

Analysis of Three-Dimensional Shapes

(IN2238, TU München, Summer 2014)

Dr. Emanuele Rodolà

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Room 02.09.058, Informatik IX

07.04.2014

Computer Vision Group

4 Post-docs

14 PhD students

Master and bachelor students welcome!



Prof. Dr. Daniel Cremers



Image-based 3D
Reconstruction



Optical Flow
Estimation



Shape Analysis



Quadrocopter



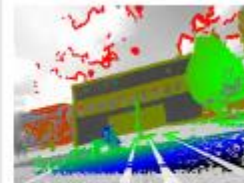
RGB-D Sensors
(Kinect)



Image Segmentation



Convex Relaxation
Methods



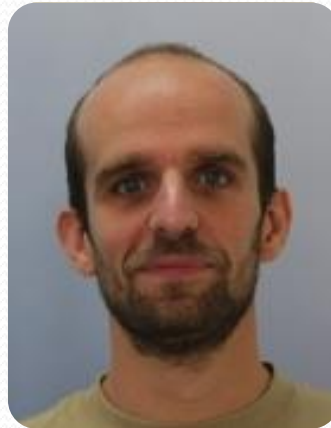
Semi-Dense SLAM for
Monocular Cameras

Formalities

- **Who?**



Dr. Emanuele Rodolà



Thomas Windheuser



Matthias Vestner

TA

- **Where?** Room 02.09.023, Informatik IX
- **When?** Mondays 10:00-12:00 *lecture*
Tuesdays 14:00-15:00 *exercises*

Other formalities

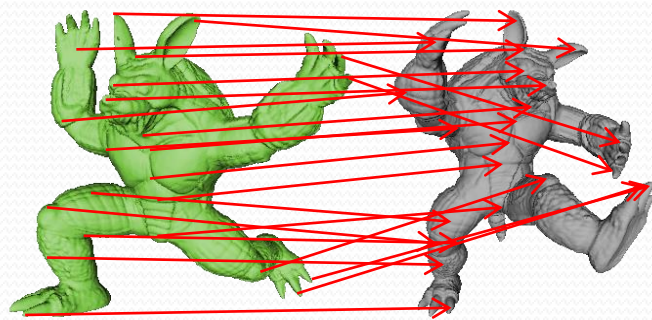
- **Mathematical problems**
- **Programming exercises** (Matlab, C++)
- **Final exam** (written or oral or both)
- **Office hours:** send me an e-mail to set up a meeting

- **Textbook** (just a suggestion)

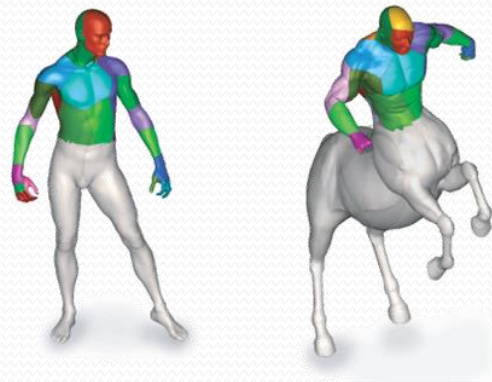
Bronstein, Bronstein, Kimmel. *Numerical geometry of non-rigid shapes*, Springer 2008

