5. Multirobot SLAM





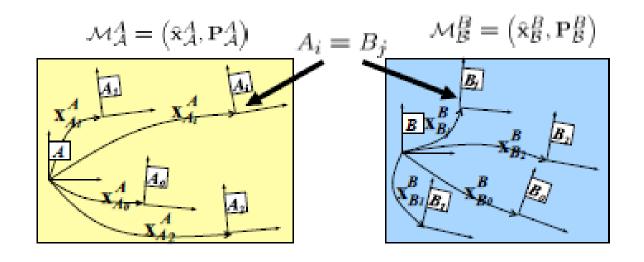
- Local Map Joining (Tardós et. al. 2002)
- Atlas (Bosse et. al. 2003)
- Constant time SLAM (Newman et. al 2003)



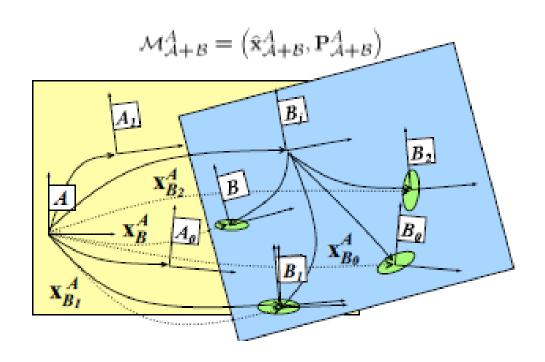


- Local Map Joining (Tardós et. al. 2002)
- Atlas (Bosse et. al. 2003)
- Constant time SLAM (Newman et. al 2003)



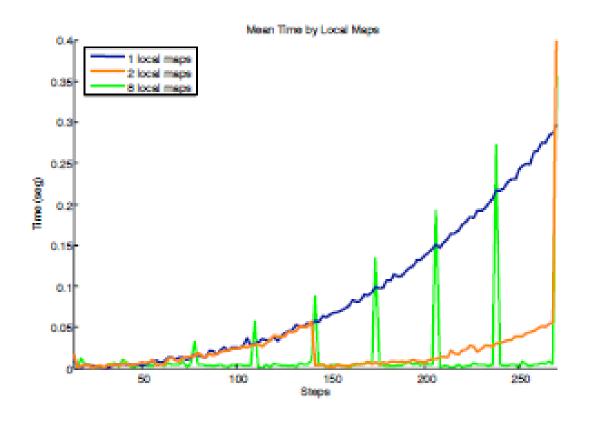








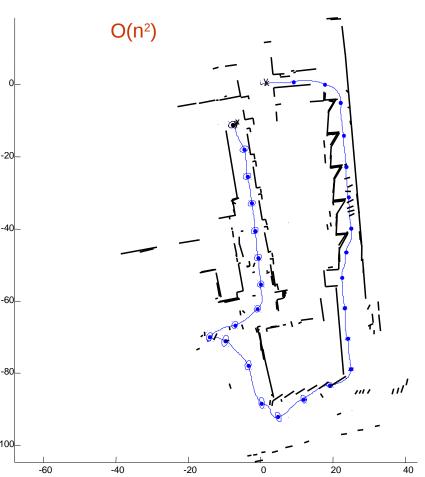
Local Maps Improve Complexity



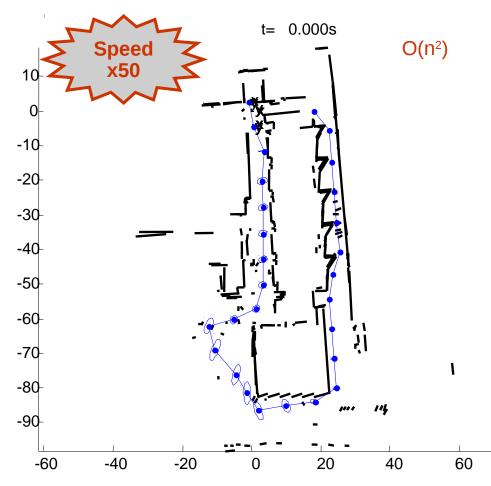


Local Maps Improve Consistency

Classic EKF-SLAM



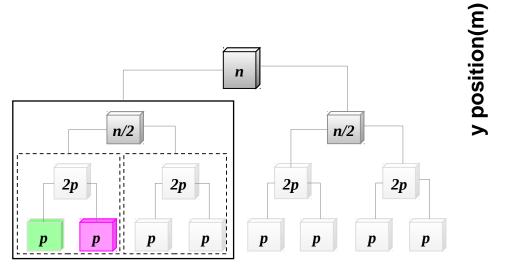
Map joining: 28 local maps

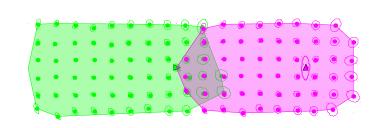


J.A. Castellanos, R. Martinez-Cantin, J.D. Tardós and J. Neira, "Robocentric Map Joining: Improving the Consistency of EKF-SLAM", Robotics and Autonomous Systems, Vol. 55, pp. 21-29, 2007



Number of Maps: 2

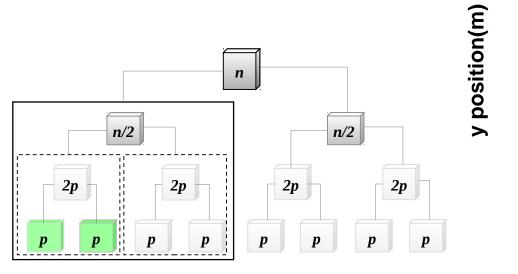


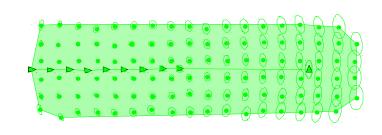


x position(m)



Number of Maps: 1

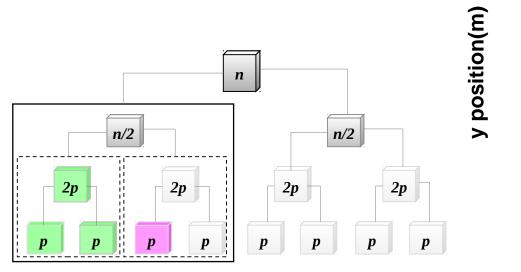


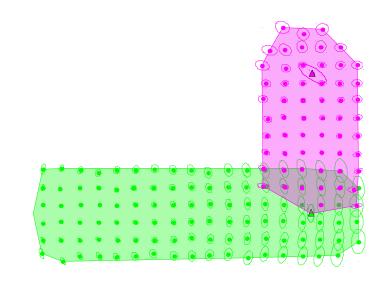


x position(m)



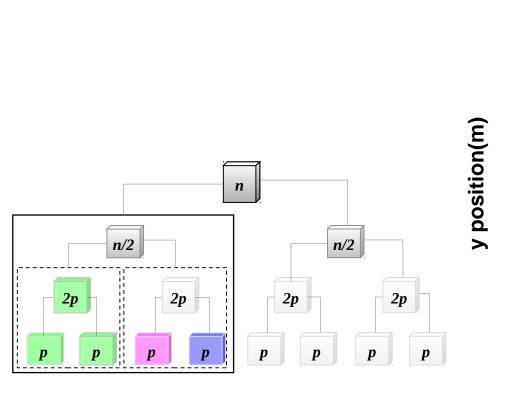
Number of Maps: 2



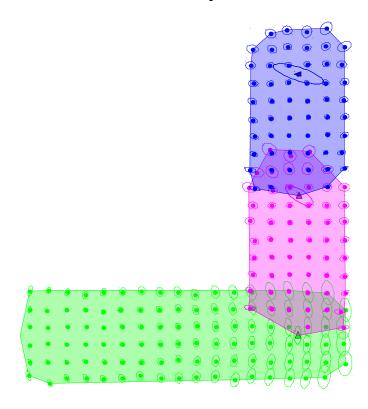


x position(m)





Number of Maps: 3

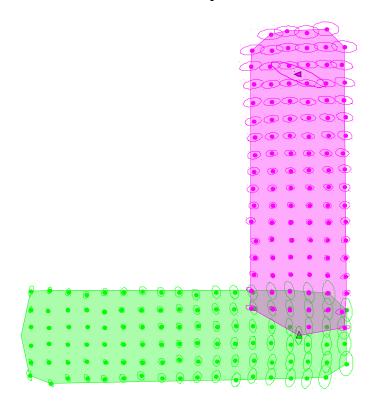


x position(m)



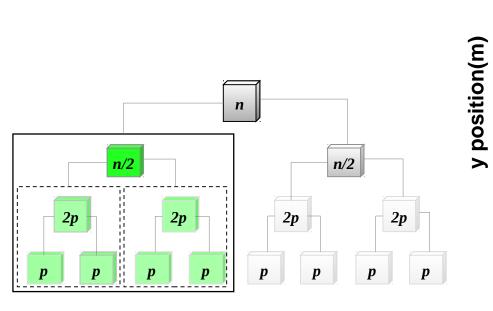
(W)uojisod X

Number of Maps: 2

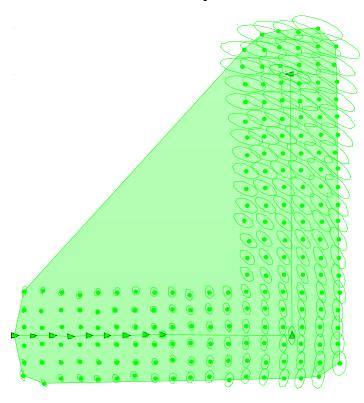


x position(m)



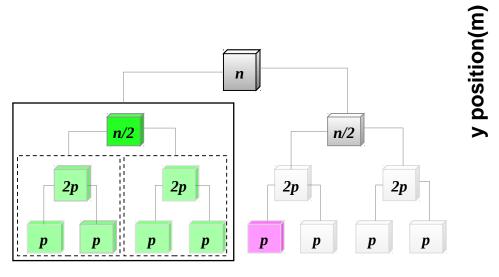


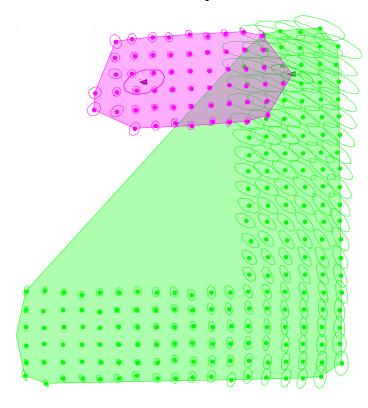
Number of Maps: 1



x position(m)

