Multiple View Geometry: Exercise Sheet 1

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Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let A be a symmetric matrix, and λ_a, λ_b eigenvalues with eigenvectors v_a and v_b. Proof: if v_a and v_b are not orthogonal, it follows: $\lambda_a = \lambda_b$.

   Hint: What can you say about $\langle Av_a, v_b \rangle$?

2. Let $A \in \mathbb{R}^{n \times n}$ with the orthonormal basis of eigenvectors $v_1, \ldots, v_n$ and eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$. Find all vectors x, that minimize the following term:

   $$\min_{||x||=1} x^T A x$$

   How many solutions exist? How can the term be maximized?

   Hint: Use the expression $x = \sum_{i=1}^{n} \alpha_i v_i$ with coefficients $\alpha_i \in \mathbb{R}$ and compute appropriate coefficients!

3. Consider the following sets:

   $$GL(n) := \{A \in \mathbb{R}^{n \times n} |\, det(A) \neq 0\} \subset \mathbb{R}^{n \times n}$$

   $$A(n) := \left\{ L = \begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix} \mid A \in GL(n), b \in \mathbb{R}^n \right\}$$

   $$O(n) := \left\{ R \in GL(n) |\, R^T R = I \right\}$$

   $$E(n) := \left\{ L = \begin{pmatrix} R & b \\ 0 & 1 \end{pmatrix} \mid R \in O(n), b \in \mathbb{R}^n \right\}$$

   $$SO(n) := \left\{ R \in GL(n) |\, R^T R = I, \det(R) = 1 \right\}$$

   $$SE(n) := \left\{ L = \begin{pmatrix} R & b \\ 0 & 1 \end{pmatrix} \mid R \in SO(n), b \in \mathbb{R}^n \right\}.$$

   The following holds: $SE(n) \subset E(n) \subset A(n) \subset GL(n+1)$. Proof:

   (a) $A(n)$ is a group with respect to multiplication.

   (b) $E(n)$ is a subgroup of $A(n)$ with respect to multiplication.

   (c) $SE(n)$ is a subgroup of $E(n)$ with respect to multiplication.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Let

$$A = \begin{pmatrix} 2 & 6 & 7 & 8 & 11 \\ 6 & 9 & 6 & 8 & 6 \\ 7 & 6 & 1 & 7 & 9 \\ 8 & 8 & 7 & 12 & 7 \\ 11 & 6 & 9 & 7 & 7 \end{pmatrix}$$

   (a) Indicate at least two possibilities how to assert via matlab if the matrix is invertible.

   (b) Compute the eigenvalue decomposition $A = P\Lambda P^{-1}$ with diagonal matrix Λ. Compute $A - P\Lambda P^{-1}$. What do you observe?

   (c) Compute the Singular Value Decomposition (SVD) $A = U\Sigma V^\top$ with diagonal matrix Σ. Compute $A - U\Sigma V^\top$. What do you observe?

Matlab-Tutorials:
`http://www.math.utah.edu/lab/ms/matlab/matlab.html`
`http://www.math.ufl.edu/help/matlab-tutorial/`