- **Scientific papers** will be suggested throughout the lecture

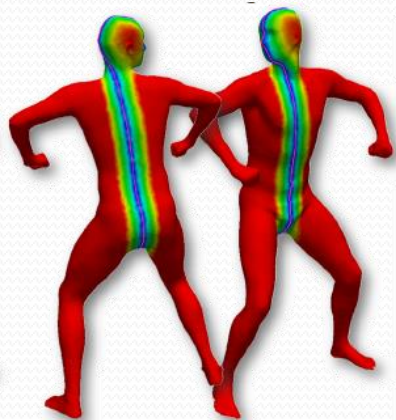
Topics



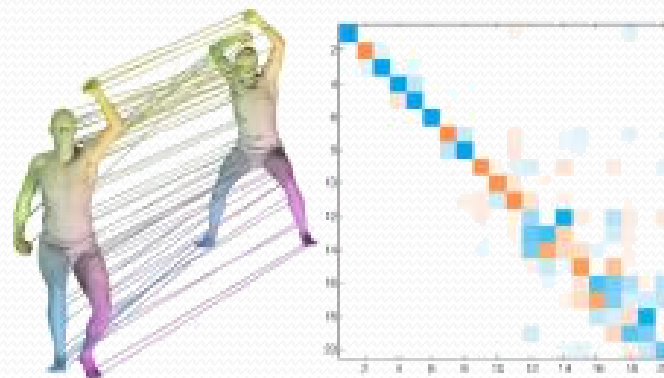
Correspondence



Partial similarity

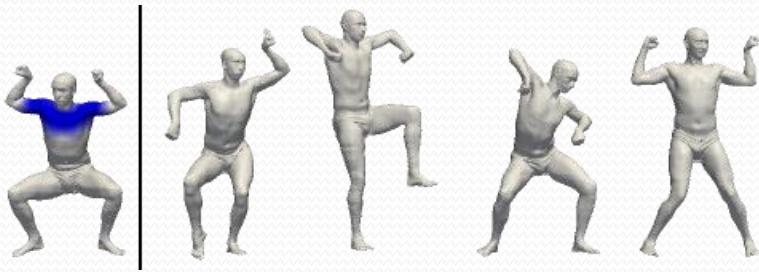


Symmetry



Representation

Topics



Analysis of shape collections



Feature detection

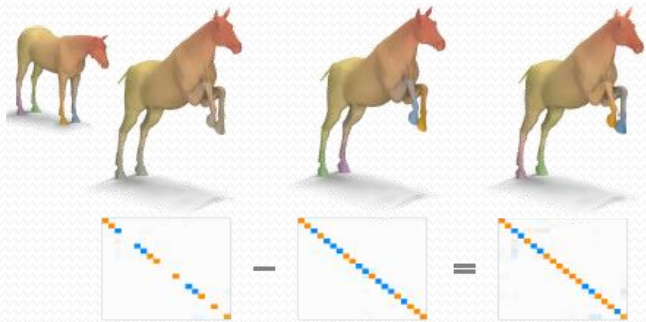


Segmentation

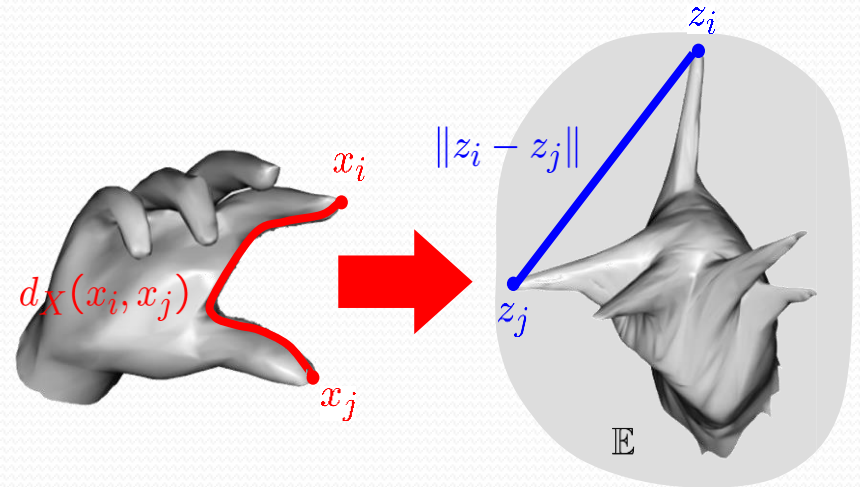


Description

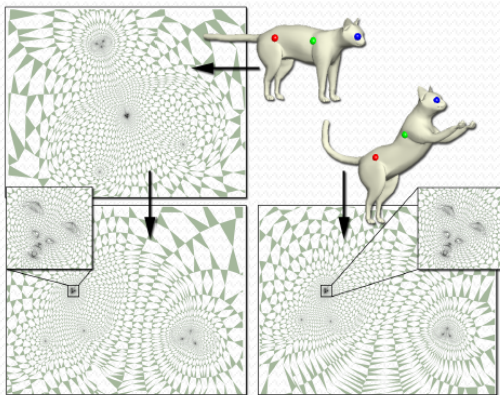
Tools



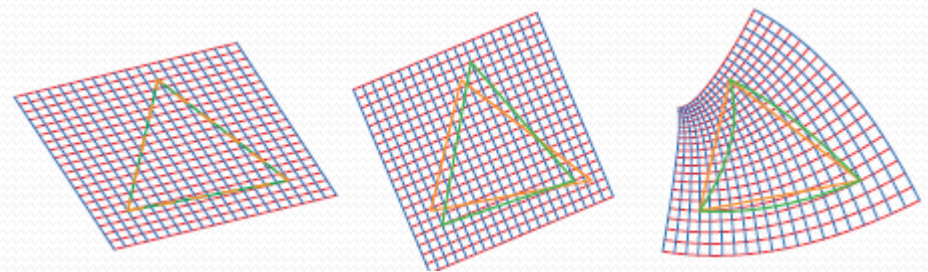
Linear algebra



Metric spaces

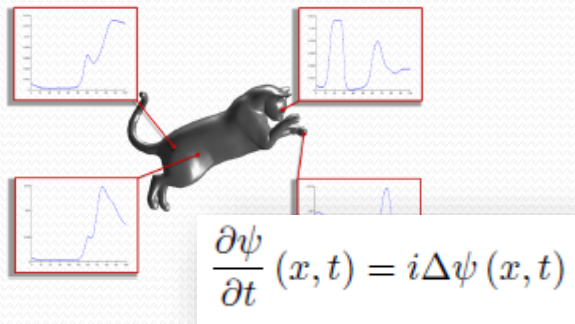