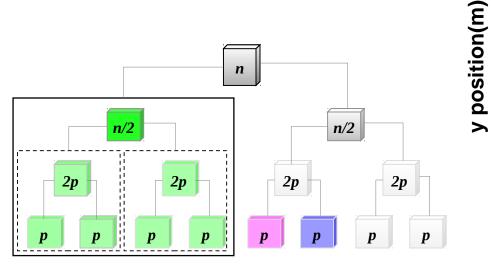




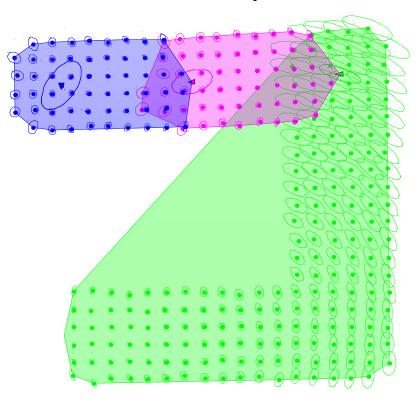


x position(m)



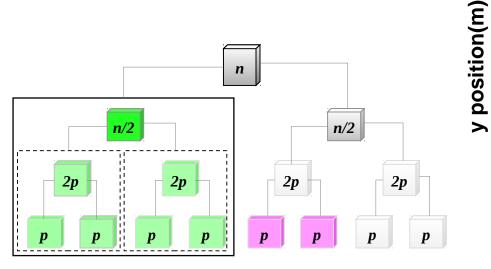


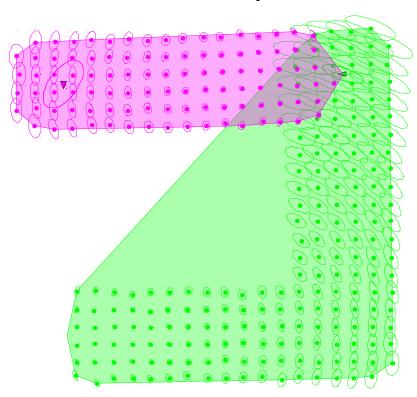
Number of Maps: 3



x position(m)

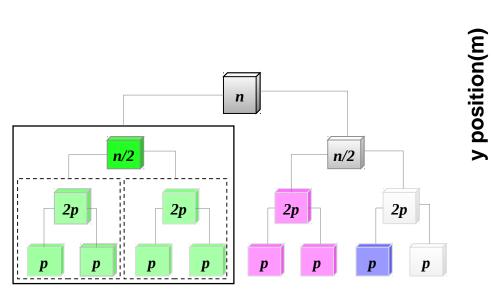


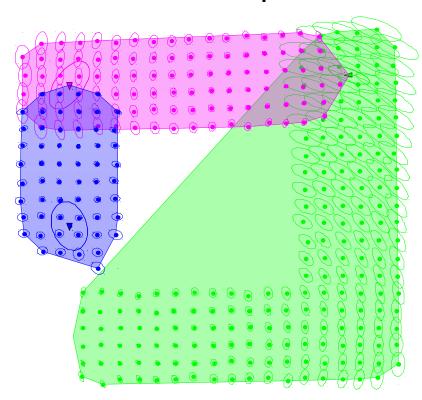




x position(m)

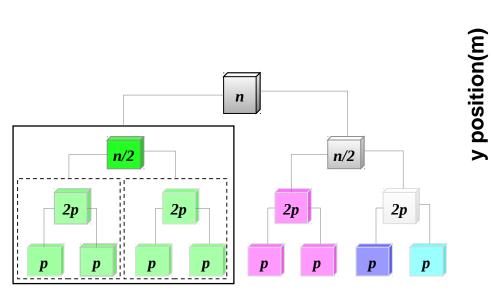


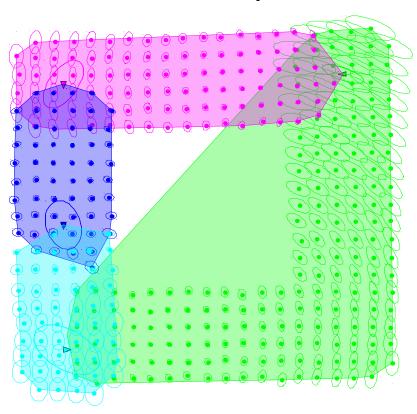




x position(m)

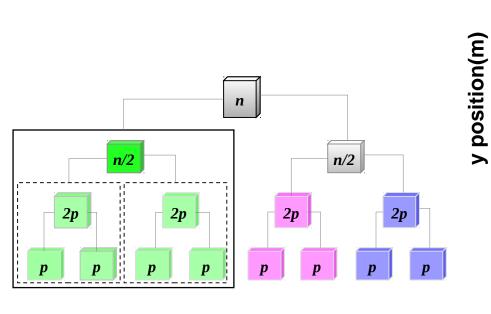




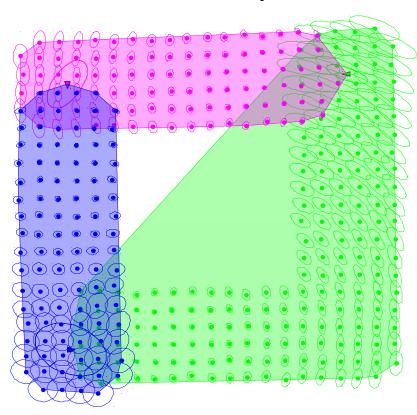


x position(m)



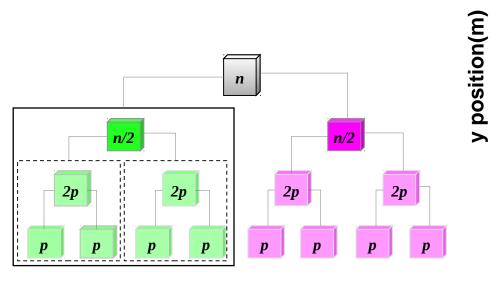


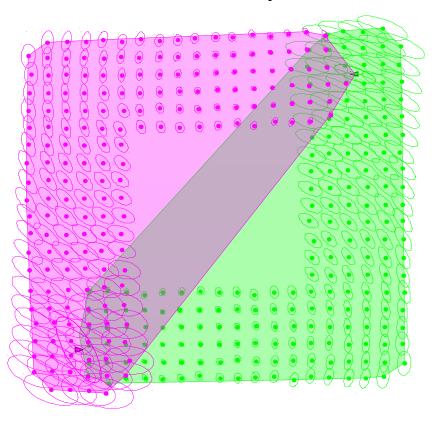
Number of Maps: 3



x position(m)



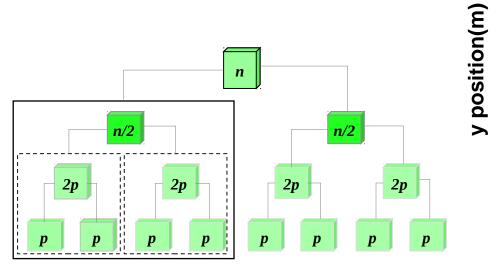




x position(m)



Number of Maps : 1

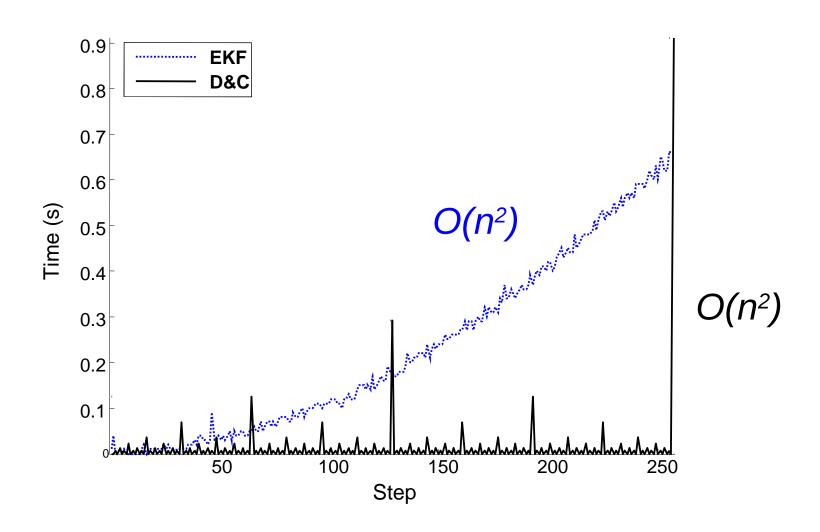


v position/ml

L.M. Paz, P. Jensfelt, J.D. Tardós and J. Neira. **EKF SLAM updates in O(n) with Divide and Conquer SLAM** 2007 IEEE Int. Conf. Robotics and Automation, April 10-14, Rome, Italy

