Multiple View Geometry: Exercise Sheet 5

Prof. Dr. Daniel Cremers, Julia Bergbauer, TU Munich

http://vision.in.tum.de/teaching/ss2013/mvg2013

Exercise: June 17th, 2013

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. The essential matrix $E = \hat{T}R$ has the singular value decomposition $E = U\Sigma V^T$. Let $R_Z\left(\pm\frac{\pi}{2}\right)$ be the rotation by $\pm\frac{\pi}{2}$ around the z-axis.

   Show the following properties:

   (a) $\hat{T} \in so(3)$ (i.e. $\hat{T}$ is a skew-symmetric matrix)

   (b) $R \in \mathrm{SO}(3)$ (i.e. R is a rotation matrix)

   Hint: Use the equalities: $\hat{T} = UR_Z\left(\pm\frac{\pi}{2}\right)\Sigma\, U^\top$ and $R = UR_Z\left(\pm\frac{\pi}{2}\right)^\top V^\top$.

2. Consider the matrices $E = \hat{T}R$ and $H = R + Tu^\top$ with $R \in \mathbb{R}^{3\times 3}$ and $T, u \in \mathbb{R}^3$.
   Show that the following holds:

   (a) $E = \hat{T}H$

   (b) $H^\top E + E^\top H = 0$

3. Let $F \in \mathbb{R}^{3\times 3}$ be the fundamental matrix for the cameras C_1 and C_2. Show that the following holds for the epipoles e_1 and e_2:

$$Fe_1 = 0 \qquad \text{and} \qquad e_2^\top F = 0$$

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

... I will complete this practical exercise during the weekend.