Machine Learning for Computer Vision

Dr. Rudolph Triebel

Lecturers

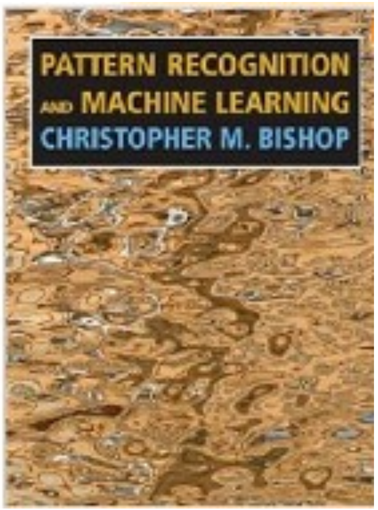
- Dr. Rudolph Triebel
- rudolph.triebel@in.tum.de
- Room number 02.09.059
- Main lecture

- Dipl. Inf. Jan Stühmer
- jan.stuehmer@in.tum.de
- Room number 02.09.057
- Assistance and exercises

Class Schedule

Date	Topic
26.4.	Introduction
3.5	Regression
10.5	Probabilistic Graphical Models I
17.5.	Probabilistic Graphical Models II
24.5	Boosting
31.5	Random Forests
7.6	Kernel Methods
14.6	Gaussian Processes I
21.6.	Gaussian Processes II
28.6.	Evaluation and Model Selection
5.7	Sampling Methods
12.7	Unsupervised Learning
19.7	Online Learning

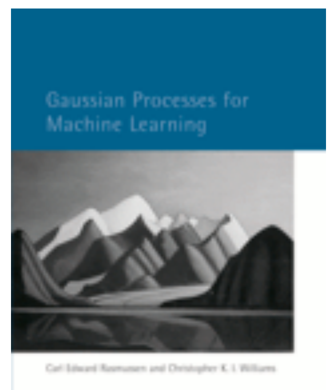
Literature

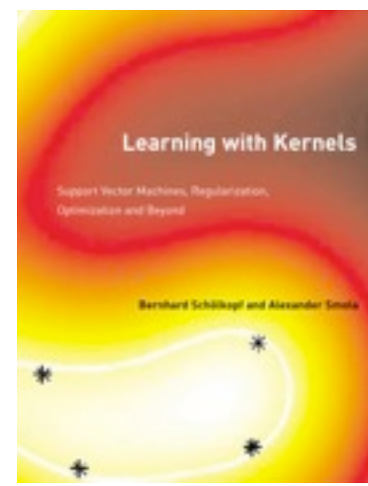


Recommended textbook for the lecture: Christopher M. Bishop: “Pattern Recognition and Machine Learning”

More detailed:

- “Gaussian Processes for Machine Learning” Rasmussen/Williams
- “Learning With Kernels” Schölkopf/Smola





The Exercises

- Bi-weekly exercise classes
- Participation in exercise classes and submission of solved exercise sheets is totally free
- The submitted solutions will be corrected and returned
- In class, you have the opportunity to present your solution
- Exercises will be theoretical and practical problems

The Exam

- No “qualification” necessary for the final exam
- Final exam will be oral
- From a given number of known questions, some will be drawn by chance
- Usually, from each part a fixed number of questions appears