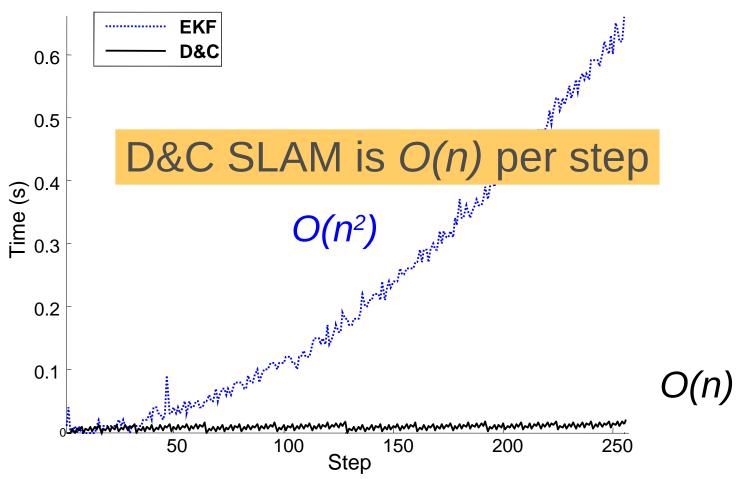


Computational cost per step





Amortized cost per step

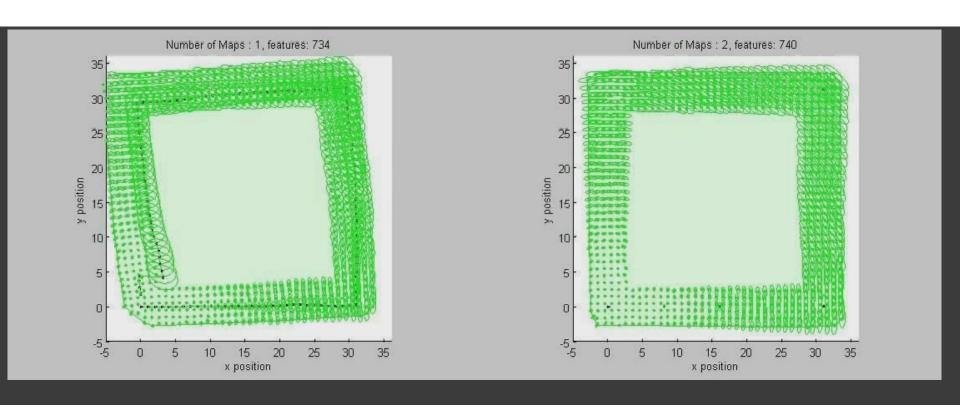


The full map can be recovered at any time in a single $O(n^2)$ step

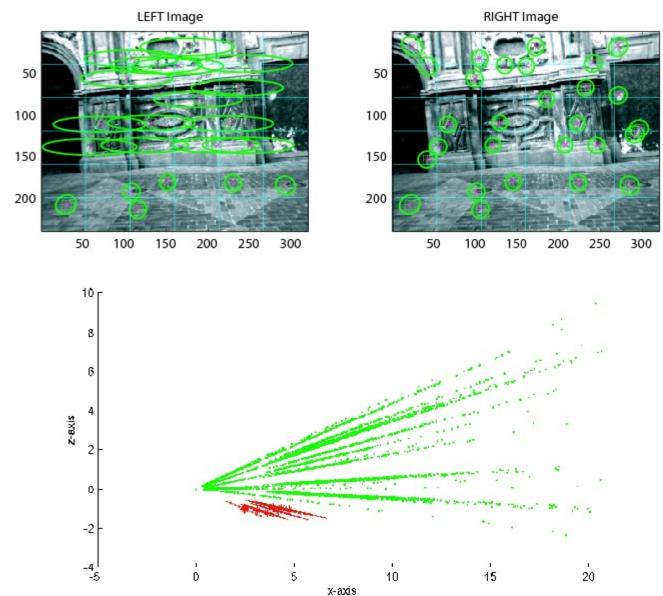
But no part of the algorithm or process requires it



Consistency: LOOP

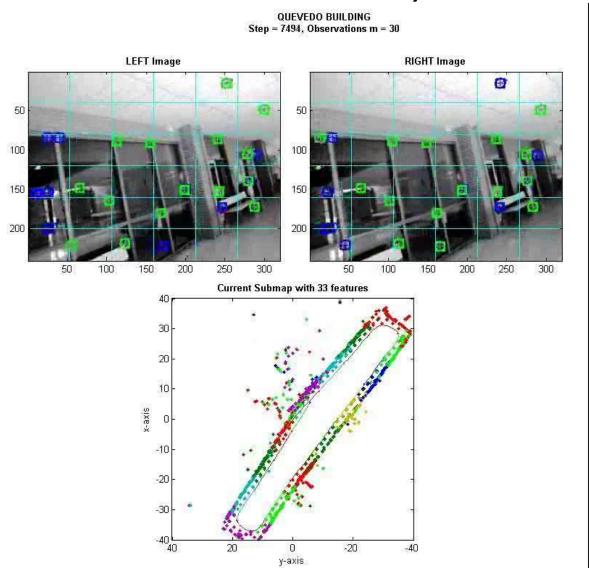


colose and Distant Points (Inverse Depth)



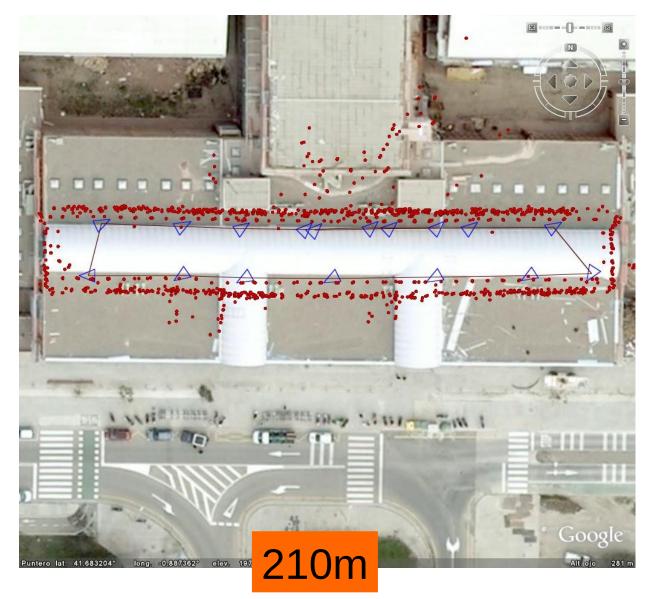


6Dof Stereo SLAM, indoors





6Dof Stereo SLAM, indoors



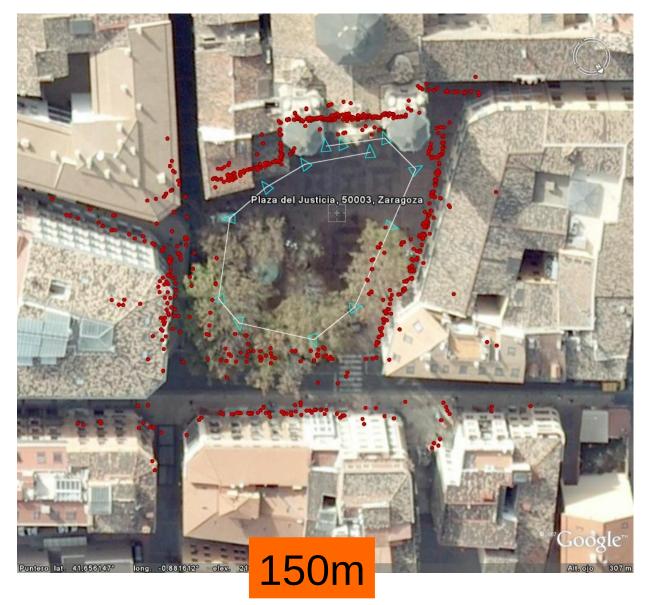


6Dof Stereo SLAM, outdoors **LEFT Image** RIGHT Image 200 200 300 Current Submap with 76 features x-axis 40 y-axis

L. Paz, P. Piníes, J. Neira and J.D. Tardós, Large Scale 6DOF SLAM with Stereo-in-Hand. IEEE Transactions on Robotics, 25(4): 946-957, Oct 2008.



6Dof Stereo SLAM, outdoors





Outline

- 1. The SLAM scaling problem: Complexity and Consistency
- 2.D&C SLAM: Independent local maps

- 3.CI-Graph SLAM: Conditionally independent maps
- 4. DBA: Decomposable Bundle Adjustment

5. Multirobot SLAM



Conditionally Independent Maps

Independence

- x e y are independent if:

$$p(\mathbf{x}, \mathbf{y}) = p(\mathbf{x})p(\mathbf{y})$$

 $p(\mathbf{x}|\mathbf{y}) = p(\mathbf{x})$

- Variable y does not provide information about x
- The maps cannot share information

Conditional Independence

x e y are independent, given z if:

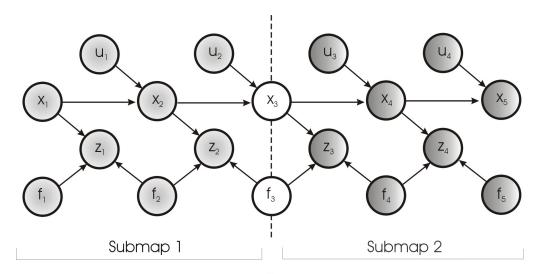
$$p(\mathbf{x}, \mathbf{y}|\mathbf{z}) = p(\mathbf{x}|\mathbf{z})p(\mathbf{y}|\mathbf{z})$$

 $p(\mathbf{x}|\mathbf{y}, \mathbf{z}) = p(\mathbf{x}|\mathbf{z})$

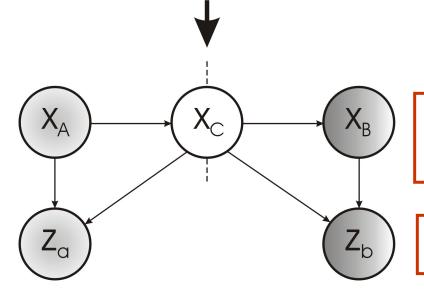
- If z is known, y does not provide additional information about x
- The maps can share information (z)



Conditionally Independent Maps



- Both maps share \mathbf{x}_3 and \mathbf{f}_3
- They are not independent
- But they are Conditionally Independent, given x₃ and f₃



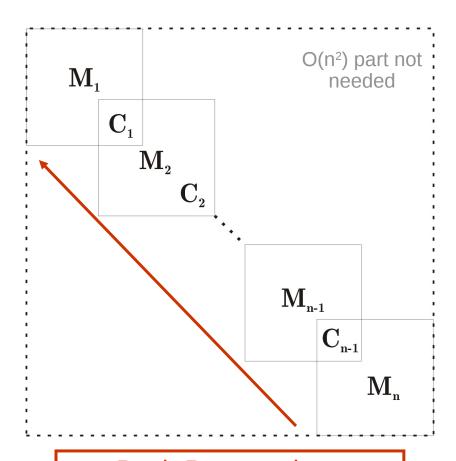
$$p(\mathbf{x}_A|\mathbf{x}_B, \mathbf{x}_C, \mathbf{z}_a, \mathbf{z}_b) = p(\mathbf{x}_A|\mathbf{x}_C, \mathbf{z}_a)$$
$$p(\mathbf{x}_B|\mathbf{x}_A, \mathbf{x}_C, \mathbf{z}_a, \mathbf{z}_b) = p(\mathbf{x}_B|\mathbf{x}_C, \mathbf{z}_b)$$