Conformal geometry

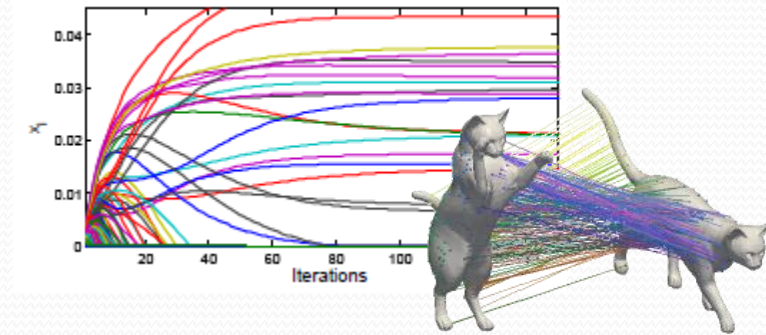


Differential geometry

Tools

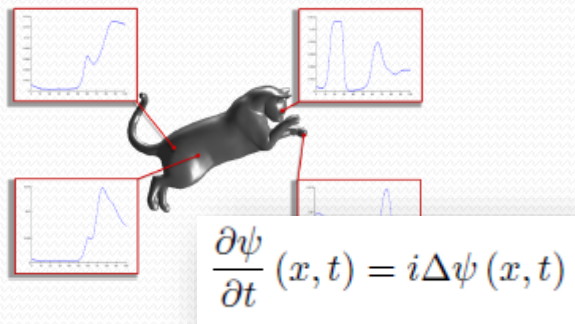


PDEs

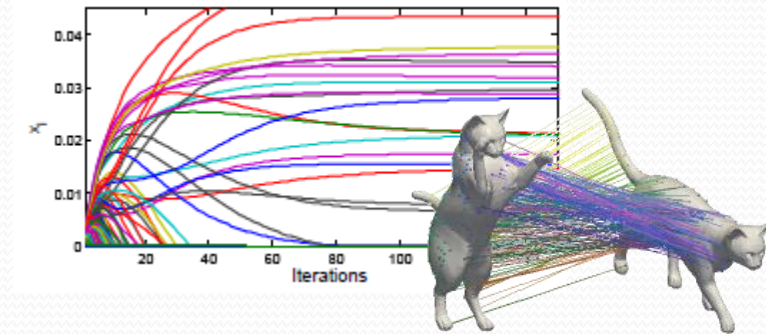


Optimization

Tools



PDEs



Optimization

Good news:

90% of the time we will be able to have a visualization of what we are doing!

Seminar

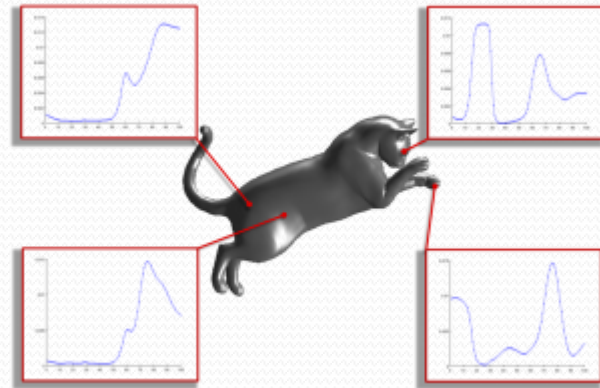
Recent Advances in the Analysis of 3D Shapes (IN2107)

When? Wednesdays, 14:00

Where? 02.09.023

First meeting: Apr 16, 14:00

Topic: Laplace-Beltrami Operator on manifolds



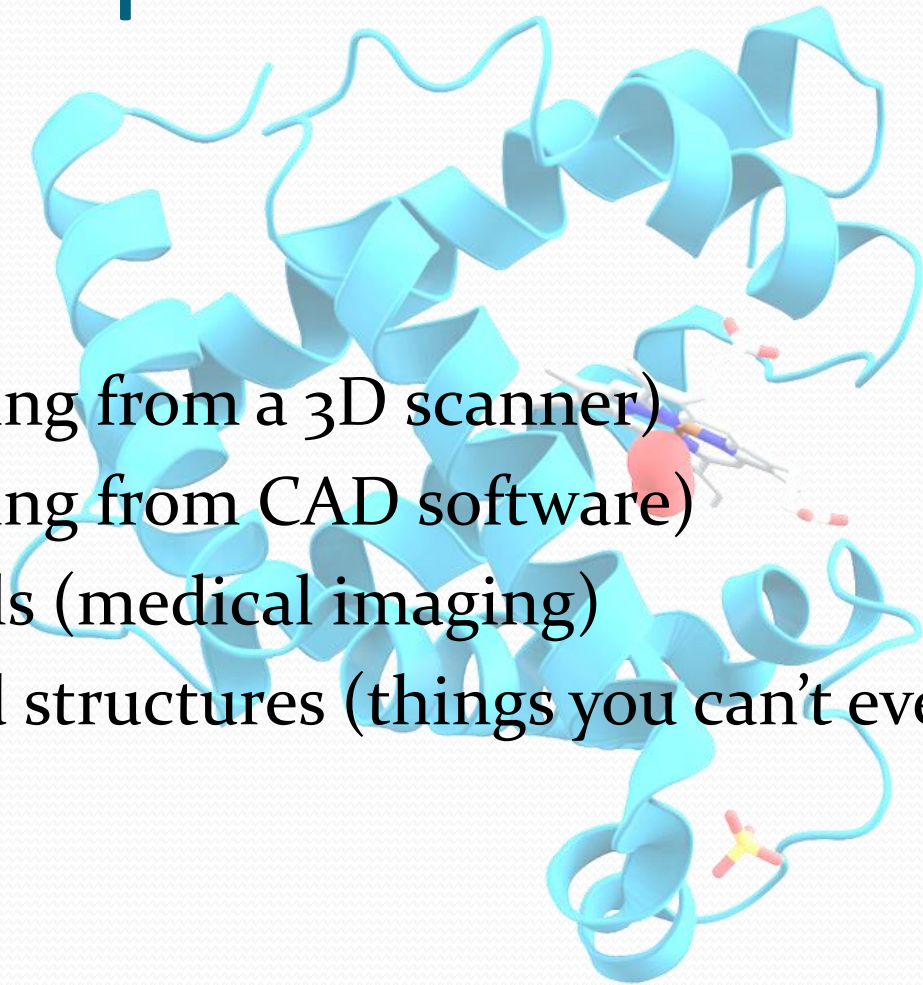
What is a shape?