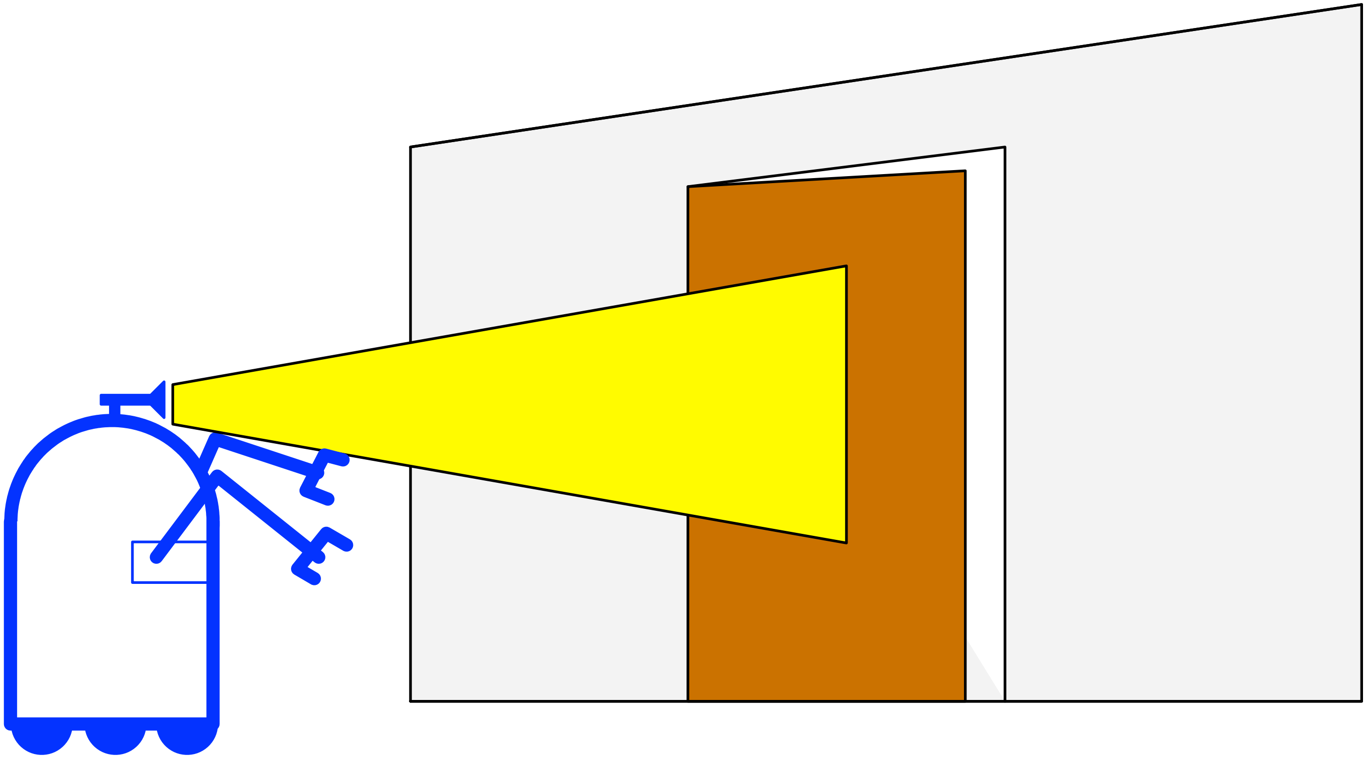
Class Webpage

http://vision.in.tum.de/teaching/ss2013/ml_ss13

1. Introduction to Learning and Probabilistic Reasoning

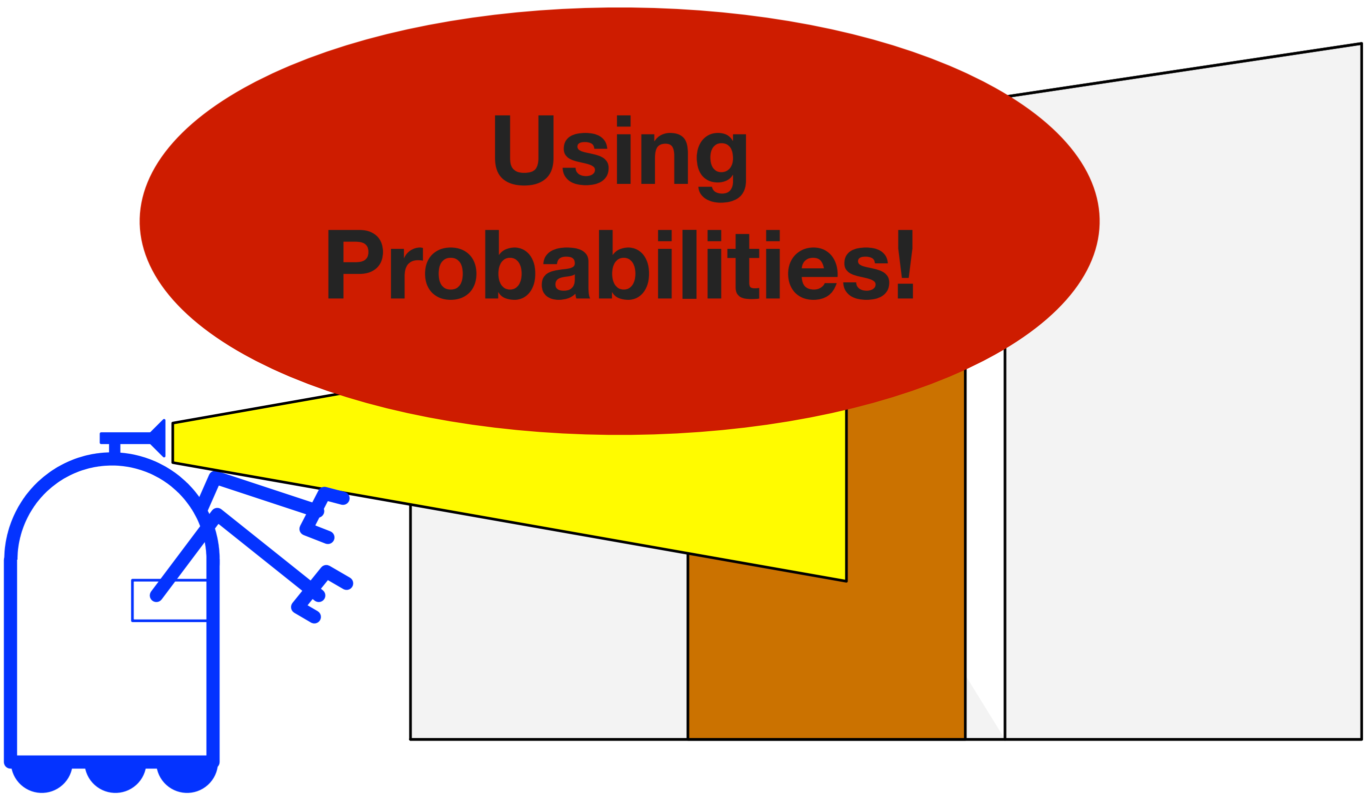
Motivation

Suppose a robot stops in front of a door. It has a **sensor** (e.g. a camera) to measure the **state** of the door (open or closed). **Problem**: the sensor may fail.



Motivation

Question: How can we obtain knowledge about the environment from sensors that may return incorrect results?



Basics of Probability Theory

Definition 1.1: A ***sample space*** $\mathcal{S}$ is a set of outcomes of a given experiment.

Examples:

a) Coin toss experiment: $\mathcal{S} = \{H, T\}$

b) Distance measurement: $\mathcal{S} = \mathbb{R}_0^+$

Definition 1.2: A ***random variable*** X is a function that assigns a real number to each element of $\mathcal{S}$.

Example: Coin toss experiment: $H = 1, T = 0$

Values of random variables are denoted with small letters, e.g.: $X = x$

Discrete and Continuous

If $\mathcal{S}$ is countable then X is a ***discrete*** random variable, else it is a ***continuous*** random variable.

The probability that X takes on a certain value x is a real number between 0 and 1. It holds:

$$\sum_x p(X = x) = 1$$

Discrete case

$$\int p(X = x) dx = 1$$

Continuous case

A Discrete Random Variable

Suppose a robot knows that it is in a room, but it does not know in ***which*** room. There are 4 possibilities:

Kitchen, Office, Bathroom, Living room

Then the random variable *Room* is discrete, because it can take on one of four values. The probabilities are, for example:

$$P(Room = \text{kitchen}) = 0.7$$

$$P(Room = \text{office}) = 0.2$$

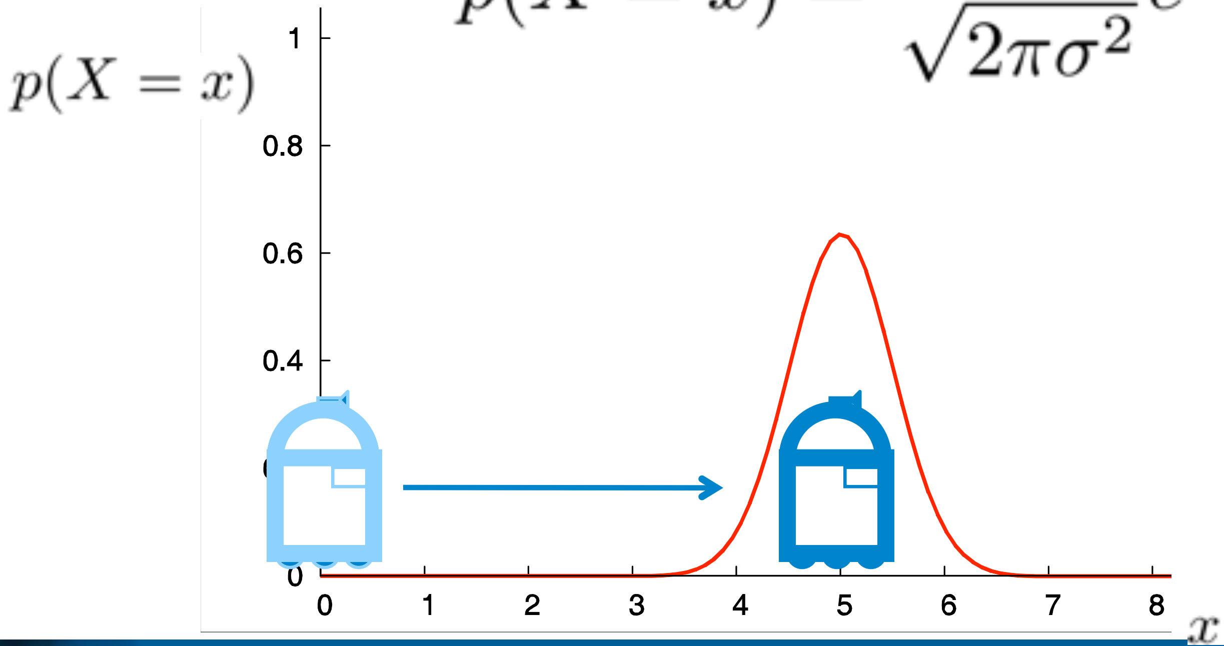
$$P(Room = \text{bathroom}) = 0.08$$

$$P(Room = \text{living room}) = 0.02$$

A Continuous Random Variable

Suppose a robot travels 5 meters forward from a given start point. Its position X is a continuous random variable with a ***Normal distribution***:

$$p(X = x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-5)^2}{\sigma^2}}$$



Shorthand:

