
Multiple View Geometry: Exercise Sheet 6

Solution of the theoretical exercises

1. The camera center o_2 is in the preimage of every x_2 .

$$\begin{aligned} \Rightarrow & \text{The epipole } e_1 \text{ (which is the projection of } o_2 \text{ to the image plane of image 1) lies on all epipolar lines} \\ L_1 = & \{x_1 \mid x_2^\top F x_1 = 0\} \text{ (for all } x_2). \\ \Rightarrow & x_2^\top F e_1 = 0 \quad \forall x_2 \\ \Rightarrow & F e_1 = 0 \\ \text{Analog: } & e_2^\top F = 0. \end{aligned}$$

2. (a) F is essential matrix, i.e. $\Sigma = \text{diag}\{\sigma, \sigma, 0\}$:

$$R_z(\pm \frac{\pi}{2})\Sigma = \begin{pmatrix} 0 & \mp 1 & 0 \\ \pm 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \mp \sigma & 0 \\ \pm \sigma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -(R_z(\pm \frac{\pi}{2})\Sigma)^\top$$

$$\begin{aligned} -\hat{T}^\top &= -(U R_z \Sigma U^\top)^\top \\ &= U(-R_z \Sigma)^\top U^\top \\ &= U R_z \Sigma U^\top \\ &= \hat{T} \end{aligned}$$

- (b) i. U, V are orthogonal matrices $\Rightarrow U^\top U = Id$ and $V V^\top = Id$
 R_z is a rotation matrix $\Rightarrow R_z R_z^\top = Id$

$$\begin{aligned} R^\top R &= (U R_z^\top V^\top)^\top (U R_z^\top V^\top) \\ &= V R_z U^\top U R_z^\top V^\top \\ &= V R_z R_z^\top V^\top \\ &= V V^\top \\ &= Id \end{aligned}$$

- ii. U and V are orthogonal matrices with $\det(U) = \det(V^\top) = \pm 1$.

$$\det(R) = \det(U R_z^\top V^\top) = \underbrace{\det(U)}_{\pm 1} \cdot \underbrace{\det(R_z^\top)}_{+1} \cdot \underbrace{\det(V^\top)}_{\pm 1} = 1$$

3. (a) $H = R + T u^\top \Leftrightarrow R = H - T u^\top$.

$$E = \hat{T} R = \hat{T}(H - T u^\top) = \hat{T} H - \underbrace{\hat{T} T}_{=T \times T=0} u^\top = \hat{T} H$$

(b)

$$\begin{aligned} H^\top E + E^\top H &= (R + T u^\top)^\top \cdot (\hat{T} R) + (\hat{T} R)^\top \cdot (R + T u^\top) \\ &= (R^\top + u T^\top) \cdot (\hat{T} R) + (R^\top \hat{T}^\top) \cdot (R + T u^\top) \\ &= R^\top \hat{T} R + u \underbrace{T^\top \hat{T}}_{=0} R + R^\top \hat{T}^\top R + R^\top \underbrace{\hat{T}^\top T}_{=0} u^\top \\ &= R^\top \hat{T} R + R^\top \hat{T}^\top R \\ &= R^\top (\hat{T} + \hat{T}^\top) R \\ &= R^\top (\hat{T} - \hat{T}) R \quad (\text{because } \hat{T} \text{ is skew-symmetric, i.e. } \hat{T}^\top = -\hat{T}) \\ &= 0 \end{aligned}$$