“There can be no such thing as a mathematical theory of shape. The very notion of shape belongs to the natural sciences.”

J. Koenderink. *Solid Shape*. MIT Press 1990.

What is a shape?

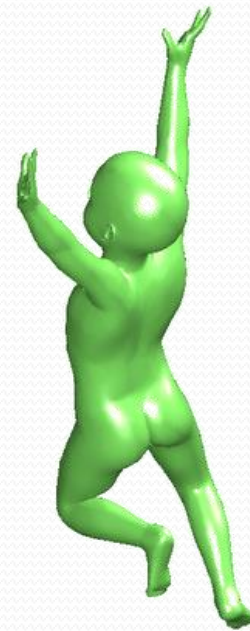
- Proteins
- Molecules
- 2D Images
- **3D models** (coming from a 3D scanner)
- **3D models** (coming from CAD software)
- Volumetric models (medical imaging)
- More complicated structures (things you can't even visualize)



Shapes vs images: domain

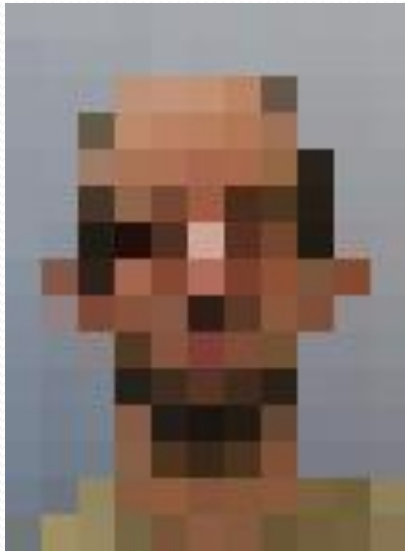


Euclidean
(flat)



Non-Euclidean
(curved)

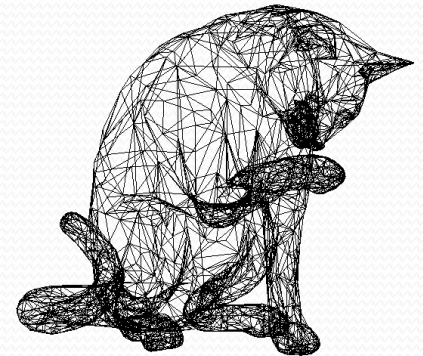
Shapes vs images: representation



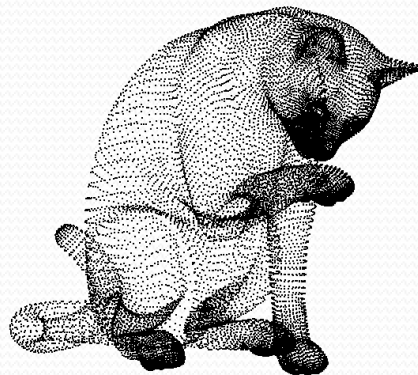
Array of pixels
(uniform grid)



