Multiple View Geometry: Exercise Sheet 8

Exercise: 19 July 2011

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Denote by $[R, T]$ the relative coordinate transformation between two camera views. Then the associated essential matrix is denoted by $F = \hat{T}R$. Denote the image of the ith center o_i in the jth view by $e_j \in \mathbb{R}^3$. Show that

   $$e_2 \sim T \;\text{ and }\; e_1 \sim R^\top e_2$$

   where equality is in the homogeneous sense.

   Hint: *Consider the relations between epipoles and the fundamental matrix, i.e.* $Fe_1 = 0$ *and* $e_2^\top F = 0$.

2. Suppose two projection matrices $\Pi = [R, T]$ and $\Pi' = [R', T'] \in \mathbb{R}^{3\times 4}$ are related by a common transformation H of the form

   $$H = \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} \in \mathbb{R}^{4\times 4} \;\text{ with } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

   That is, $[R, T]H_p \sim [R', T']$ are equal up to scale. Show that Π and Π' give the same essential matrices (up to a scale factor).

3. Consider m image points $x_1, ..., x_m$ with respect to m camera frames specified by $\Pi_1, ..., \Pi_m$ with $\Pi_t = [R_t, T_t] \; \forall t = 1, ..., m$ (with the first camera frame being the reference, i.e. $R_1 = Id, T_1 = 0$). Show that

   $$\text{rank}(N_p) \leq m + 3 \;\Rightarrow\; \text{rank}(M_p) \leq 1$$

4. Show that the only solution corresponding to the equation

   $$\text{rank}(M_p) = 0$$

   is that all the camera centers $o_1, o_2, \ldots, o_m$ lie on a line and the point p can be anywhere on this line.

5. Given two coimages of a line $l_1, l_2 \in \mathbb{R}^3$ and the relative motion (R, T) of the camera between the two vantage points, what is the 3D location of the line with respect to each camera frame? Express the direction of the line and its distance to the center of the camera in terms of l_1, l_2 and (R, T). Under what conditions are such a distance and direction not uniquely determined?

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Download the package `mvg_ex08.tgz` from the website and extract the five images and calibration file.
2. Show the images with matlab and click at corresponding points. You should click at the same points in each image in the same order.
3. Take the first image as reference frame and compute the matrix M_p for each series of corresponding points. Test if the rank constraint is fulfilled. You can retrieve the calibration data using the matlab file `readcalibration(filename)`. The function returns the intrinsic calibration matrices K_i, the rotation matrices R_i and the translational vectors T_i for each camera i.
4. Compute the 3D reconstruction of the points (see slide 33).
5. Recover the camera motions R_i and T_i from the scene structure. To this end, build the linear equation system from slide 32 and solve it with SVD.

Matlab-Tutorials:
`http://www.math.utah.edu/lab/ms/matlab/matlab.html`
`http://www.math.ufl.edu/help/matlab-tutorial/`
`http://www.glue.umd.edu/~nsw/ench250/matlab.htm`