If x_c is known, x_A and z_a do not provide additional information about x_B and z_b



Only the block-diagonal Covariance is computed

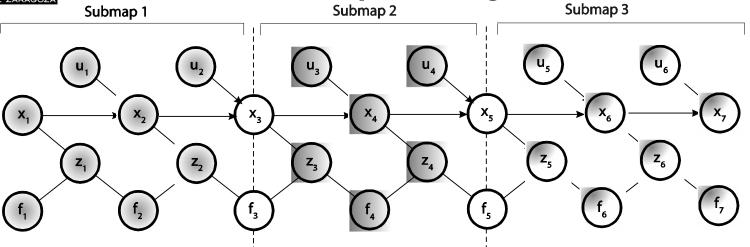
Local Maps in O(1)

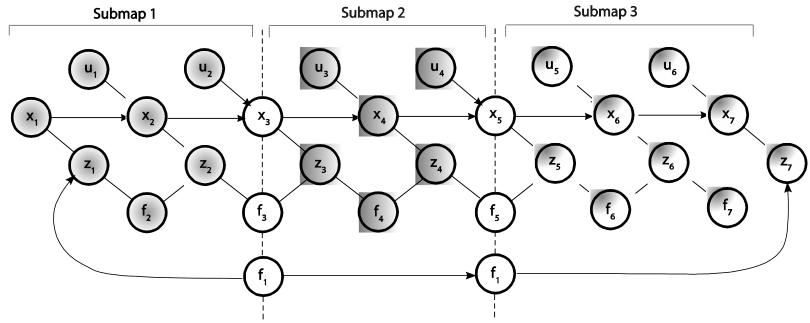


Back-Propagation:
Optimal Global Map in O(n)



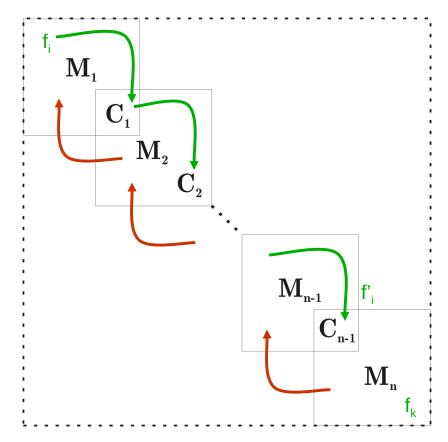
Loop Closing







Loop Closing



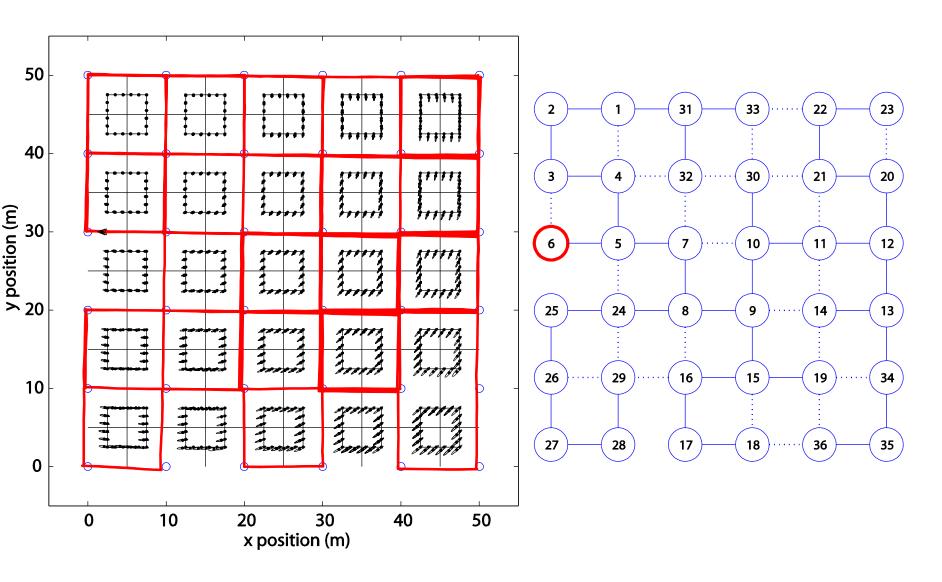
 Detect loop closing (f_i = f_k)

- Copy f_i to the common part of every map
- Impose $f'_i = f_k$ in the last map

Optimal Global Map in O(n) Back-propagate the correction

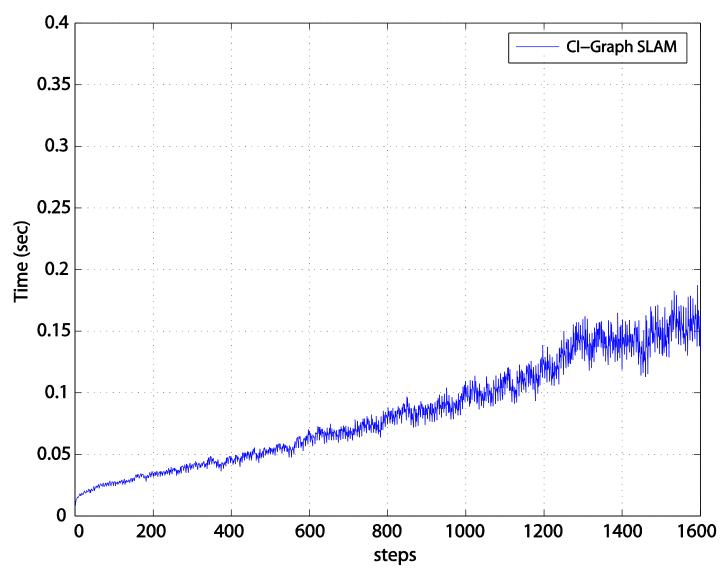


Graph of maps + Spanning tree





Cl-Graph: close to O(n)





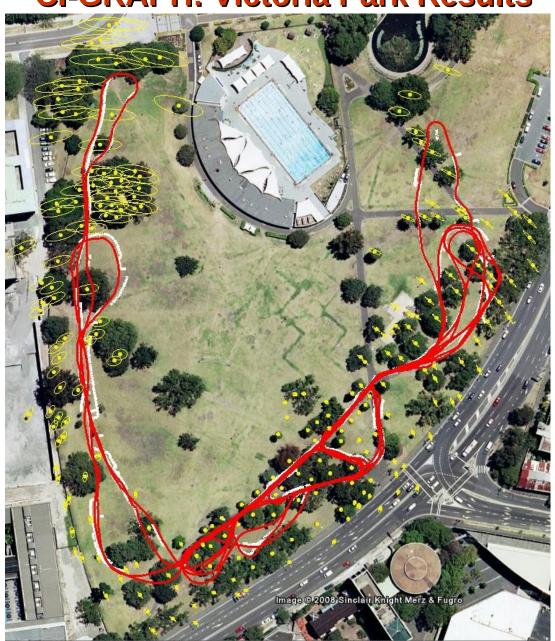
Victoria Park: Graph and Spanning Tree



P. Piniés, L. Paz and J.D. Tardós, CI-Graph: an efficient approach for Large Scale SLAM. IEEE ICRA 2009 Kobe, Japan

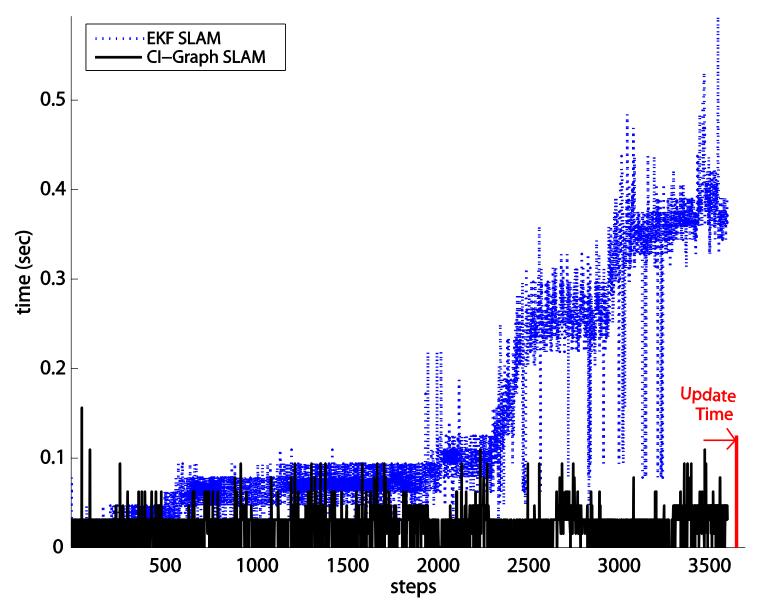


CI-GRAPH: Victoria Park Results





CI-GRAPH: Victoria Park Results



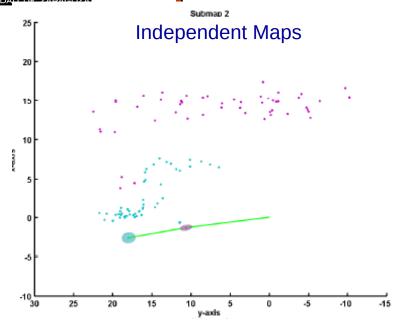


Results: Public Square (150m loop)

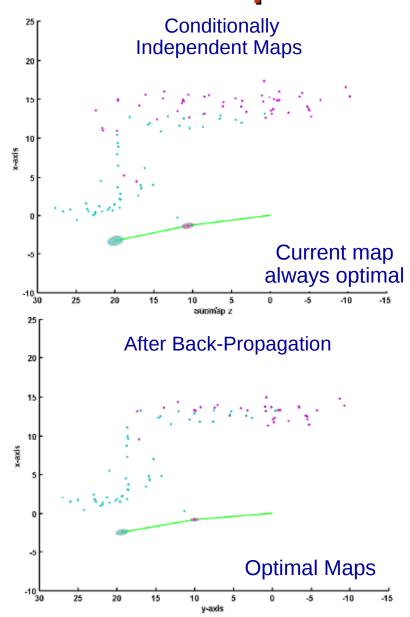


P. Piniés, J. D. Tardós. Large Scale SLAM Building Conditionally Independent Local Maps: Application to Monocular Vision. IEEE Transactions on Robotics 24(5): 1094 - 1106, Oct. 2008