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$\downarrow$$

$$\mathcal{N}(x; \mu, \sigma^2)$$

Joint and Conditional Probability

The ***joint probability*** of two random variables X and Y is the probability that the events $X = x$ and $Y = y$ occur at the same time:

$$p(X = x \text{ and } Y = y)$$

Shorthand: $p(X = x) \longrightarrow p(x)$

$p(X = x \text{ and } Y = y) \longrightarrow p(x, y)$

Definition 1.3: The ***conditional probability*** of X given Y is defined as:

$$p(X = x \mid Y = y) = p(x \mid y) := \frac{p(x, y)}{p(y)}$$

Independency, Sum and Product Rule

Definition 1.4: Two random variables X and Y are *independent* iff:

$$p(x, y) = p(x)p(y)$$

For independent random variables X and Y we have:

$$p(x \mid y) = \frac{p(x, y)}{p(y)} = \frac{p(x)p(y)}{p(y)} = p(x)$$

Furthermore, it holds:

$$p(x) = \sum_y p(x, y) \qquad p(x, y) = p(y \mid x)p(x)$$

"Sum Rule" **"Product Rule"**

Law of Total Probability

Theorem 1.1: For two random variables X and Y it holds:

$$p(x) = \sum_y p(x \mid y) p(y) \qquad p(x) = \int p(x \mid y) p(y) dy$$

Discrete case — Continuous case

The process of obtaining $p(x)$ from $p(x, y)$ by summing or integrating over all values of y is called

Marginalisation

Bayes Rule

Theorem 1.2: For two random variables X and Y it holds:

$$p(x \mid y) = \frac{p(y \mid x)p(x)}{p(y)}$$

"Bayes Rule"

Proof:

I. $p(x \mid y) = \dfrac{p(x, y)}{p(y)}$ *(definition)*

II. $p(y \mid x) = \dfrac{p(x, y)}{p(x)}$ *(definition)*

III. $p(x, y) = p(y \mid x)p(x)$ *(from II.)*

Bayes Rule: Background Knowledge

For $p(y \mid z) \neq 0$ it holds:

Background knowledge

$$p(x \mid y, z) = \frac{p(y \mid x, z)p(x \mid z)}{p(y \mid z)}$$

Shorthand: $p(y \mid z)^{-1} \longrightarrow \eta$

"Normalizer"

$$p(x \mid y, z) = \eta \, p(y \mid x, z)p(x \mid z)$$

Computing the Normalizer

$$p(x \mid y) = \frac{p(y \mid x)p(x)}{p(y)}$$

Bayes rule

$$p(y) = \sum_x p(y \mid x)p(x)$$

Total probability

$$p(x \mid y) = \frac{p(y \mid x)p(x)}{\sum_{x'} p(y \mid x')p(x')}$$

$p(x \mid y)$ can be computed without knowing $p(y)$

Conditional Independence

Definition 1.5: Two random variables X and Y are *conditional independent* given a third random variable Z iff:

$$p(x, y \mid z) = p(x \mid z)p(y \mid z)$$

This is equivalent to:

$$p(x \mid z) = p(x \mid y, z) \quad \text{and}$$
$$p(y \mid z) = p(y \mid x, z)$$

Expectation and Covariance

Definition 1.6: The ***expectation*** of a random variable X is defined as:

$$E[X] = \sum_x x\ p(x) \qquad \text{(discrete case)}$$

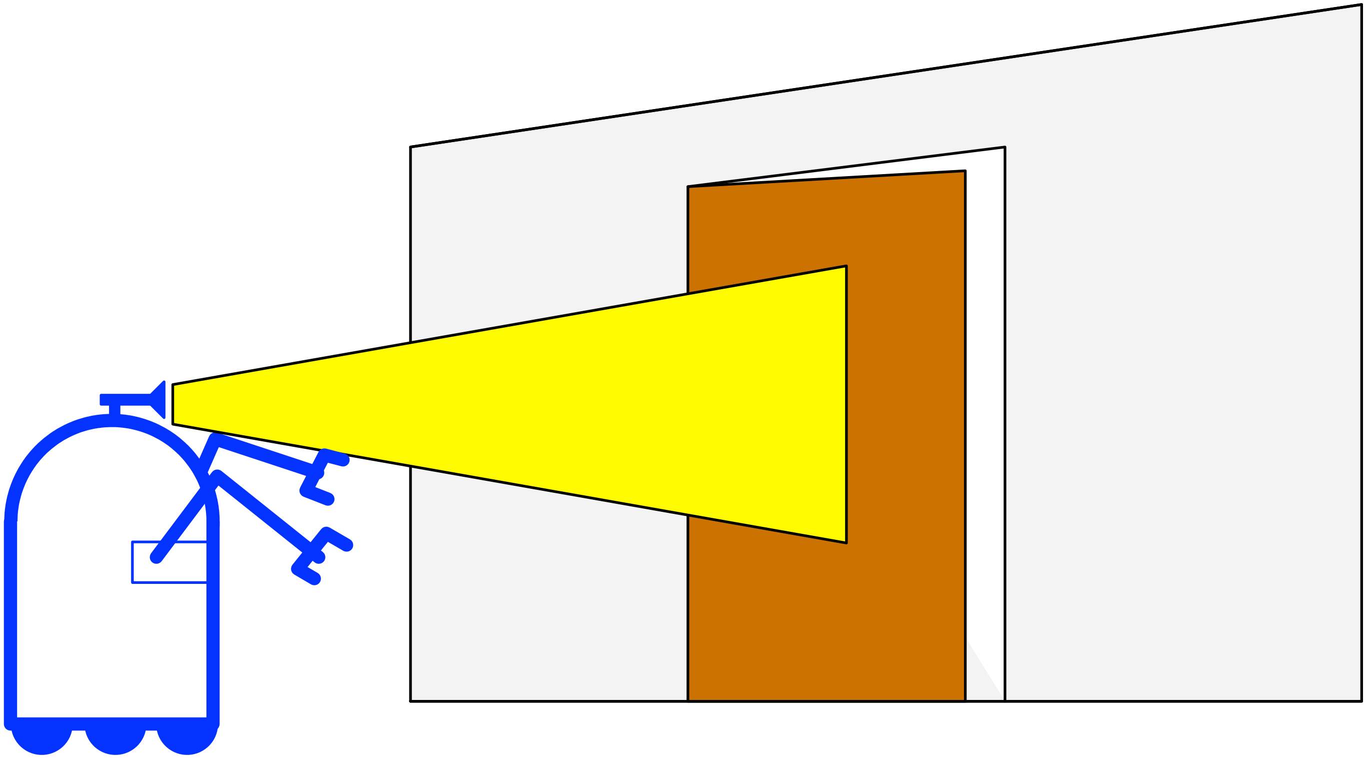
$$E[X] = \int x\ p(x) dx \qquad \text{(continuous case)}$$

Definition 1.7: The ***covariance*** of a random variable X is defined as:

$$Cov[X] = E[X - E[X]]^2 = E[X^2] - E[X]^2$$

Mathematical Formulation of Our Example

We define two binary random variables: z and open, where z is "light on" or "light off". Our question is: What is $p(\text{open} \mid z)$?