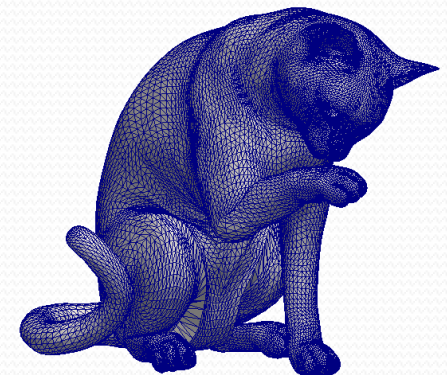
Splines



Graph

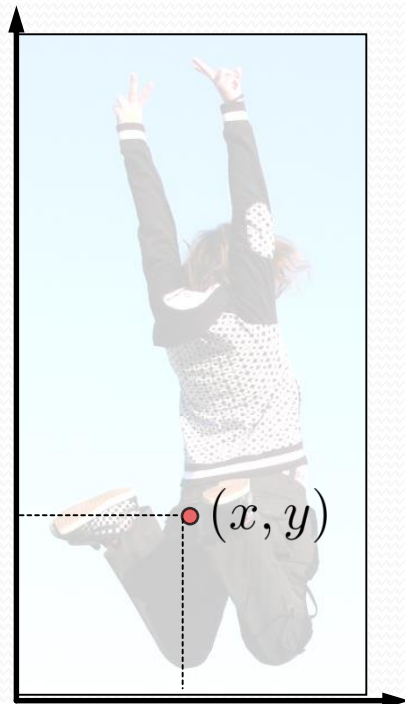


Point cloud

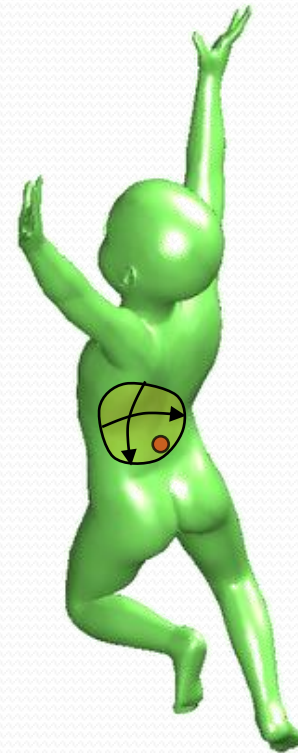


Triangular mesh

Shapes vs images: parametrization

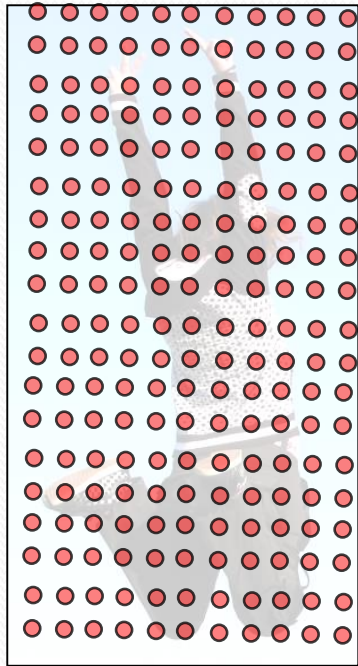


Global

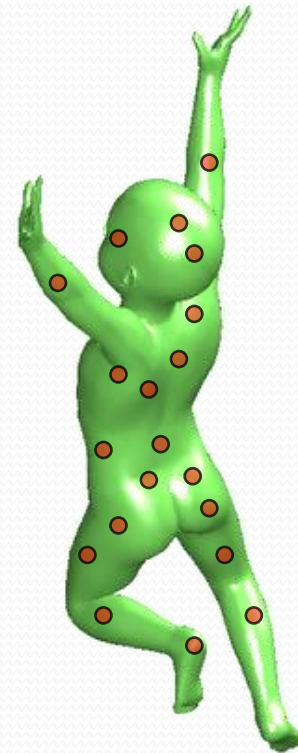


Local

Shapes vs images: sampling



Uniform



“Uniform” is not
well-defined

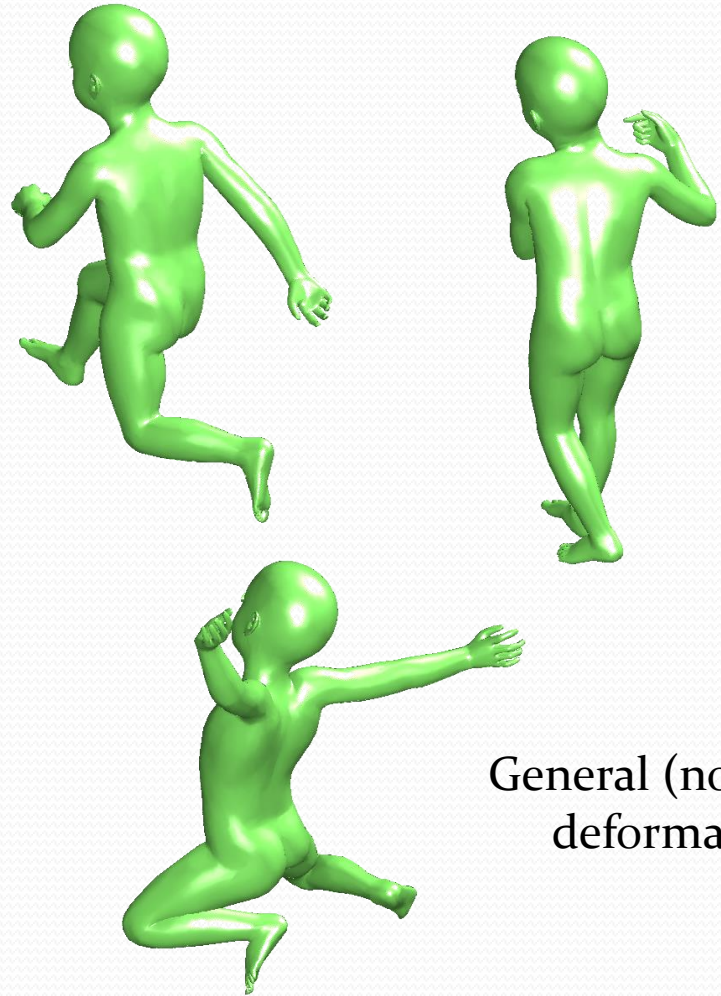
Shapes vs images: transformations



Perspective



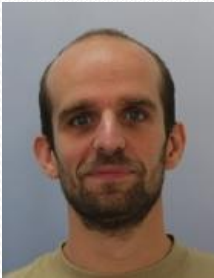