Independence .vs. Conditional Independence

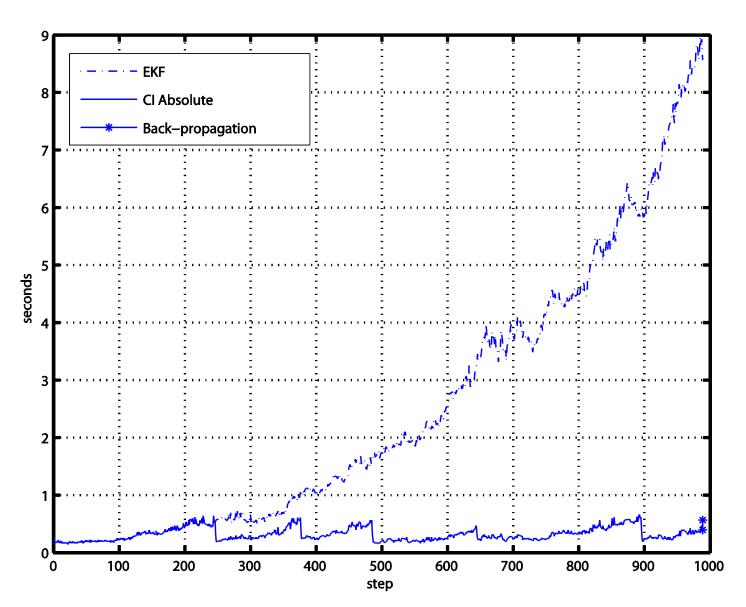


- Features and vehicle states cannot be shared between maps
- Sub-optimal maps
- Different map scales





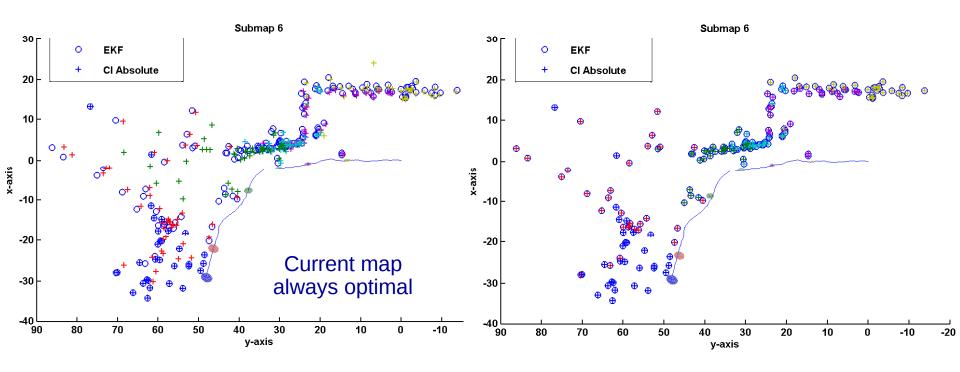
CI-SLAM: Complexity





Conditionally Independent Maps

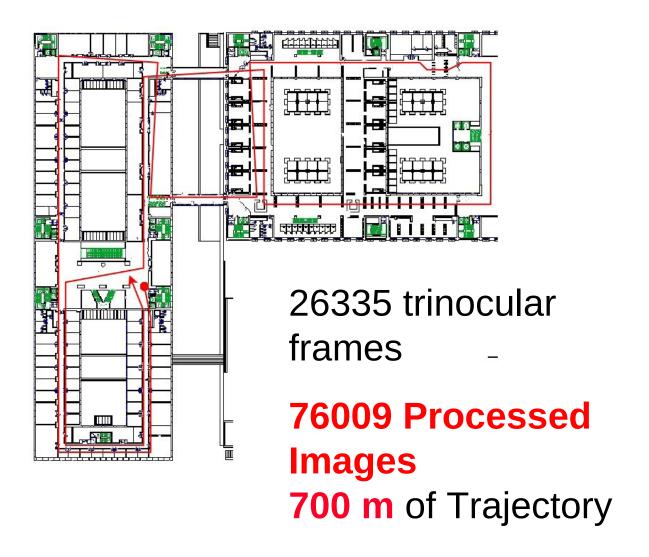
After Back-Propagation





RAWSEEDS dataset: Bicocca_2009-02-25b

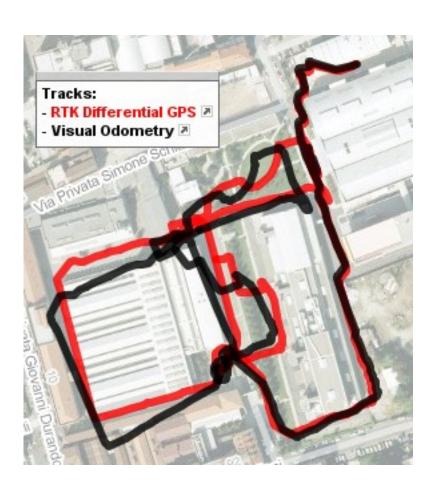
www.rawseeds.org





RAWSEEDS dataset: Bovisa

www.rawseeds.org



34173 trinocular frames

102519 Processed Images

1.365 km of Trajectory

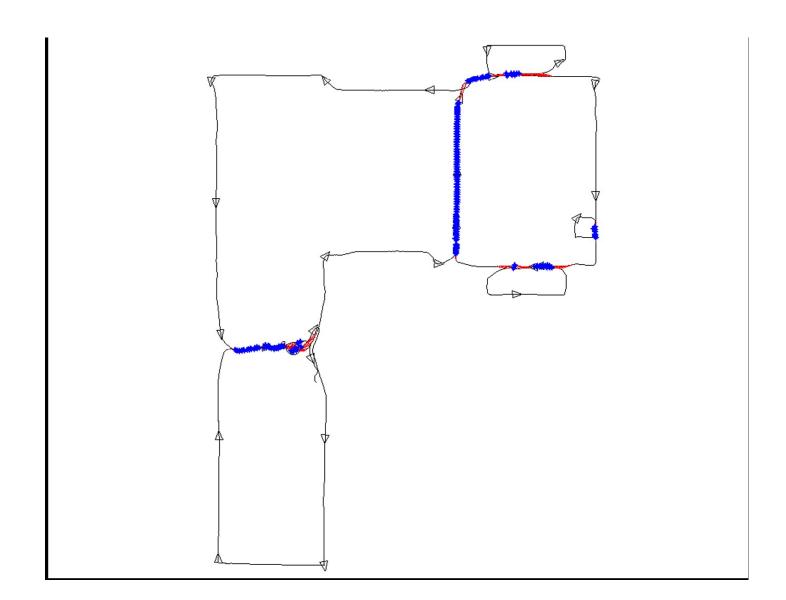


Appareance-Based Loop Detection

- Bag of words representation
 - SURF features clustered into visual words
 - Hierarchical vocabulary tree (Nister 2006)
 - Vocabulary trained off-line with a RAWSEEDS mixed dataset
- On-line loop detection
 - Learn one image per second
 - Match with previous learned images
 - Find the loop closing transformation solving the multi-view geometry

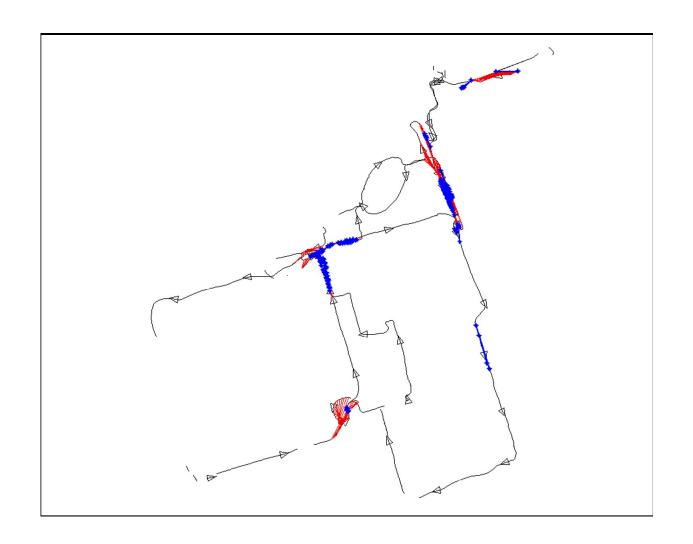


Appareance-Based Loop Detection





Appareance-Based Loop Detection





Example of Loop Successfully Detected







Missed Loop Closure







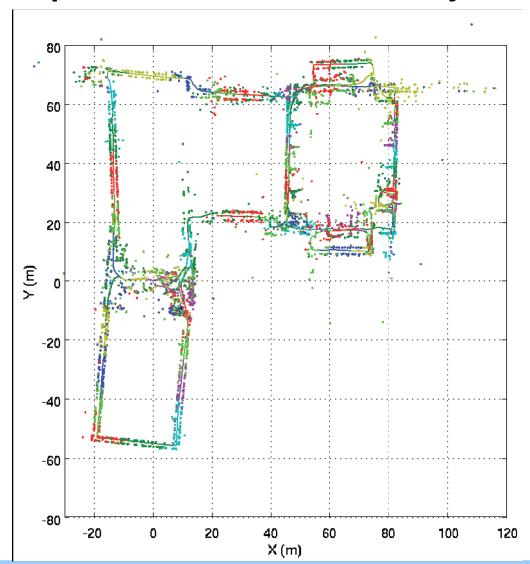
Example of Loop Successfully Detected







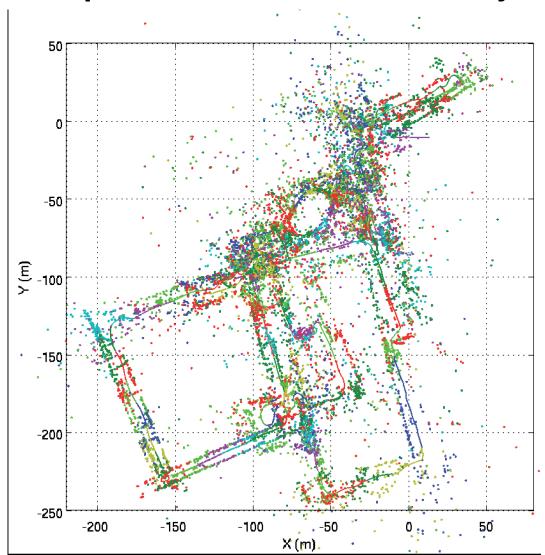
CI-Graph Results: 700 mts of Trajectory



P. Piniés, L. M. Paz, D. Gálvez, J. D. Tardós: **CI-Graph SLAM for 3D** reconstruction of large and complex environments using a multicamera system. Journal of Field Robotics, Oct 2010