Causal vs. Diagnostic Reasoning

- Searching for $p(\text{open} \mid z)$ is called ***diagnostic reasoning***
- Searching for $p(z \mid \text{open})$ is called ***causal reasoning***
- Often causal knowledge is easier to obtain
- Bayes rule allows us to use causal knowledge:

$$
\begin{aligned}
p(\text{open} \mid z) &= \frac{p(z \mid \text{open})p(\text{open})}{p(z)} \\
&= \frac{p(z \mid \text{open})p(\text{open})}{p(z \mid \text{open})p(\text{open}) + p(z \mid \neg\text{open})p(\neg\text{open})}
\end{aligned}
$$

Example with Numbers

Assume we have this ***sensor model:***

$$p(z \mid \text{open}) = 0.6 \qquad p(z \mid \neg\text{open}) = 0.3$$

and: $p(\text{open}) = p(\neg\text{open}) = 0.5$ ***"Prior prob."***

then:

$$\begin{aligned} p(\text{open} \mid z) &= \frac{p(z \mid \text{open})p(\text{open})}{p(z \mid \text{open})p(\text{open}) + p(z \mid \neg\text{open})p(\neg\text{open})} \\ &= \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67 \end{aligned}$$

" z raises the probability that the door is open"

Combining Evidence

Suppose our robot obtains another observation z_2 , where the index is the point in time.

Question: How can we integrate this new information?

Formally, we want to estimate $p(\text{open} \mid z_1, z_2)$.
Using Bayes formula with background knowledge:

$$p(\text{open} \mid z_1, z_2) = \frac{p(z_2 \mid \text{open}, z_1)\, p(\text{open} \mid z_1)}{p(z_2 \mid z_1)}$$

? ?

Markov Assumption

"If we know the state of the door at time $t = 1$ then the measurement z_1 does not give any further information about z_2."

Formally: "z_1 and z_2 are conditional independent given open." This means:

$$p(z_2 \mid \text{open}, z_1) = p(z_2 \mid \text{open})$$

This is called the ***Markov Assumption.***

Example with Numbers

Assume we have a second sensor:

$$p(z_2 \mid \text{open}) = 0.5 \qquad p(z_2 \mid \neg\text{open}) = 0.6$$

$$p(\text{open} \mid z_1) = \tfrac{2}{3}$$ (from above)

Then: $p(\text{open} \mid z_1, z_2) =$

$$\frac{p(z_2 \mid \text{open})p(\text{open} \mid z_1)}{p(z_2 \mid \text{open})p(\text{open} \mid z_1) + p(z_2 \mid \neg\text{open})p(\neg\text{open} \mid z_1)}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + \frac{3}{5} \cdot \frac{1}{3}} = \frac{5}{8} = 0.625$$

“ z_2 lowers the probability that the door is open”

General Form

Measurements: $z_1, \ldots, z_n$

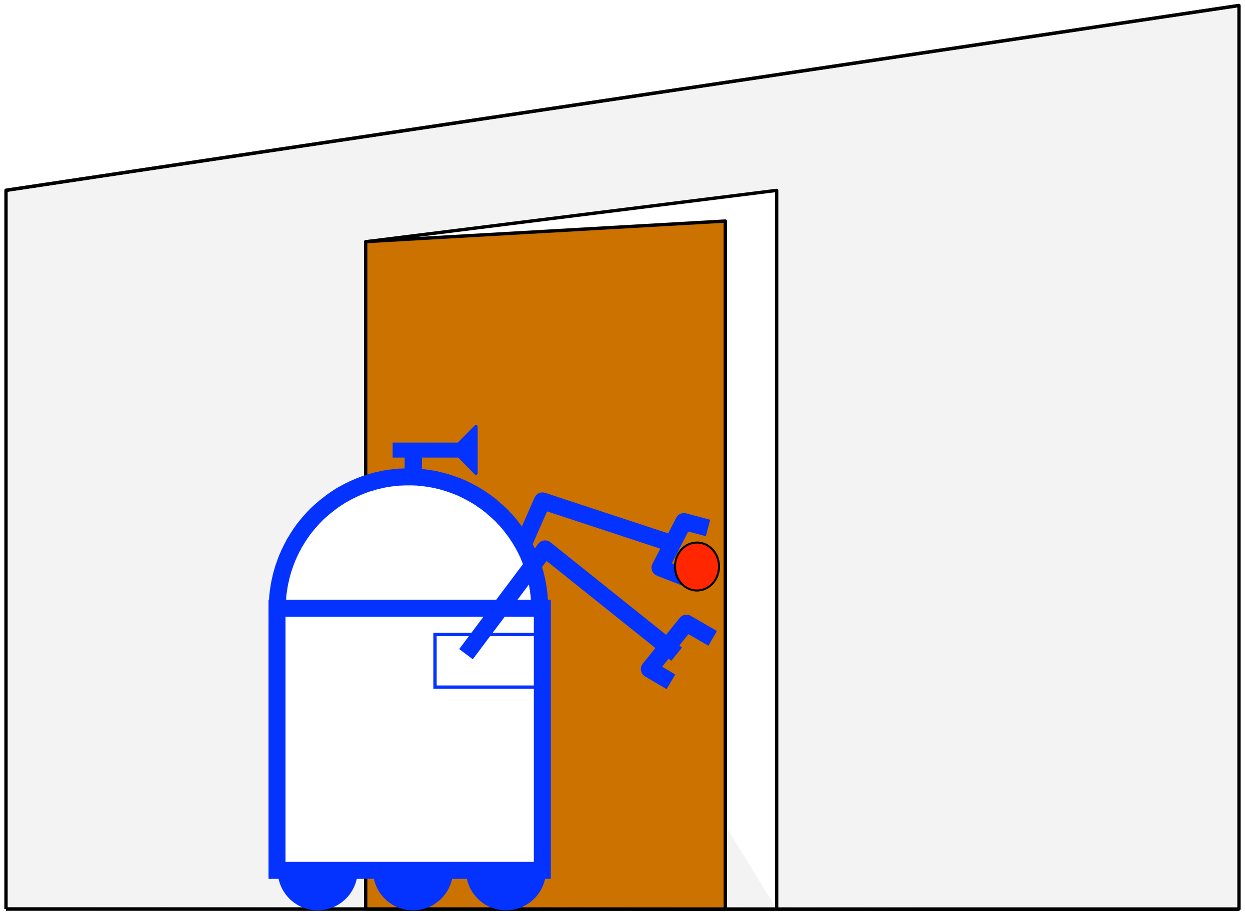
Markov assumption: z_n and $z_1, \ldots, z_{n-1}$ are conditionally independent given the state x.

$$
\begin{aligned}
p(x \mid z_1, \ldots, z_n) &= \frac{p(z_n \mid x) p(x \mid z_1, \ldots, z_{n-1})}{p(z_n \mid z_1, \ldots, z_{n-1})} \\
&= \eta_n \, p(z_n \mid x) p(x \mid z_1, \ldots, z_{n-1}) \\
&= \prod_{i=1}^{n} \eta_i \, p(z_i \mid x) p(x)
\end{aligned}
$$

