



# Multiple View Geometry: Solution Exercise Sheet 6

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<http://vision.in.tum.de/teaching/ss2013/mvg2013>

## Part I: Theory

1.

$$\begin{aligned} e_2^\top F &= 0 && \text{(Exercise Sheet 5, Nr. 3)} \\ \Rightarrow F^\top e_2 &= 0 \\ \Rightarrow (\hat{T}R)^\top e_2 &= 0 \\ \Rightarrow R^\top \hat{T}^\top e_2 &= 0 \\ \Rightarrow R^\top (-T \times e_2) &= 0 && \text{(because } \hat{T} \text{ is skew-symmetric)} \\ \Rightarrow -T \times e_2 &= 0 \\ \Rightarrow e_2 &\sim T \end{aligned}$$

$$\begin{aligned} Fe_1 &= 0 \\ \Rightarrow \hat{T}Re_1 &= 0 \\ \Rightarrow T \times Re_1 &= 0 \\ \Rightarrow T &\sim Re_1 \\ \Rightarrow e_1 &\sim R^\top T \\ \Rightarrow e_1 &\sim R^\top e_2 \end{aligned}$$

2. (a)  $l$  is coimage of  $L$ , and therefore  $l$  is normal vector to the plane that is determined by the camera position and  $L$ .

$$\begin{aligned} \Rightarrow l^\top x_1 &= 0 \\ \Rightarrow l^\top x_2 &= 0. \\ \Rightarrow l &\sim x_1 \times x_2 = \hat{x}_1 x_2. \end{aligned}$$

$l_1$  and  $l_2$  are normal vectors to the planes through camera position and  $L_1, L_2$  respectively.

$$\begin{aligned} \Rightarrow l_1^\top x &= 0 \\ \Rightarrow l_2^\top x &= 0 \\ \Rightarrow x &\sim l_1 \times l_2 = \hat{l}_1 l_2. \end{aligned}$$

- (b) i.  $l_1 \sim \hat{x}u$  :

$$\begin{aligned} x \text{ is in the preimage of } L_1. &\Rightarrow l_1^\top x = 0. \\ \exists \text{ point } u \neq p \text{ in } L_1. &\Rightarrow l_1^\top u = 0 \\ \Rightarrow l_1 &\sim \hat{x}u. \end{aligned}$$

- ii.  $l_2 \sim \hat{x}v$  : analog to i.

- iii.  $x_1 \sim \hat{l}r$  :

$$\begin{aligned} x_1 \text{ is in the preimage of } L. &\Rightarrow x_1^\top l = 0 \\ \exists \text{ a line } L' \text{ through } p_1 \text{ with coimage } r \neq l. &\Rightarrow x_1^\top r = 0. \\ \Rightarrow x_1 &\sim \hat{l}r. \end{aligned}$$

- iv.  $x_2 \sim \hat{l}s$  : analog to iii.

$$3. \quad \text{rank} \begin{pmatrix} \hat{x}_1 \Pi_1 \\ \hat{x}_2 \Pi_2 \end{pmatrix} \leq 3$$

$$\Rightarrow \exists X \in \mathbb{R}^4 \setminus \{0\} \text{ with } \begin{pmatrix} \hat{x}_1 \Pi_1 \\ \hat{x}_2 \Pi_2 \end{pmatrix} X = 0.$$

$$\Rightarrow \hat{x}_1 \Pi_1 X = 0 \quad \wedge \quad \hat{x}_2 \Pi_2 X = 0,$$

$$\Rightarrow x_1 \times \Pi_1 X = 0 \quad \wedge \quad x_2 \times \Pi_2 X = 0.$$

$\Rightarrow x_1$  and  $\Pi_1 X$  are linearly dependent; and  $x_2$  and  $\Pi_2 X$  are linearly dependent.

$$\Rightarrow \exists \lambda_1, \lambda_2 \in \mathbb{R} \text{ with } \Pi_1 X = \lambda_1 x_1 \quad \wedge \quad \Pi_2 X = \lambda_2 x_2$$

$\Rightarrow x_1$  and  $x_2$  are projections of  $X$ .

$$4. \quad \exists \lambda \in \mathbb{R} : [R', T'] = \lambda [R, T] H = \lambda [R, T] \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} = \lambda [R + T v^\top, T v_4]$$

$$\begin{aligned} E' &= \hat{T}' R' \\ &= (\widehat{\lambda v_4 \hat{T}}) \cdot (\lambda (R + T v^\top)) \\ &= \lambda^2 v_4 \hat{T} (R + T v^\top) \\ &= \lambda^2 v_4 \hat{T} R + \lambda^2 v_4 \underbrace{\hat{T} T}_{=0} v^\top \\ &= \lambda^2 v_4 \hat{T} R \\ &= \lambda^2 v_4 E \quad \text{with } \lambda^2 v_4 \in \mathbb{R} \end{aligned}$$

$$\Rightarrow E' \sim E$$