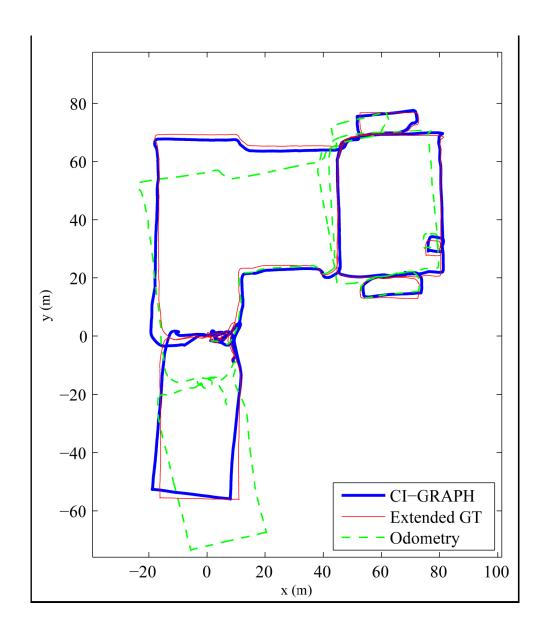
CI-Graph Results: 1.365 km of Trajectory



P. Piniés, L. M. Paz, D. Gálvez, J. D. Tardós: **CI-Graph SLAM for 3D** reconstruction of large and complex environments using a multicamera system. Journal of Field Robotics, Oct 2010

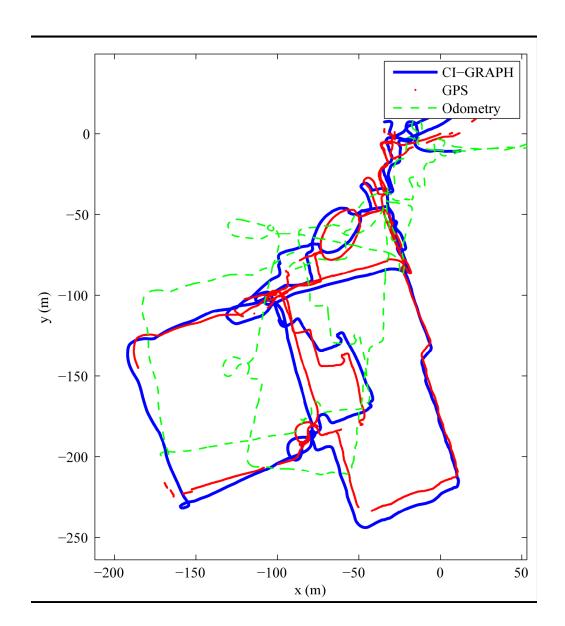


CI-Graph Results: 700 mts of Trajectory





CI-Graph Results: 1.365 km of Trajectory





SLAM in Large Environments

AD DE -	ABACO74							
	Method	Publication	Memory	T local	T Loop	Aproxim.	Coord.	
1	. EKF	Smith 1986	$O(n^2)$	$O(n^2)$	O(n ²)		Abs	
2	SEIF	Thrun 2004	O(n)	O(1) amort	O(n ²)	Sparsif.	Abs	
3	ESEIF	Walter 2007	O(n)	O(1)	O(n ²)	Kidnap	Abs	
7	ESDF	Eustice 2006	O(p)	O(1)	O(p ²)		Abs	
8	Postpon.	Knight 2001	$O(n^2)$	O(n)	O(n²)		Abs	
g	CEKF	Guivant 2001	$O(n^2)$	O(n)	O(n ²)		Abs	
13	CLSF	Williams 2002	$O(n^2)$	O(1)	O(n ²)		Local	
14	Map Joining	Tardós 2002	$O(n^2)$	O(1)	O(n²)		Local	
4	D&C SLAM	Paz 2007	O(n ²)	O(1)	O(n) amort.		Local	
5	CTS	Newman 2003	O(n)	O(1)		FD, Loop	Local	
6	Atlas	Bosse 2004	O(n)	O(1)		FD, Loop	Local	
10	H-SLAM	Estrada 2005	O(n)	O(1)	O(n)	FD	Local	
11	TJTF	Paskin 2003	O(n)	O(1)	O(n)	Sparsif.	Abs	
12	Treemap	Frese 2006	O(n)	O(1)	O(log n); O(n)		Abs	
17	Graph SLAM	Thrun 2006	$O(p^2)$		$O(p^3)$		Abs	
	SAM	Dellaert 2006	O(n+p)		O(n+p) ??		Abs	
	FastSLAM	Montemerlo 20	(O(Kn)	O(K)	O(K)		Abs	
	CI-SLAM	Piniés 2007	O(n)	O(1)	O(n)		Local	
	CI-GRAPH	Piniés 2009	O(n)	O(1)	O(n) ??		Local	

Linearization errors are reduced

Can use Joint Compatibility for D.A.



Outline

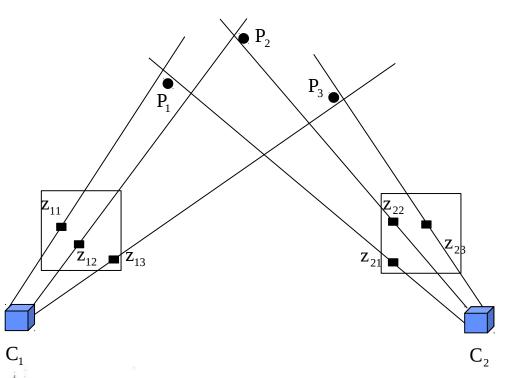
- 1. The SLAM scaling problem: Complexity and Consistency
- 2.D&C SLAM: Independent local maps

- 3.CI-Graph SLAM: Conditionally independent maps
- 4.DBA: Decomposable Bundle Adjustment

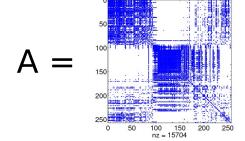
5. Multirobot SLAM

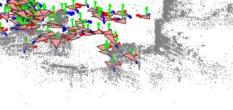


Context



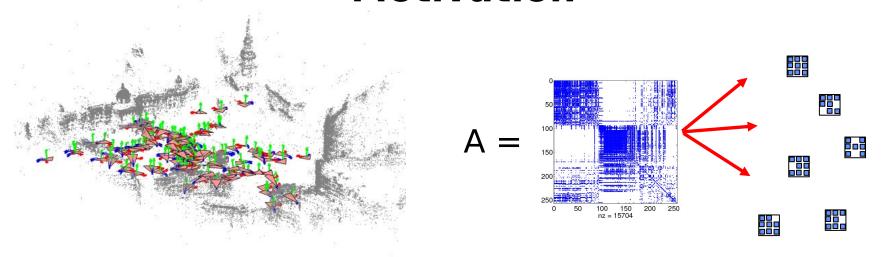
Solve at each iteration A x = b







Motivation



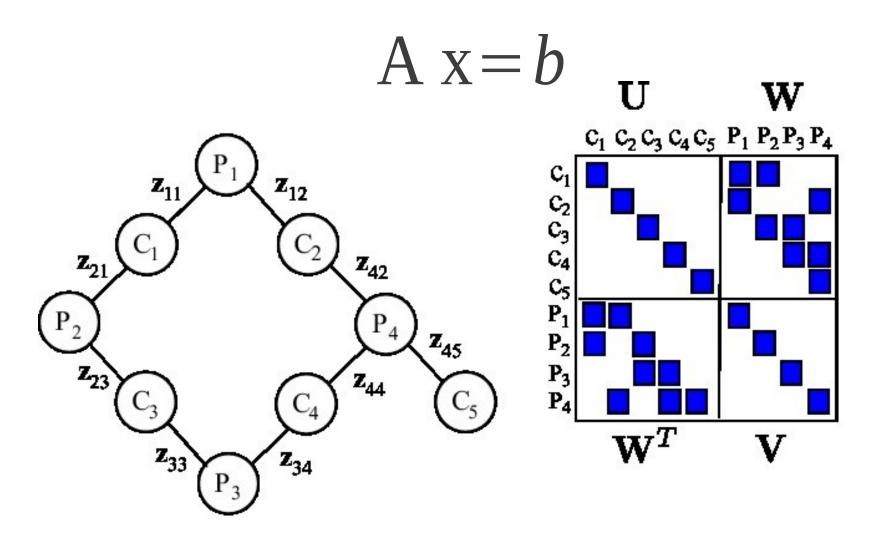
Goal: Develop a distributed algorithm

Examples:

- distributed BA using a team of robots
- solve big systems in a cluster of computers

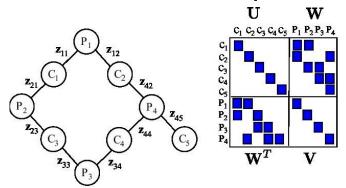


Primary structure



ROBOTICS Benefits of the Primary structure

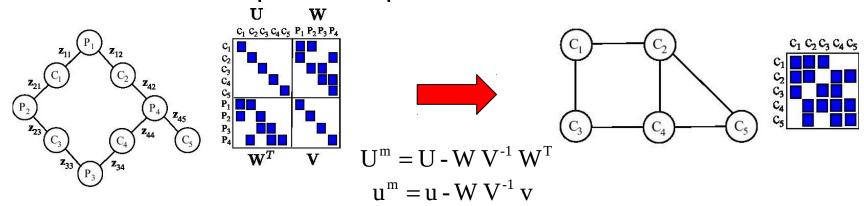
1. Get rid of the points (np >> nc):