Recursion

Example: Sensing and Acting

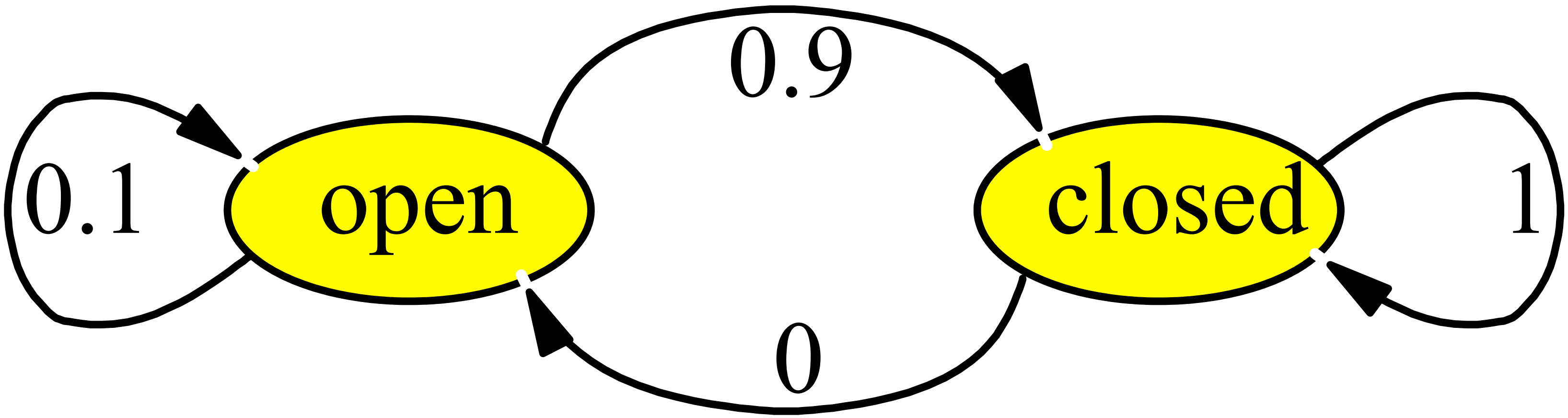
Now the robot ***senses*** the door state and ***acts*** (it opens or closes the door).



State Transitions

The ***outcome*** of an action is modeled as a random variable U where $U = u$ in our case means "state after closing the door".

State transition example:



If the door is open, the action "close door" succeeds in 90% of all cases.

The Outcome of Actions

For a given action u we want to know the probability $p(x \mid u)$. We do this by integrating over all possible previous states x'.

If the state space is discrete:

$$p(x \mid u) = \sum_{x'} p(x \mid u, x')p(x')$$

If the state space is continuous:

$$p(x \mid u) = \int p(x \mid u, x')p(x')dx'$$

Back to the Example

$$
\begin{aligned}
p(\text{open} \mid u) &= \sum_{x'} p(\text{open} \mid u, x')p(x') \\
&= p(\text{open} \mid u, \text{open}')p(\text{open}') + \\
&\quad\; p(\text{open} \mid u, \neg\text{open}')p(\neg\text{open}') \\
&= \frac{1}{10} \cdot \frac{5}{8} + 0 \cdot \frac{3}{8} \\
&= \frac{1}{16} = 0.0625
\end{aligned}
$$

$$
p(\neg\text{open} \mid u) = 1 - p(\text{open} \mid u) = \frac{15}{16} = 0.9375
$$

Sensor Update and Action Update

So far, we learned two different ways to update the system state:

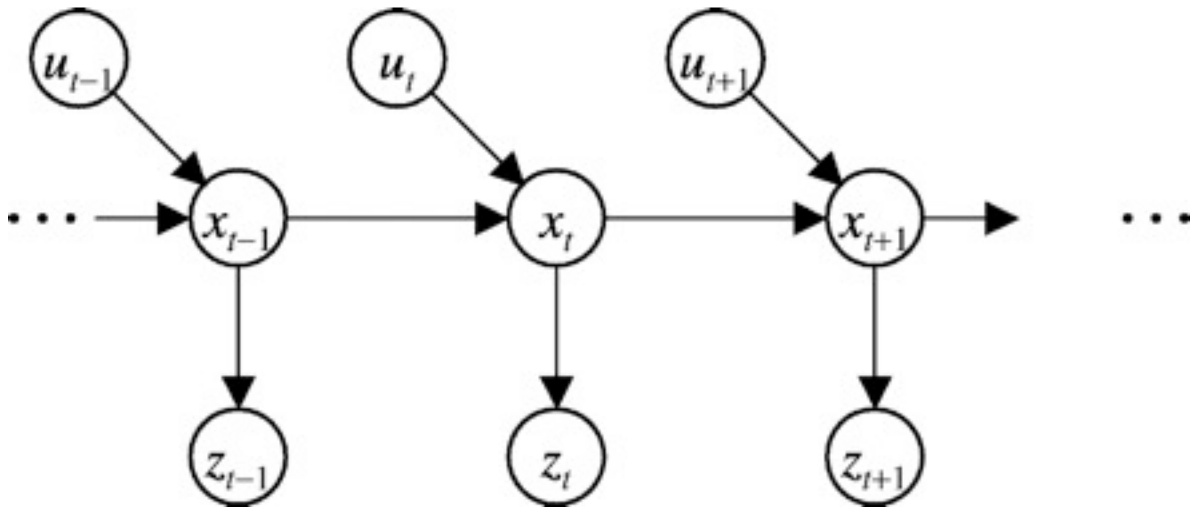
- Sensor update: $p(x \mid z)$
- Action update: $p(x \mid u)$
- Now we want to combine both:

Definition 2.1: Let $D_t = u_1, z_1, \ldots, u_t, z_t$ be a sequence of sensor measurements and actions until time t. Then the ***belief*** of the current state x_t is defined as

$$\mathrm{Bel}(x_t) = p(x_t \mid u_1, z_1, \ldots, u_t, z_t)$$

Graphical Representation

We can describe the overall process using a ***Dynamic Bayes Network:***



This incorporates the following Markov assumptions:

$$p(z_t \mid x_{0:t}, u_{1:t}, z_{1:t}) = p(z_t \mid x_t) \quad \text{(measurement)}$$

$$p(x_t \mid x_{0:t-1}, u_{1:t}, z_{1:t}) = p(x_t \mid x_{t-1}, u_t) \quad \text{(state)}$$

The Overall Bayes Filter

$$\text{Bel}(x_t) = p(x_t \mid u_1, z_1, \ldots, u_t, z_t)$$

(Bayes) $$= \eta \; p(z_t \mid x_t, u_1, z_1, \ldots, u_t) p(x_t \mid u_1, z_1, \ldots, u_t)$$

(Markov) $$= \eta \; p(z_t \mid x_t) p(x_t \mid u_1, z_1, \ldots, u_t)$$

(Tot. prob.) $$= \eta \; p(z_t \mid x_t) \int p(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \, p(x_{t-1} \mid u_1, z_1, \ldots, u_t) dx_{t-1}$$

(Markov) $$= \eta \; p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) p(x_{t-1} \mid u_1, z_1, \ldots, u_t) dx_{t-1}$$

(Markov) $$= \eta \; p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) p(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) dx_{t-1}$$

$$= \eta \; p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) \text{Bel}(x_{t-1}) dx_{t-1}$$

The Bayes Filter Algorithm

$$\text{Bel}(x_t) = \eta \, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) \text{Bel}(x_{t-1}) dx_{t-1}$$

Algorithm Bayes_filter $(\text{Bel}(x), d)$:

1. if d is a sensor measurement z then
2. $\eta = 0$
3. for all x do
4. $\text{Bel}'(x) \leftarrow p(z \mid x)\text{Bel}(x)$
5. $\eta \leftarrow \eta + \text{Bel}'(x)$
6. for all x do $\text{Bel}'(x) \leftarrow \eta^{-1}\text{Bel}'(x)$
7. else if d is an action u then
8. for all x do $\text{Bel}'(x) \leftarrow \int p(x \mid u, x')\text{Bel}(x')dx'$
9. return $\text{Bel}'(x)$