Affine



General (non-rigid)
deformations

Shapes vs images: calculus

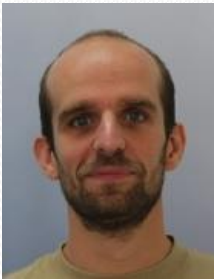
$$\frac{1}{2}$$



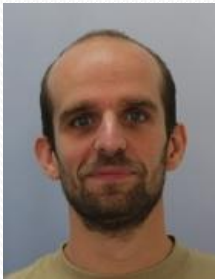
Shapes vs images: calculus

$$\frac{1}{2} \text{ } \langle \text{Image of a man with a beard} \rangle + \frac{1}{2} \text{ } \langle \text{Image of a man with short brown hair} \rangle =$$

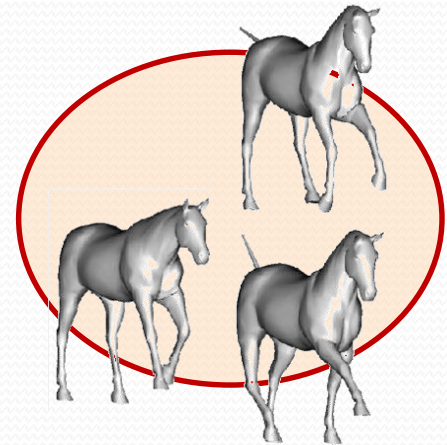
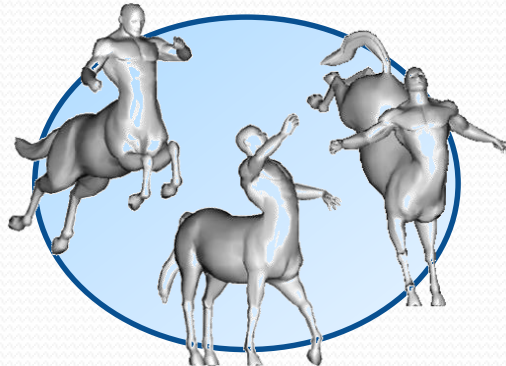
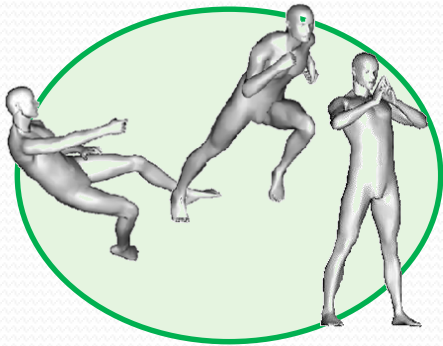
Shapes vs images: calculus

