
Multiple View Geometry: Exercise Sheet 2

Solution of the theoretical exercises

1. (a) $\begin{pmatrix} I & T \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} R & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} R & RT \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_1 t_x \\ r_{21} & r_{22} & r_{23} & r_2 t_y \\ r_{31} & r_{32} & r_{33} & r_3 t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$, with r_1, r_2, r_3 : row vectors of R .

2. " \Rightarrow ":

$$M_1 - M_2 := \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

$$\begin{aligned} & x^\top M_1 x = x^\top M_2 x \\ \Leftrightarrow & x^\top M_1 x - x^\top M_2 x = 0 \\ \Leftrightarrow & x^\top (M_1 - M_2) x = 0 \\ \Leftrightarrow & m_{11}x_1x_1 + m_{12}x_1x_2 + m_{13}x_1x_3 \\ & + m_{21}x_2x_1 + m_{22}x_2x_2 + m_{23}x_2x_3 \\ & + m_{31}x_3x_1 + m_{32}x_3x_2 + m_{33}x_3x_3 = 0 \\ \Leftrightarrow & m_{11}x_1x_1 \\ & + m_{22}x_2x_2 \\ & + m_{33}x_3x_3 \\ & + (m_{12} + m_{21})x_1x_2 \\ & + (m_{13} + m_{31})x_1x_3 \\ & + (m_{23} + m_{32})x_2x_3 = 0 \end{aligned}$$

$$\Rightarrow m_{11} = 0 \wedge m_{22} = 0 \wedge m_{33} = 0 \wedge m_{12} = -m_{21} \wedge m_{13} = -m_{31} \wedge m_{23} = -m_{32}$$

$$\Rightarrow M_1 - M_2 \in so(3).$$

” \Leftarrow ”:

$$M_1 - M_2 := \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$

$$\begin{aligned} x^\top M_1 x - x^\top M_2 x &= x^\top (M_1 - M_2) x \\ &= (x_1 \ x_2 \ x_3) \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= ax_1 x_2 + bx_1 x_3 - ax_2 x_1 + cx_3 x_2 - bx_1 x_3 - cx_2 x_3 \\ &= 0 \\ \Rightarrow x^\top M_1 x &= x^\top M_2 x \end{aligned}$$

3. We will show that $x \in \text{kern}(A^\top A) \Leftrightarrow x \in \text{kern}(A)$, i.e. $A^\top A x = 0 \Leftrightarrow Ax = 0 \ \forall x \in \mathbb{R}^m$.

” \Rightarrow ”:

$$Ax = \sum_{i=1}^n a_i x_i, \quad \text{with } a_1, \dots, a_n : \text{column vectors of } A.$$

$$\begin{aligned} A^\top A x &= \begin{pmatrix} a_1 \sum_i a_i x_i \\ \vdots \\ a_n \sum_i a_i x_i \end{pmatrix} \stackrel{!}{=} 0 \\ \Rightarrow a_1 \sum_i a_i x_i &= 0 \wedge \cdots \wedge a_n \sum_i a_i x_i = 0 \\ \Rightarrow \left(a_1 = 0 \vee \underbrace{\sum_i a_i x_i}_\text{Ax} = 0 \right) \wedge \cdots \wedge \left(a_n = 0 \vee \underbrace{\sum_i a_i x_i}_\text{Ax} = 0 \right) \\ \Rightarrow Ax &= 0 \end{aligned}$$

” \Leftarrow ”: $Ax = 0 \Rightarrow A^\top A x = A^\top \cdot 0 \Rightarrow A^\top A x = 0$