# Multiple View Geometry: Exercise Sheet 3

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## Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $R \in \mathrm{SO}(3)$ be a diagonalizable $3 \times 3$-matrix with eigenvalues $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}$.

   (a) Show that $|\lambda_i| = 1$ holds for all eigenvalues $\lambda_i$.

   (b) Show that for an eigenvalue $\lambda$ its corresponding complex conjugate $\bar{\lambda}$ is also an eigenvalue.

   (c) Show that $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 1$ holds.

   (d) Show that one of the eigenvalues is 1. What does this mean for its corresponding eigenvector?

   **Hint:** *For* $z \in \mathbb{C}$: $z\bar{z} = (a+ib)(a-ib) = a^2 + b^2 = |z|^2$

2. Consider a vector $\omega \in \mathbb{R}^3$ with $\|\omega\| = 1$ and its corresponding skew-symmetric matrix $\hat{\omega}$.

   (a) Show that $\hat{\omega}^2 = \omega\omega^\top - I$ and $\hat{\omega}^3 = -\hat{\omega}$.

   (b) Following the result of (a), find simple rules for the calculation of $\hat{\omega}^n$ and proof your result. Distinguish between odd and even numbers $n$.

   (c) Derive the Rodrigues' formula for a skew-symmetric matrix $\hat{\omega}$ corresponding to an arbitrary vector $\omega \in \mathbb{R}^3$ (i.e. $\|\omega\|$ does not have to be equal to 1):

$$e^{\hat{\omega}} = I + \frac{\hat{\omega}}{\|\omega\|}\sin(\|\omega\|) + \frac{\hat{\omega}^2}{\|\omega\|^2}(1 - \cos(\|\omega\|))$$

3. Consider a continuous family of rigid body transformations:

$$g(t) = \begin{pmatrix} R(t) & T(t) \\ 0 & 1 \end{pmatrix} \in SE(3) \subset \mathbb{R}^{4\times4}$$

   (a) Calculate $\dot{g}(t)$ and $g^{-1}(t)$

   (b) Calculate the twist $\dot{g}(t) \cdot g^{-1}(t)$

## Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. (a) Write a function which takes a vector $w \in \mathbb{R}^3$ as input and returns its corresponding element $R = e^{\hat{w}} \in SO(3) \subset \mathbb{R}^{3\times3}$ from the Lie group. Hence, the function will be a concatenation of the hat operator $\hat{}: \mathbb{R}^3 \to so(3)$ and the exponential mapping.

   (b) Implement another function which performs the corresponding inverse transformation and test the two functions on some examples.

   (c) Implement similar functions which calculate the transformation for twists.
   I.e. from $\xi \in \mathbb{R}^6$ to $e^{\hat{\xi}} \in SE(3) \subset \mathbb{R}^{4x4}$ and the other way around.

2. Let $R \in SO(3)$ be a rotation matrix generated by a rotation about a unit vector $\omega$ by $\theta$ radians that satisfies $R = \exp(\hat{\omega}\theta)$. Suppose $R$ is given as

$$R = \begin{pmatrix} 0.1729 & -0.1468 & 0.9739 \\ 0.9739 & 0.1729 & -0.1468 \\ -0.1468 & 0.9739 & 0.1729 \end{pmatrix} \quad (1)$$

   (a) Use the formula given in the lecture to compute the rotation axis and the associated angle.

   (b) Use the Matlab function `eig` to compute the eigenvalues and eigenvectors of the rotation matrix $R$. What is the eigenvector associated with the unit eigenvalue? Give its form and explain its meaning.

3. In Part II: Practical Exercises of Exercise Sheet 2 you have already implemented a function which rotates a model around a given point.
   Extend this code by a function `W = rotate_around_ray(V,a,b,angle)` which rotates a model – given as vertex list (matrix) `V` – around a ray defined by the offset vector `a` and direction vector `b`. The rotation angle is given by `angle`. The function output is the transformed vertex list `W`.
   Make use of the Rodrigues' formula and check whether the created matrix is orthogonal. Test your method with the model given in `model.off`. For instance, rotate the model by 50 degrees around the ray through point `a = (0.5 0 0.45)`$^\top$ and direction `b = (0 1 0)`$^\top$.

**Matlab-Tutorials:**
http://www.math.utah.edu/lab/ms/matlab/matlab.html
http://www.math.ufl.edu/help/matlab-tutorial/