 $\frac{1}{2}$  $+$
 $\frac{1}{2}$  $=$ 

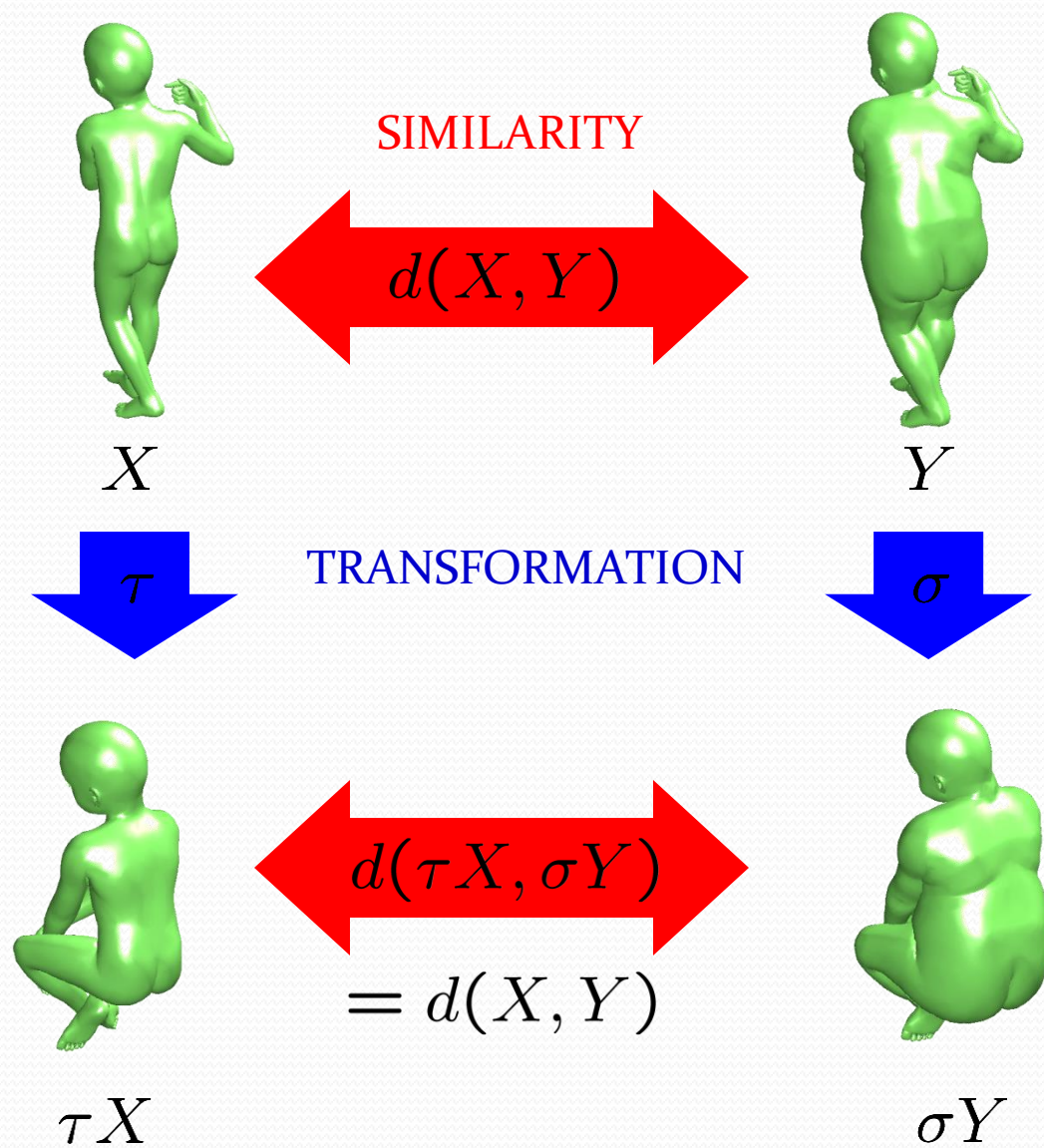
Shapes vs images: calculus

 $\frac{1}{2}$  $+$ $\frac{1}{2}$  $=$  $\frac{1}{2}$  $+$ $\frac{1}{2}$  $= ?$

Shape similarity



Is there something like a “space of shapes”?



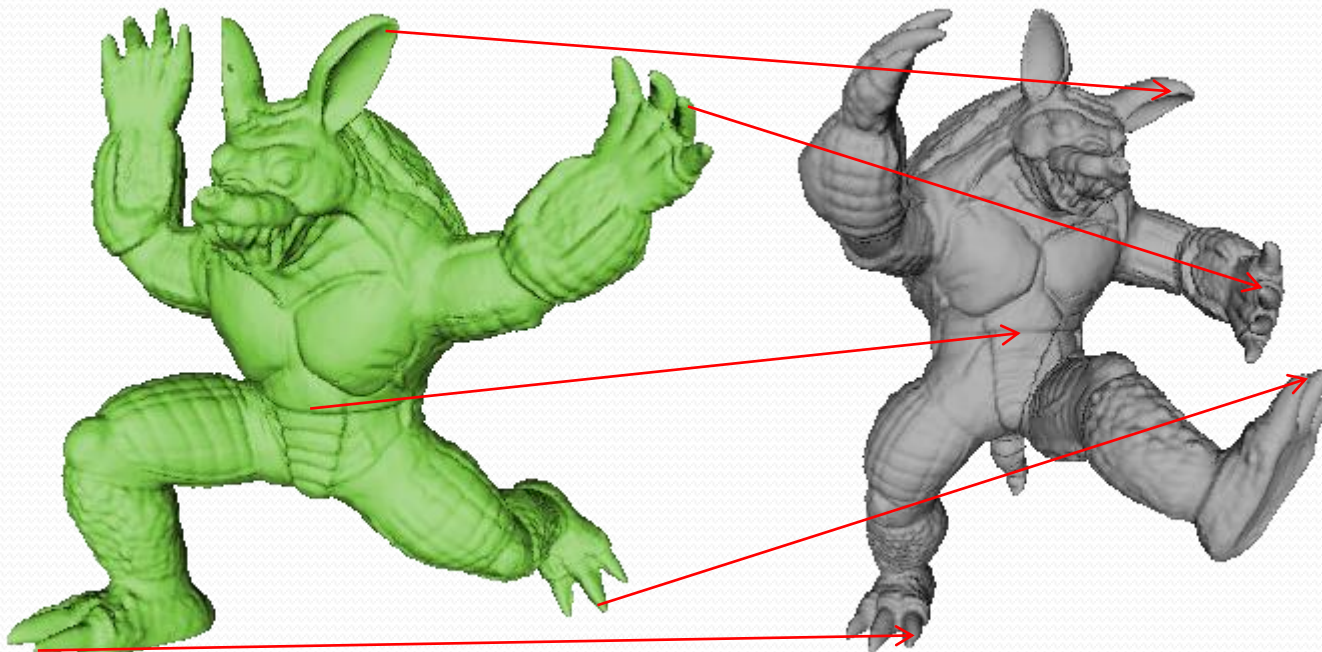
Shape matching

- Given a pair of shapes, let's try to find a **correspondence** between them.

