Multiple View Geometry: Exercise Sheet 2

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`http://vision.in.tum.de/teaching/ss2013/mvg2013`

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Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Indicate the matrices $M \in SE(3)$ representing the following transformations:
   (a) Translation by the vector $T = (t_x\ t_y\ t_z)^\top$.
   (b) Rotation by the rotation matrix R.
   (c) Rotation by R followed by the translation T.
   (d) Translation by T followed by the rotation R.

2. Let $M_1, M_2 \in \mathbb{R}^{3\times3}$. Please prove the following:

$$\begin{array}{ccc} \mathbf{x}^T M_1 \mathbf{x} = \mathbf{x}^T M_2 \mathbf{x} & \text{iff} & M_1 - M_2 \text{ is skew-symmetric} \\ \text{for all} \quad \mathbf{x} \in \mathbb{R}^3 & & (\text{i.e. } M_1 - M_2 \in so(3)) \end{array}$$

   Info: The group $SO(3)$ is called a **Lie group**.
   The space $so(3) = \{\hat{\omega} \mid \omega \in \mathbb{R}^3\}$ of skew-symmetric matrices is called its **Lie algebra**.

3. Let $A \in \mathbb{R}^{m\times n}$. Prove that $\text{kernel}(A) = \text{kernel}(A^\top A)$.

   Hint: Consider a) $x \in \text{kernel}(A) \Rightarrow x \in \text{kernel}(A^\top A)$
   and b) $x \in \text{kernel}(A^\top A) \Rightarrow x \in \text{kernel}(A)$.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Download the package `mvg_exerciseSheet_02.zip` and use `openOFF.m` to load the 3D model `model.off`.

2. Write a function that rotates the model around its center (i.e. the mean of its vertices) for given rotation angles α, β and γ around the x-, y- and z-axis. Use homogeneous coordinates and describe the overall transformation by a single matrix. The rotation matrices around the respective axes are as follows:

   rotation matrix (x-axis) rotation matrix (y-axis) rotation matrix (z-axis)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix} \quad \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \quad \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. Rotate the model first 5 degrees around the x-axis and then 25 degrees around the z-axis. Now start again by doing the same rotation around the z-axis first followed by the x-axis rotation. What do you observe?

4. Perform a translation in addition to the rotation. Find a suitable matrix from $SE(3)$ for this purpose and add it to your function from 2. Translate the model by the vector $(0.5 \;\; 0.2 \;\; 0.1)^\top$.

Matlab-Tutorials:

`http://www.math.utah.edu/lab/ms/matlab/matlab.html`
`http://www.math.ufl.edu/help/matlab-tutorial/`