Multiple View Geometry: Exercise Sheet 4
Solution of the theoretical exercises

1. Let $p = (X\ Y\ Z)$ be a point on the smaller object and $p' = (X'\ Y'\ Z')$ a point on the larger object. Since the p' is twice as far away, we have $Z' = 2Z$, and twice as big we have $X' = 2X$ and $Y' = 2Y$. From the intercept theorem it follows that p and p' lie on the same projection ray.

$$\pi(p') = \pi \begin{pmatrix} 2X \\ 2Y \\ 2Z \end{pmatrix} = \begin{pmatrix} 2X/2Z \\ 2Y/2Z \end{pmatrix} = \begin{pmatrix} X/Z \\ Y/Z \end{pmatrix} = \pi \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \pi(p)$$

2.

$$R = \begin{pmatrix} \bar{R} & 0 \\ 0 & 1 \end{pmatrix},\ T = \begin{pmatrix} I & \bar{T} \\ 0 & 1 \end{pmatrix},\ \Pi_0 = \begin{pmatrix} I & 0 \end{pmatrix},\ K = \begin{pmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} P &= K \cdot \Pi_0 \cdot T \cdot R \\ &= K \cdot \Pi_0 \cdot \begin{pmatrix} \bar{R} & \bar{T} \\ 0 & 1 \end{pmatrix} \\ &= K \cdot \begin{pmatrix} \bar{R} & \bar{T} \end{pmatrix} \\ &= \begin{pmatrix} K\bar{R} & K\bar{T} \end{pmatrix} \end{aligned}$$

3. (a)

$$\pi(P1 \cdot X) = \pi(-3\ \ 0\ \ 4)^\top = (-0.75\ \ 0)^\top$$
$$\pi(P2 \cdot X) = \pi(1\ \ 0\ \ 4)^\top = (0.25\ \ 0)^\top$$

(b) $\hat{x} = (-1\ \ 0)^\top, \hat{y} = (0\ \ 0)^\top$

$$\text{Preimage}(\hat{x}) = \left\{ X_0 = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} : \pi(P_1 X_0) = \hat{x} \right\}$$

$$\text{Preimage}(\hat{y}) = \left\{ X_0 = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} : \pi(P_2 X_0) = \hat{y} \right\}$$

$$\pi(P_1 X_0) = \hat{x} \wedge \pi(P_2 X_0) = \hat{y}$$

$$\begin{aligned} \Rightarrow \quad (X-3)/Z &= \hat{x_1} \\ Y/Z &= \hat{x_2} \\ (X+1)/Z &= \hat{y_1} \\ Y/Z &= \hat{y_2} \end{aligned}$$