ROBOTICS

Benefits of the Primary structure

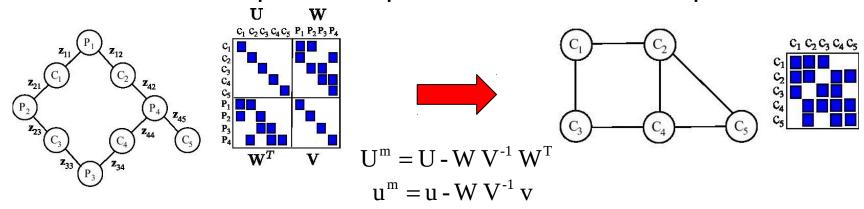
1. Get rid of the points (np >> nc): Variable elimination



ROBO

Benefits of the Primary structure

1. Get rid of the points (np >> nc): Schur complement



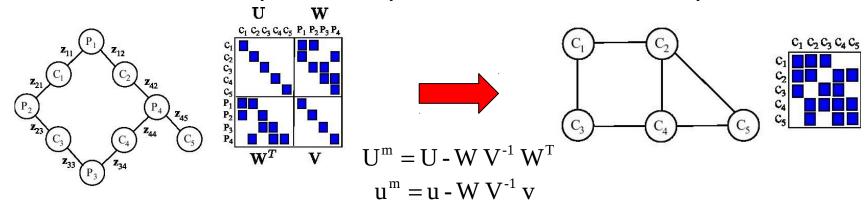
2. Solve for the reduced camera matrix

$$U^m x_C = u^m$$

ROBOT

Benefits of the Primary structure

1. Get rid of the points (np >> nc): Schur complement



- 2. Solve for the reduced camera matrix $U^m x_c = u^m$
- Solution linear in the number of points
- Reduced camera matrix decomposable
- 3. Back-substitution to obtain points solution $V x_p = v W^T x_C^{sol}$



DBA

Algorithm proposed:

1. Use the decomposable structure of the reduced camera matrix to distribute the calculations

2. Take advantage of the natural Primary Structure to efficiently solve for the points



Decompose Camera Graph

An ordered method to decompose the camera graph: **Junction Tree**



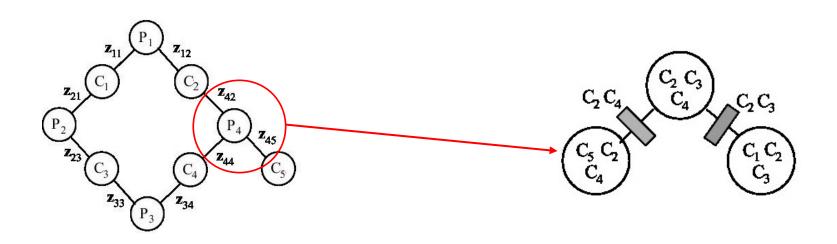
All cliques of the original graph are at least in one of the tree nodes

M. A. Paskin and G. D. Lawrence, "Junction tree algorithms for solving sparse linear systems", Computer Science Division (EECS), University of California, Berkeley, California 94720, Tech. Rep. UCB/CSD-03-1271, September 2003.



Locally maintain a Primary Structure

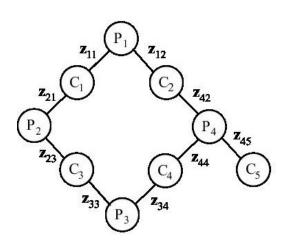
Distribute points and measurements to corresponding clique nodes

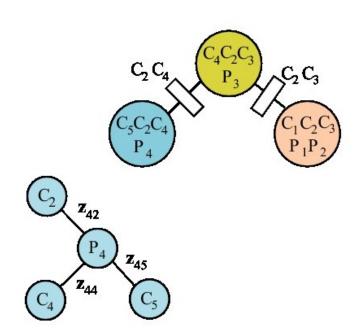




Locally maintain a Primary Structure

Each tree node maintains a Primary Structure: Solving for points is linear.



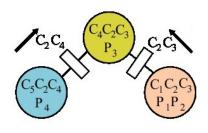




 For each tree node obtain the reduced camera matrix (Shur points)

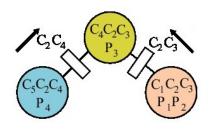


- For each tree node obtain the reduced camera matrix (Shur points)
- 2. Collect common camera information from leaves to root (Schur cam)





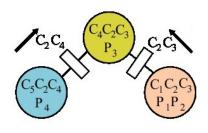
- For each tree node obtain the reduced camera matrix (Shur points)
- 2. Collect common camera information from leaves to root (Schur cam)



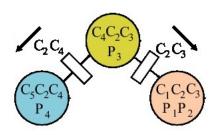
3. Solve at root



- For each tree node obtain the reduced camera matrix (Shur points)
- 2. Collect common camera information from leaves to root (Schur cam)

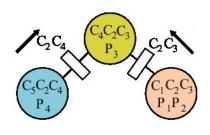


- 3. Solve at root
- 4. Distribute common camera information from root to leaves (Back cam)

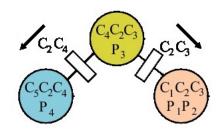




- For each tree node obtain the reduced camera matrix (Shur points)
- 2. Collect common camera information from leaves to root (Schur cam)



- 3. Solve at root
- 4. Distribute common camera information from root to leaves (Back cam)

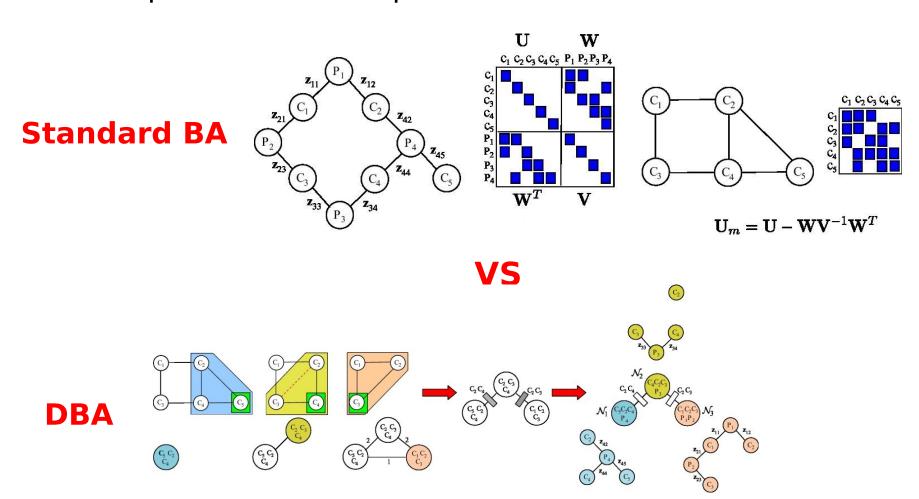


5. Solve for the points at each node (Back points)



Results

We compare two own implementations:





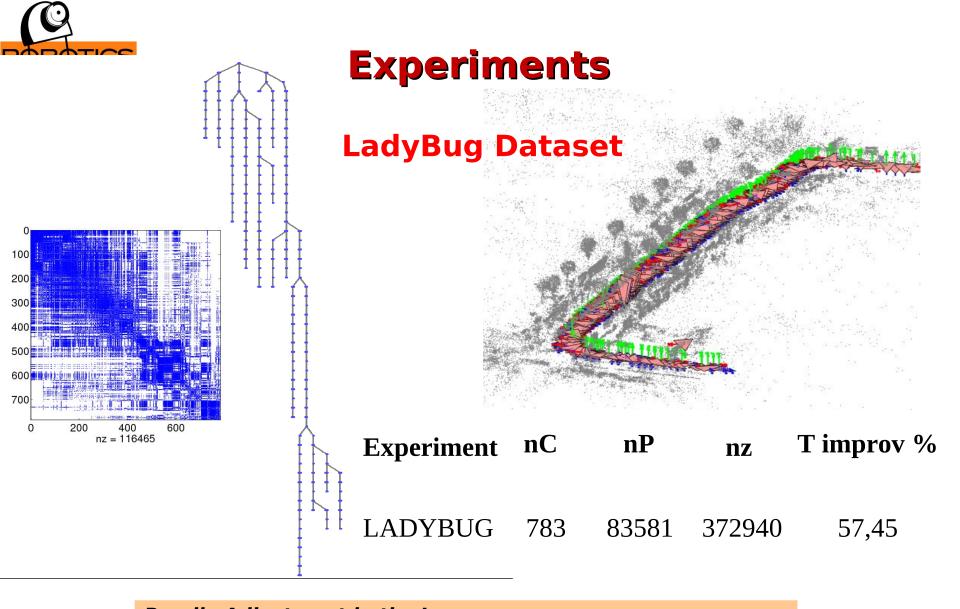
Results

Implementation details: Pentium Core Quad 2.6 GHz, 4Gb RAM

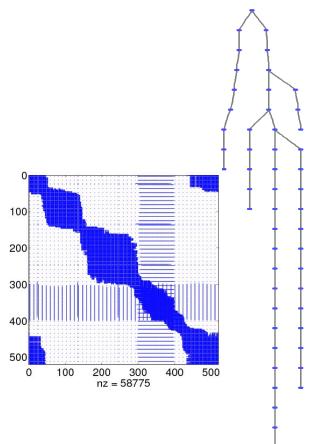
For both algorithms matrices are built using C mex functions

For standard BA: U, V and W are sparse and we use the MATLAB's CHOLMOD built-in solver

For DBA: Matrices in each tree node are usually small so we use a dense representation and MATLAB dense CHOLESKY solver.