Bayes Filter Variants

$$\text{Bel}(x_t) = \eta \, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) \text{Bel}(x_{t-1}) dx_{t-1}$$

The Bayes filter principle is used in

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

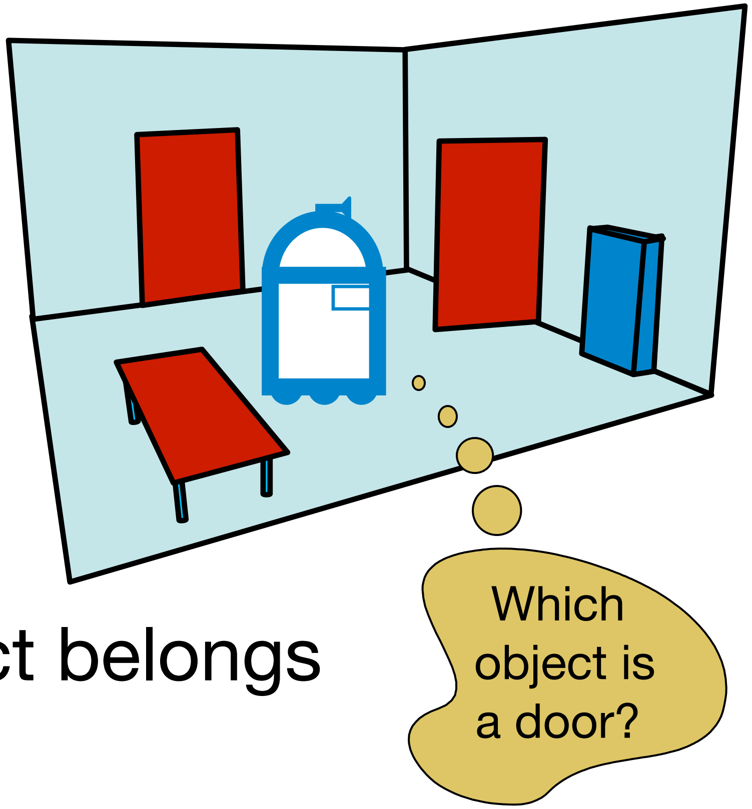
Summary

- ***Probabilistic reasoning*** is necessary to deal with uncertain information, e.g. sensor measurements
- Using ***Bayes rule***, we can do diagnostic reasoning based on causal knowledge
- The outcome of a robot‘s action can be described by a ***state transition diagram***
- Probabilistic state estimation can be done recursively using the ***Bayes filter*** using a sensor and a motion update
- A graphical representation for the state estimation problem is the ***Dynamic Bayes Network***

2. Introduction to Learning

Motivation

- Most objects in the environment can be classified, e.g. with respect to their size, functionality, dynamic properties, etc.
- Robots need to ***interact*** with the objects (move around, manipulate, inspect, etc.) and with humans
- For all these tasks it is necessary that the robot knows to which ***class*** an object belongs



Learning

- A natural way to do object classification is to first **learn** the categories of the objects and then **infer** from the learned data a possible class for a new object.
- The area of **machine learning** deals with the formulization and investigates methods to do the learning automatically.
- Nowadays, machine learning algorithms are more and more used in robotics and computer vision

Mathematical Formulation

Suppose we are given a set $\mathcal{X}$ of objects and a set $\mathcal{Y}$ of object categories (classes). In the learning task we search for a mapping $\varphi : \mathcal{X} \to \mathcal{Y}$ such that **similar** elements in $\mathcal{X}$ are mapped to **similar** elements in $\mathcal{Y}$.

Examples:

- Object classification: chairs, tables, etc.
- Optical character recognition
- Speech recognition

Important problem: Measure of similarity!

Categories of Learning

Learning

Unsupervised Learning

clustering, density estimation

Supervised Learning

learning from a training data set, **inference** on the test data

Reinforcement Learning

no supervision, but a **reward function**

Discriminant Function

no prob. formulation, learns a **function** from objects $\mathcal{X}$ to labels $\mathcal{Y}$.

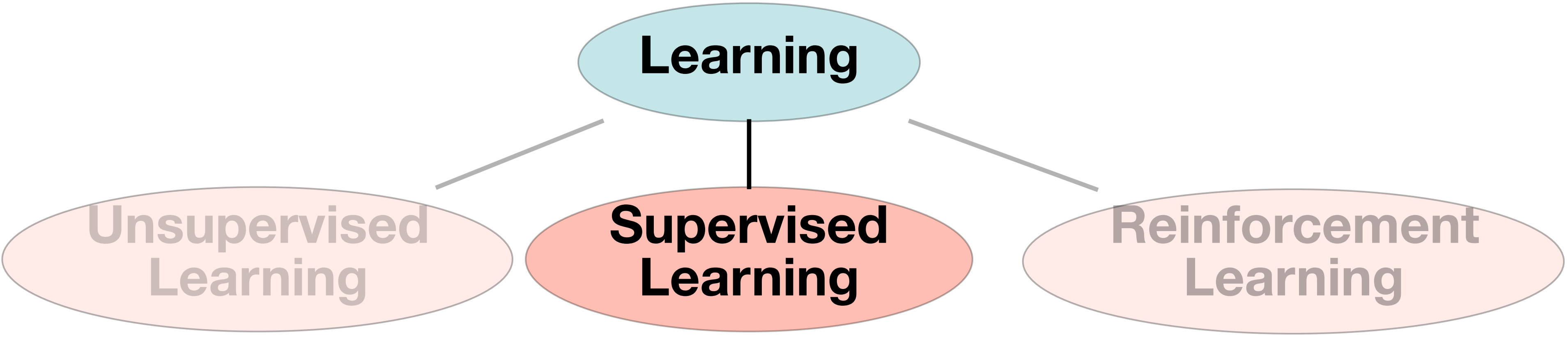
Discriminative Model

estimates the **posterior** $p(y_k \mid \mathbf{x})$ for each class

Generative Model

est. the **likelihoods** $p(\mathbf{x} \mid y_k)$ and use Bayes rule for the post.

Categories of Learning



Unsupervised Learning	Supervised Learning	Reinforcement Learning
clustering, density estimation	**learning** from a training data set, **inference** on the test data	no supervision, but a **reward function**

Supervised Learning is the main topic of this lecture!

Methods used in Computer Vision include:

- Regression
- Conditional Random Fields
- Boosting
- Support Vector Machines
- Gaussian Processes
- Hidden Markov Models

Categories of Learning

Learning

- **Unsupervised Learning**: clustering, density estimation
- **Supervised Learning**: **learning** from a training data set, **inference** on the test data
- **Reinforcement Learning**: no supervision, but a **reward function**

Most Unsupervised Learning methods are based on Clustering.

