## Multiple View Geometry: Exercise Sheet 1

## Part I: Theory

The following exercises should be solved at home. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $A$ be a symmetric matrix, and $\lambda_{a}, \lambda_{b}$ eigenvalues with eigenvectors $v_{a}$ and $v_{b}$. Proof: if $v_{a}$ and $v_{b}$ are not orthogonal, it follows: $\lambda_{a}=\lambda_{b}$.
Hint: What can you say about $\left\langle A v_{a}, v_{b}\right\rangle$ ?
2. Let $A \in \mathbb{R}^{n \times n}$ with the orthonormal basis of eigenvectors $v_{1}, \ldots, v_{n}$ and eigenvalues $\lambda_{1} \geq$ $\ldots \geq \lambda_{n}$. Find all vectors $x$, that minimize the following term:

$$
\min _{\|x\|=1} x^{T} A x
$$

How many solutions exist? How can the term be maximized?
Hint: Use the expression $x=\sum_{i=1}^{n} \alpha_{i} v_{i}$ with coefficients $\alpha_{i} \in \mathbb{R}$ and compute appropriate coefficients!
3. Consider the following sets:

$$
\begin{aligned}
& G L(n) \subset \mathbb{R}^{n \times n} \\
& A(n):=\left\{\left.L=\left(\begin{array}{ll}
A & b \\
0 & 1
\end{array}\right) \right\rvert\, A \in \mathbb{R}^{n \times n}, b \in \mathbb{R}^{n}\right\} \\
& O(n):=\left\{R \in G L(n) \mid R^{T} R=I\right\} \\
& E(n):=\left\{\left.L=\left(\begin{array}{ll}
R & b \\
0 & 1
\end{array}\right) \right\rvert\, R \in O(n), b \in \mathbb{R}^{n}\right\} \\
& S O(n):=\left\{R \in G L(n) \mid R^{T} R=I, \operatorname{det}(R)=1\right\} \\
& S E(n):=\left\{\left.L=\left(\begin{array}{ll}
R & b \\
0 & 1
\end{array}\right) \right\rvert\, R \in S O(n), b \in \mathbb{R}^{n}\right\} .
\end{aligned}
$$

The following holds: $S E(n) \subset E(n) \subset A(n) \subset G L(n+1)$. Proof:
(a) $A(n)$ is a group with respect to multiplication.
(b) $E(n)$ is a subgroup of $A(n)$ with respect to multiplication.
(c) $S E(n)$ is a subgroup of $E(n)$ with respect to multiplication.

## Part II: Practical Exercises

This exercise is to be solved during the tutorial.

1. Let

$$
A=\left(\begin{array}{ccccc}
2 & 6 & 7 & 8 & 11 \\
6 & 9 & 6 & 8 & 6 \\
7 & 6 & 1 & 7 & 9 \\
8 & 8 & 7 & 12 & 7 \\
11 & 6 & 9 & 7 & 7
\end{array}\right)
$$

(a) Indicate two possibilities how to assert via matlab if the matrix is invertible.
(b) Compute the eigenvalue decomposition $A=P \Lambda P^{-1}$ with diagonal matrix $\Lambda$. Compute $A-P \Lambda P^{-1}$. What do you observe?
(c) Compute the SVD $A=U \Sigma V^{\top}$ with diagonal matrix $\Sigma$. Compute $A-U \Sigma V^{\top}$. What do you observe?

## Matlab-Tutorials:

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http://www.math.utah.edu/lab/ms/matlab/matlab.html
http://www.math.ufl.edu/help/matlab-tutorial/
http://www.glue.umd.edu/~nsw/ench250/matlab.htm
```

