Multiple View Geometry: Exercise Sheet 5
Solution of the theoretical exercises

1. First:

$$\frac{d}{dx} x^T M x = (M^T + M) x$$

Therefore for symmetric M:

$$\frac{d}{dx} x^T M x = 2 M x$$

Furthermore:

$$\frac{d}{dx} b^T x = b$$

(a)

$$\begin{aligned}
& \frac{d}{db} \int (\nabla I(x)^T b + I_t(x))^2 dx \\
= \; & \frac{d}{db} b^T \left(\left(\int \nabla I(x) \nabla I(x)^T dx \right) b + 2 \left(\int I_t(x) \nabla I(x)^T dx \right) b + \int I_t(x)^2 dx \right) \\
= \; & 2 \left(\int \nabla I(x) \nabla I(x)^T dx \right) b + 2 \left(\int I_t(x) \nabla I(x) dx \right) \stackrel{!}{=} 0 \\
\Leftrightarrow \; b = \; & \left(\int \nabla I(x) \nabla I(x)^T dx \right)^{-1} \left(\int I_t(x) \nabla I(x) dx \right)
\end{aligned}$$

(b)

$$\begin{aligned}
& \frac{d}{dp} \int (\nabla I(x)^T S(x) p + I_t(x))^2 dx \\
= \; & \frac{d}{dp} p^T \left(\left(S(x)^T \nabla I(x) \nabla I(x)^T S(x) dx \right) p + 2 \left(\int I_t(x) \nabla I(x)^T S(x) dx \right) p + \int I_t(x)^2 dx \right) \\
= \; & 2 \left(\int S(x)^T \nabla I(x) \nabla I(x)^T S(x) dx \right) p + 2 \left(\int I_t(x) \nabla S(x)^T I(x) dx \right) \stackrel{!}{=} 0 \\
\Leftrightarrow \; p = \; & - \left(\int S(x)^T \nabla I(x) \nabla I(x)^T S(x) dx \right)^{-1} \left(\int I_t(x) \nabla S(x)^T I(x) dx \right)
\end{aligned}$$