➡ Will be handled at the end of this semster

Categories of Learning

Learning

Unsupervised Learning	**Supervised Learning**	**Reinforcement Learning**
clustering, density estimation	**learning** from a training data set, **inference** on the test data	no supervision, but a **reward function**

Reinforcement Learning requires an ***action***

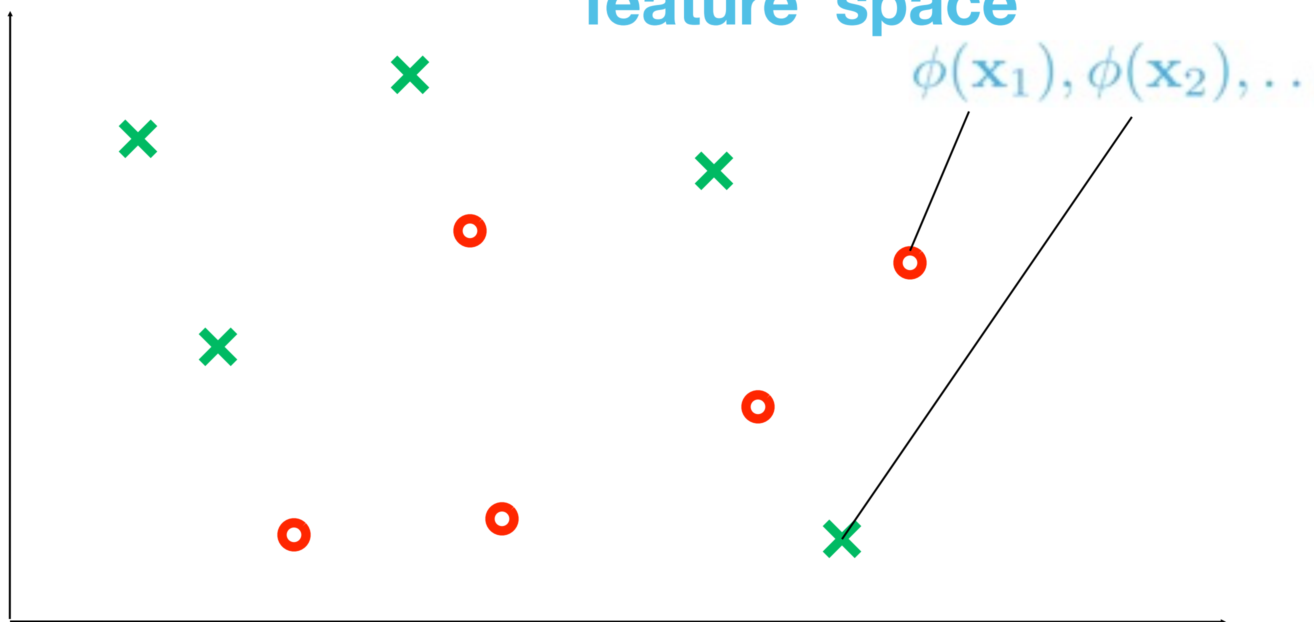
- the reward defines the quality of an action
- mostly used in robotics (e.g. manipulation)
- can be dangerous, actions need to be "tried out"
- not handled in this course

Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \ldots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

1. Training instances in feature space

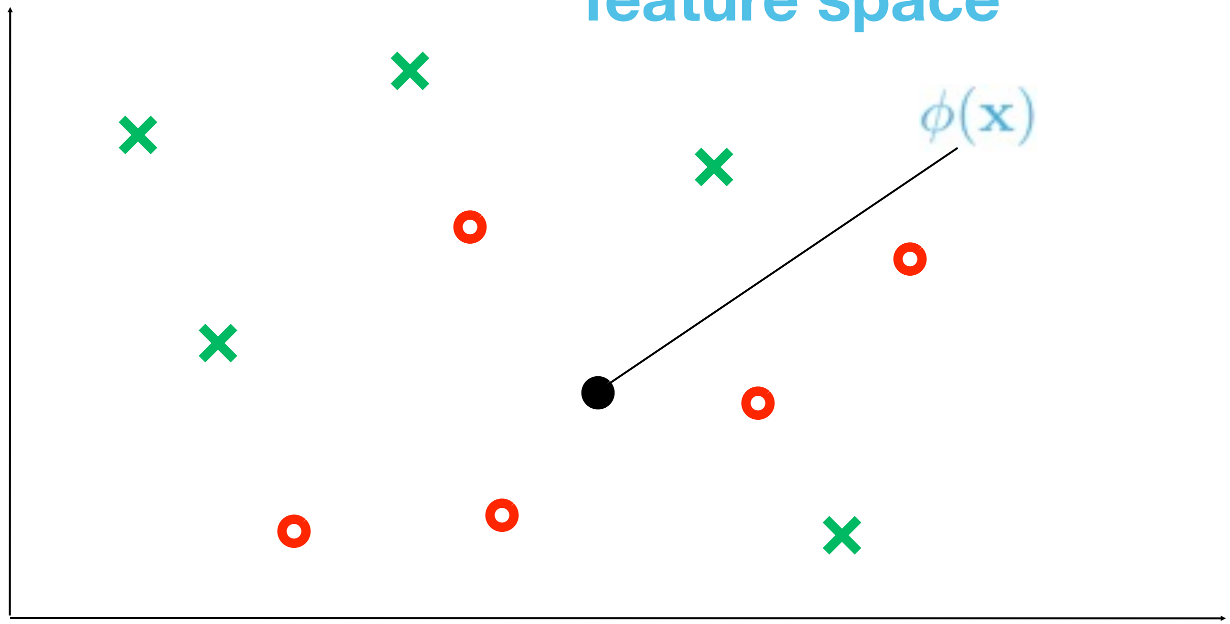


Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \ldots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

2. Map new data point into feature space

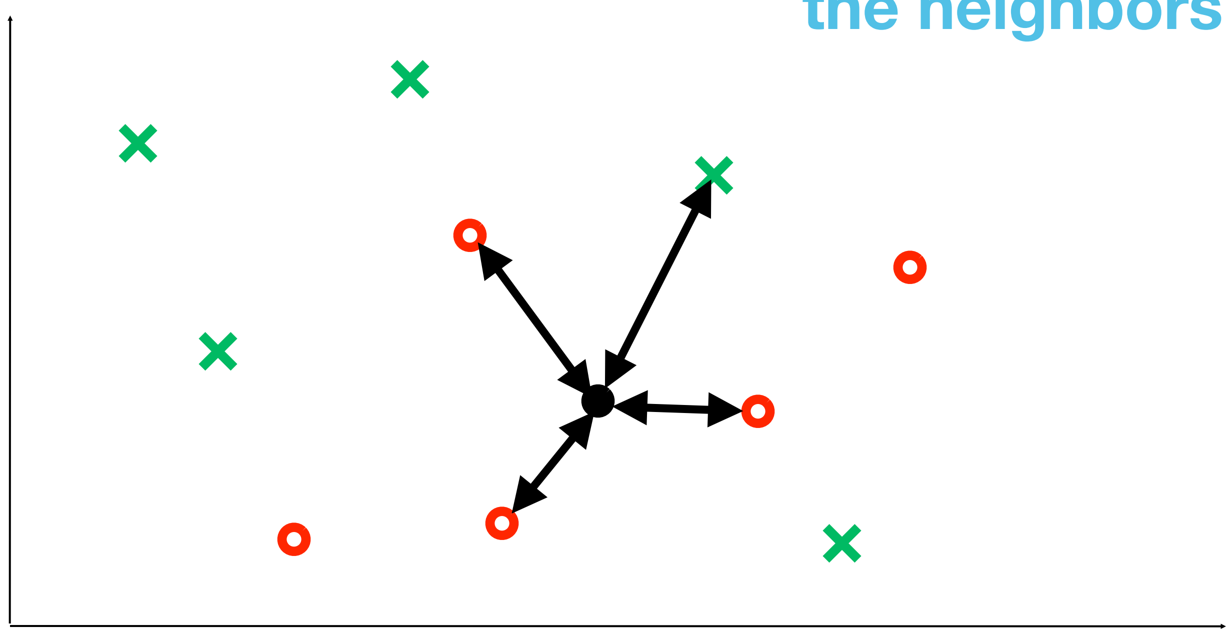


Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \dots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