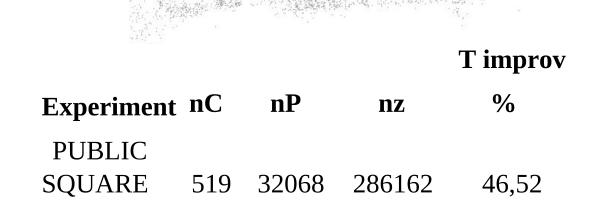


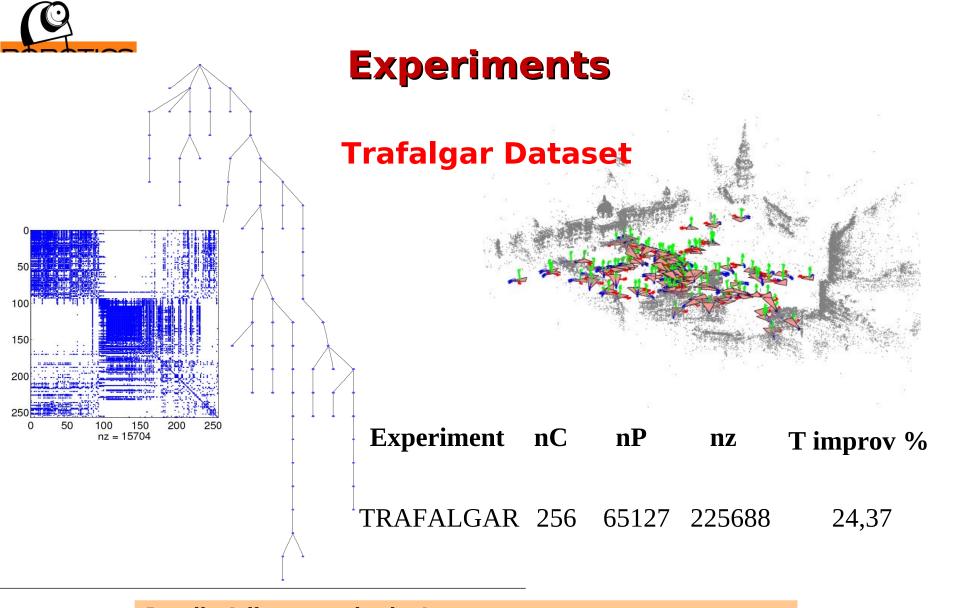




Experiments

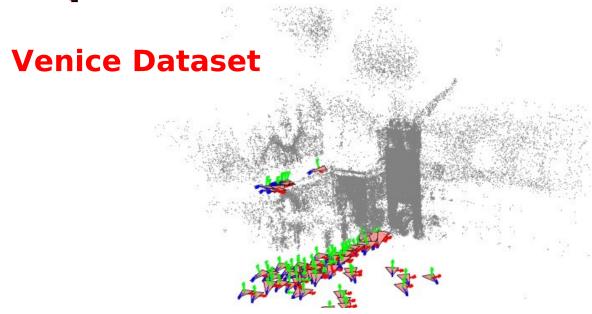








Experiments



Experiment	nC	nP	nz	T improv %
VENICE	87	110844	554826	9,86



Real experiments

				T improv
Experiment	nC	nP	nz	%
LADYBUG PUBLIC	783	83581	372940	57,45
SQUARE	519	32068	286162	46,52
TRAFALGAR	256	65127	225688	24,37
VENICE	87	110844	554826	9,86

Why is the DBA implementation faster?

- In fact is slower when solving for the reduced camera matrix (Sparse CHOLMOD is hard to beat)
- The costly operation in our standard BA implementation is:

$$U^{m} = U - W V^{-1} W^{T}$$



Conclusions and Future Work

- An easy to implement distributed algorithm for visual mapping applications (Schur-Back Subs.)
- Good Time performance (without considering communication). It is able to beat a simple standard BA implementation.
- Solution at each iteration is identical to a centralized system
- Develop methods to build balanced Junction Trees according to the available number of platforms



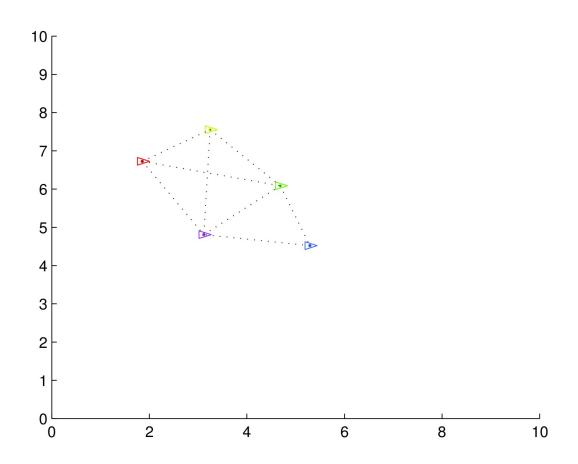
Outline

- 1. The SLAM scaling problem: Complexity and Consistency
- 2.D&C SLAM: Independent local maps

- 3.CI-Graph SLAM: Conditionally independent maps
- 4. DBA: Decomposable Bundle Adjustment

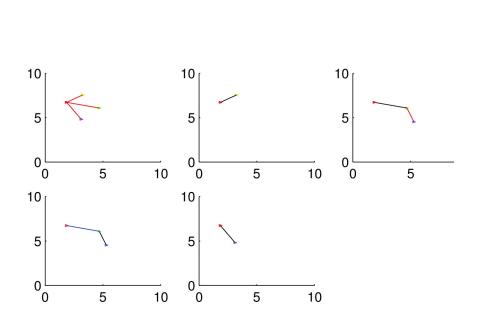
5. Multirobot SLAM

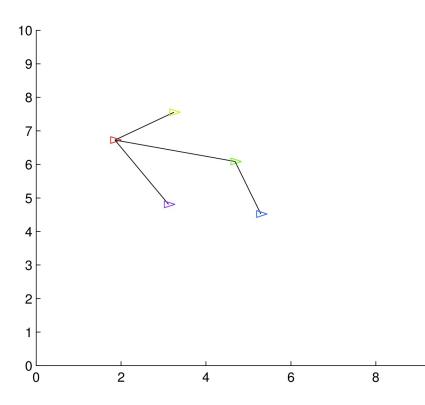




Communication Graph among 5 robots



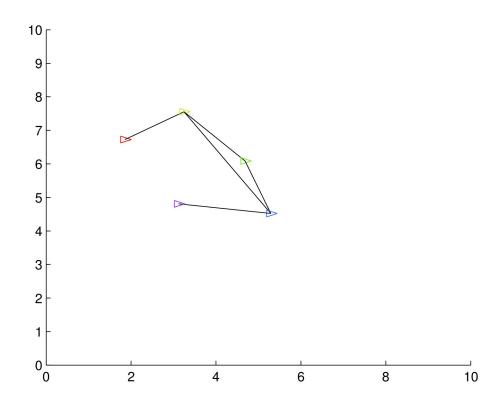




Distributed Spanning Tree

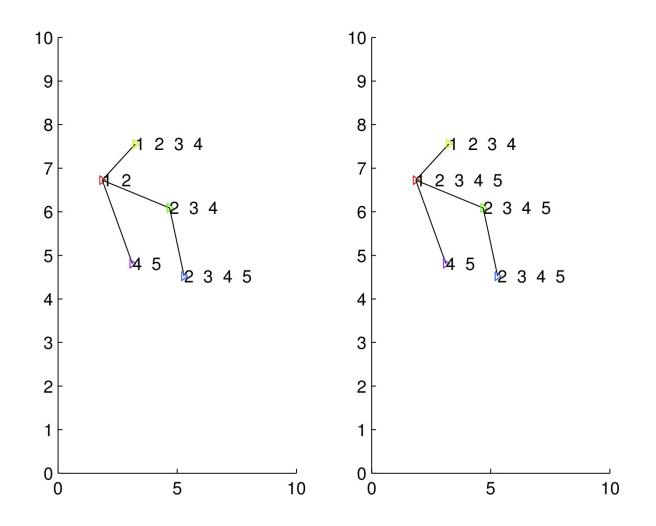
Centralized Spanning Tree





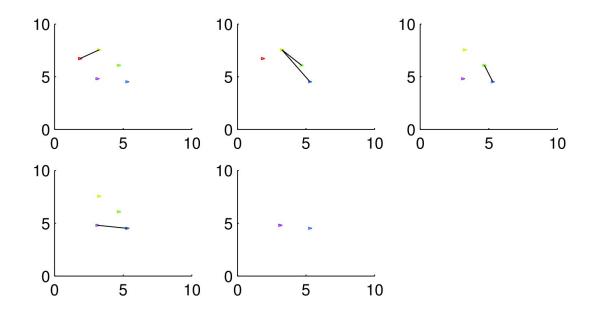
Pose Graph (Should not be the same to the connection Graph)





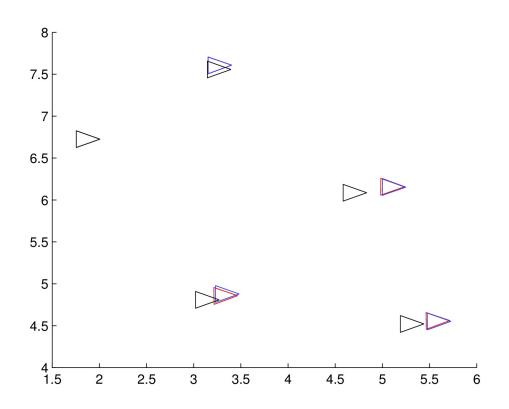
Junction Tree





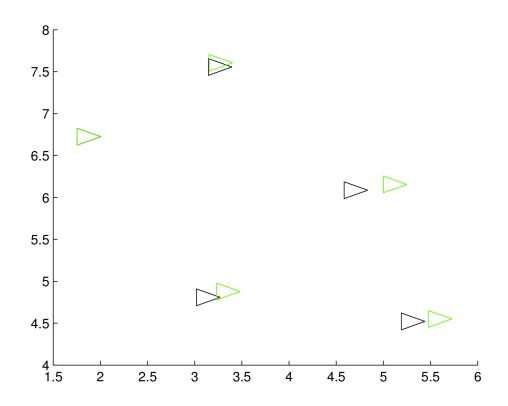
SubSystems





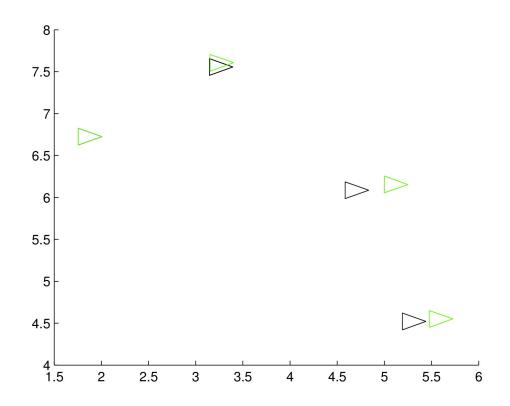
Global Solution





Solution of SubSystem 1





Solution of SubSystem 3



My Current interests

- Multi-robot SLAM research
- Dense 3D Reconstructions potentially for Real Time
- Object Categorization / Recognition
- Compression of object models



Thank you