Multiple View Geometry: Solution Exercise Sheet 7

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Part I: Theory

1. (a) Euler-Lagrange equation for E_1:

$$\begin{aligned}\frac{dE}{du} &= \frac{\partial \mathcal{L}}{\partial u} - \text{div}\frac{\partial \mathcal{L}}{\partial \nabla u}, \qquad \text{with } \mathcal{L}(u,\nabla u) = \frac{\lambda}{2}(u-f)^2 + \frac{1}{2}|\nabla u|^2 \\ &= \frac{\lambda}{2}\cdot 2(u-f) - \text{div}(\frac{1}{2}\cdot 2\cdot \nabla u) \\ &= \lambda(u-f) - \text{div}(\nabla u)\end{aligned}$$

Gradient descent update step for E_1:

$$\begin{aligned}u^{t+1} &= u^t - \text{ dt}\cdot\frac{dE}{du^t} \\ &= u^t + \text{ dt }(\lambda(f-u^t) + \text{div}(\nabla u^t)), \quad \text{with time step dt.}\end{aligned}$$

$$\text{div}(\nabla u) = \text{div}\begin{pmatrix} u_x \\ u_y \end{pmatrix} = u_{xx} + u_{yy} \quad \text{with second derivatives } u_{xx} \text{ and } u_{yy}.$$

(b)

$$\begin{aligned}\nabla u &= \begin{pmatrix} u_x \\ u_y \end{pmatrix} \\ |\nabla u| &= \sqrt{u_x^2 + u_y^2} \\ |\nabla u|_\epsilon &= \sqrt{u_x^2 + u_y^2 + \epsilon^2} = \sqrt{|\nabla u|^2 + \epsilon^2} = \left(|\nabla u|^2 + \epsilon^2\right)^{\frac{1}{2}}\end{aligned}$$

Euler-Lagrange equation for E_2:

$$\begin{aligned}\frac{dE}{du} &= \frac{\partial \mathcal{L}}{\partial u} - \text{div}\frac{\partial \mathcal{L}}{\partial \nabla u}, \qquad \text{with } \mathcal{L}(u,\nabla u) = \frac{\lambda}{2}(u-f)^2 + |\nabla u|_\epsilon \\ &= \frac{\lambda}{2}\cdot 2(u-f) - \text{div}\left(\tfrac{1}{2}\left(|\nabla u|^2 + \epsilon^2\right)^{-\frac{1}{2}}\cdot 2\cdot\nabla u\right) \\ &= \lambda(u-f) - \text{div}\left(\frac{\nabla u}{(|\nabla u|^2 + \epsilon^2)^{\frac{1}{2}}}\right) \\ &= \lambda(u-f) - \text{div}\left(\frac{\nabla u}{|\nabla u|_\epsilon}\right)\end{aligned}$$

Gradient descent update step for E_2:

$$
\begin{aligned}
u^{t+1} &= u^t - \mathrm{dt} \cdot \frac{dE}{du^t} \\
&= u^t + \mathrm{dt} \left(\lambda(f - u^t) + \operatorname{div}\left(\frac{\nabla u^t}{|\nabla u^t|_\epsilon} \right) \right), \quad \text{with time step dt.}
\end{aligned}
$$