3. Compute the distances to the neighbors

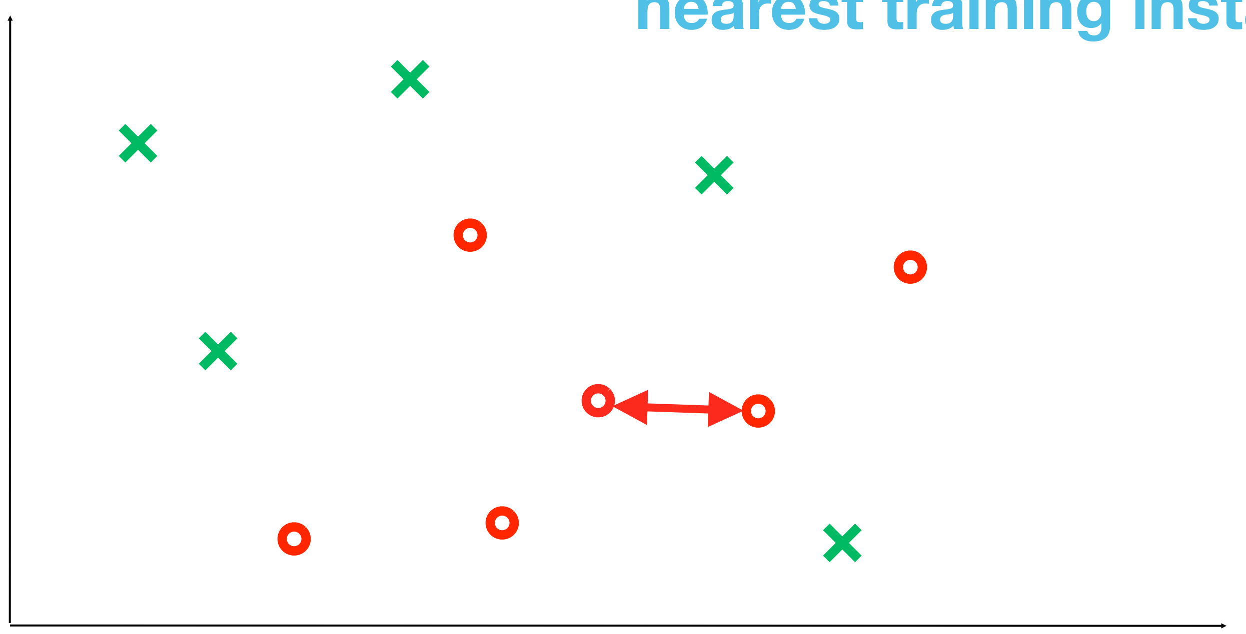


Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \ldots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

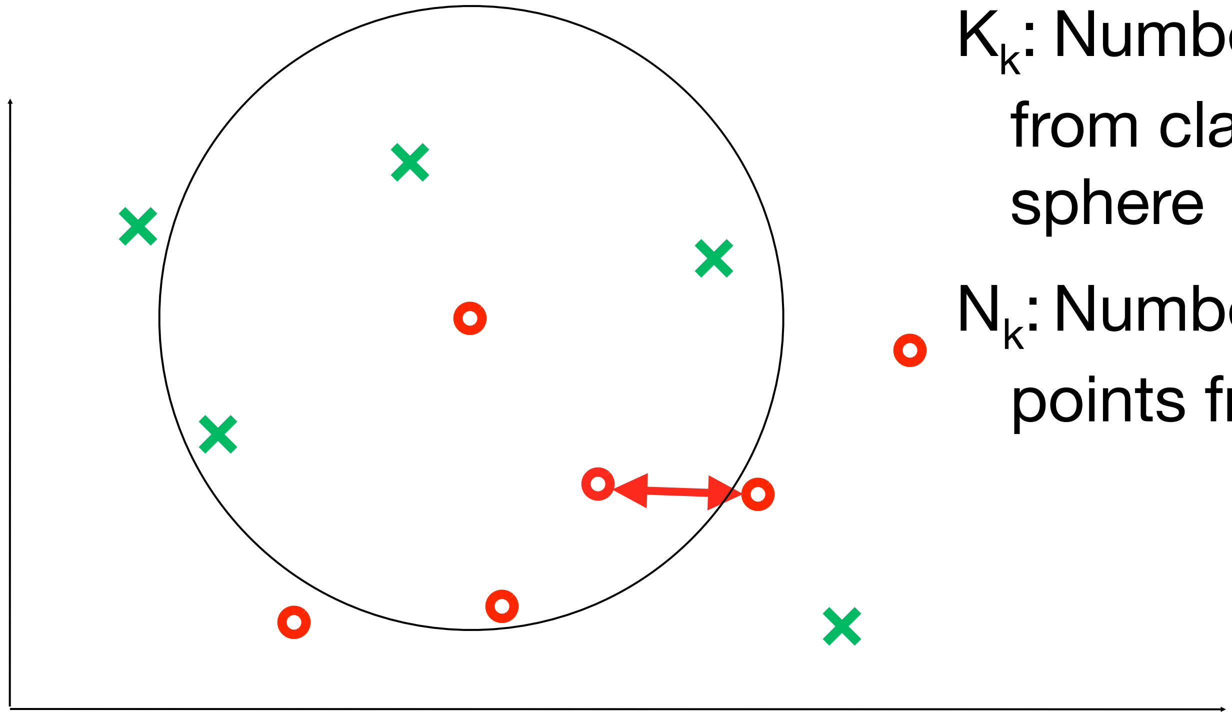
4. Assign the label of the nearest training instance



Generative Model: Example

Nearest-neighbor classification:

- General case: K nearest neighbors
- We consider a sphere around each training instance that has a fixed volume V.



K_k: Number of points from class k inside sphere

N_k: Number of all points from class k

Generative Model: Example

Nearest-neighbor classification:

- General case: K nearest neighbors
- We consider a sphere around each training instance that has a fixed volume V.
- With this we can estimate:

$$p(\mathbf{x} \mid y = k) = \frac{K_k}{N_k V}$$

"likelihood"

- and likewise:

$$p(\mathbf{x}) = \frac{K}{NV}$$

points in sphere

all points

"uncond. prob."

- using Bayes rule:

$$p(y = k \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid y = k)p(y = k)}{p(\mathbf{x})} = \frac{K_k}{K}$$

"posterior"

Generative Model: Example

Nearest-neighbor classification:

- General case: K nearest neighbors

$$p(y = k \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid y = k)p(y = k)}{p(\mathbf{x})} = \frac{K_k}{K}$$

- To classify the new data point $\mathbf{x}$ we compute the posterior for each class k = 1,2,… and assign the label that maximizes the posterior.

$$t := \arg\max_k p(y = k \mid \mathbf{x})$$

Summary

- Learning is a two-step process consisting in a ***training*** and an ***inference*** step
- Learning is useful to extract ***semantic*** information, e.g. about the objects in an environment
- There are three main categories of learning: ***unsupervised***, ***supervised*** and ***reinforcement*** learning
- Supervised learning can be split into ***discriminant function***, ***discriminant model***, and ***generative model*** learning
- An example for a generative model is ***nearest neighbor classification***