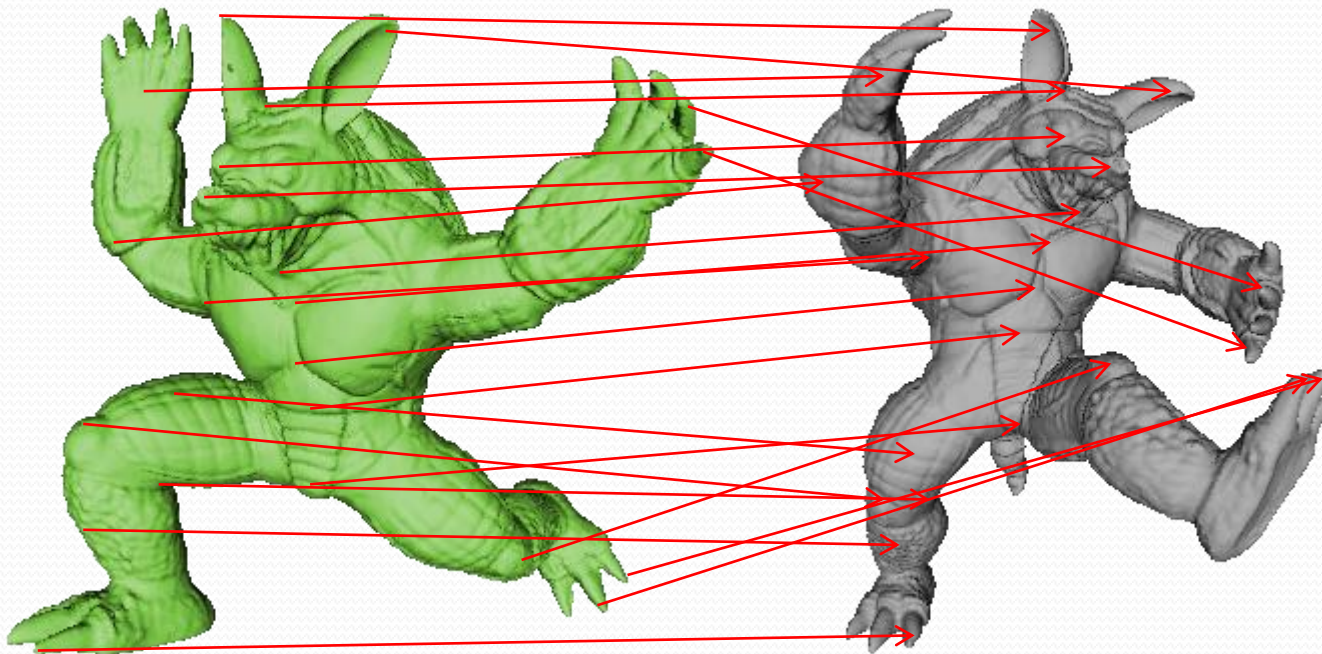


Shape matching

- Find the **best** alignment/map/correspondence.



Shape matching



In the real world



In the real world



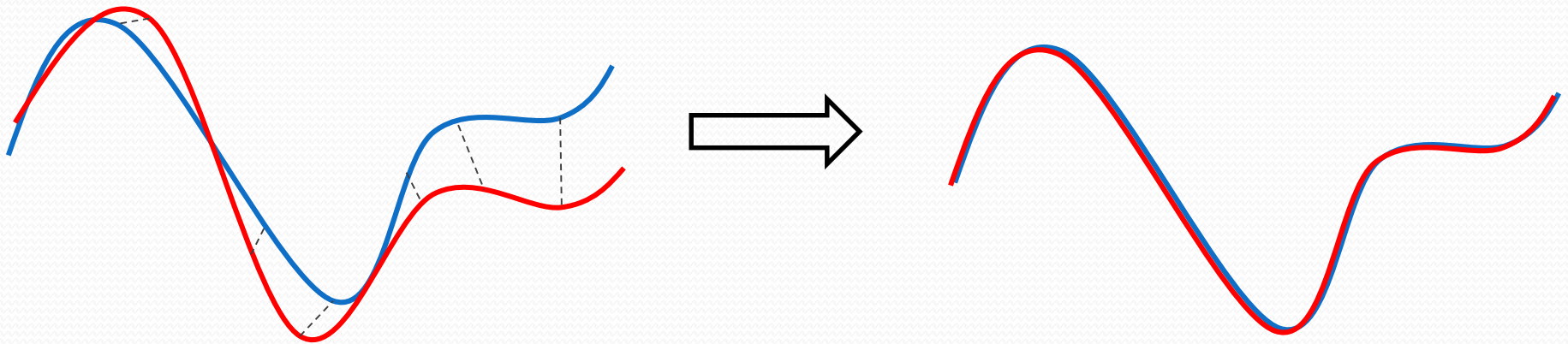
Taxonomy

| | | |
|---------------------------------------|-----|-------------------------------------|
| Local refinement (e.g. ICP) | vs. | Global alignment (search) |
|---------------------------------------|-----|-------------------------------------|

| | | |
|---------------------------------------|-----|--|
| Rigid rotation, translation | vs. | Deformable general deformation |
|---------------------------------------|-----|--|

| | | |
|---------------------------|-----|--------------------------------------|
| Pair two shapes | vs. | Collection multiple shapes |
|---------------------------|-----|--------------------------------------|

Pairwise rigid correspondence



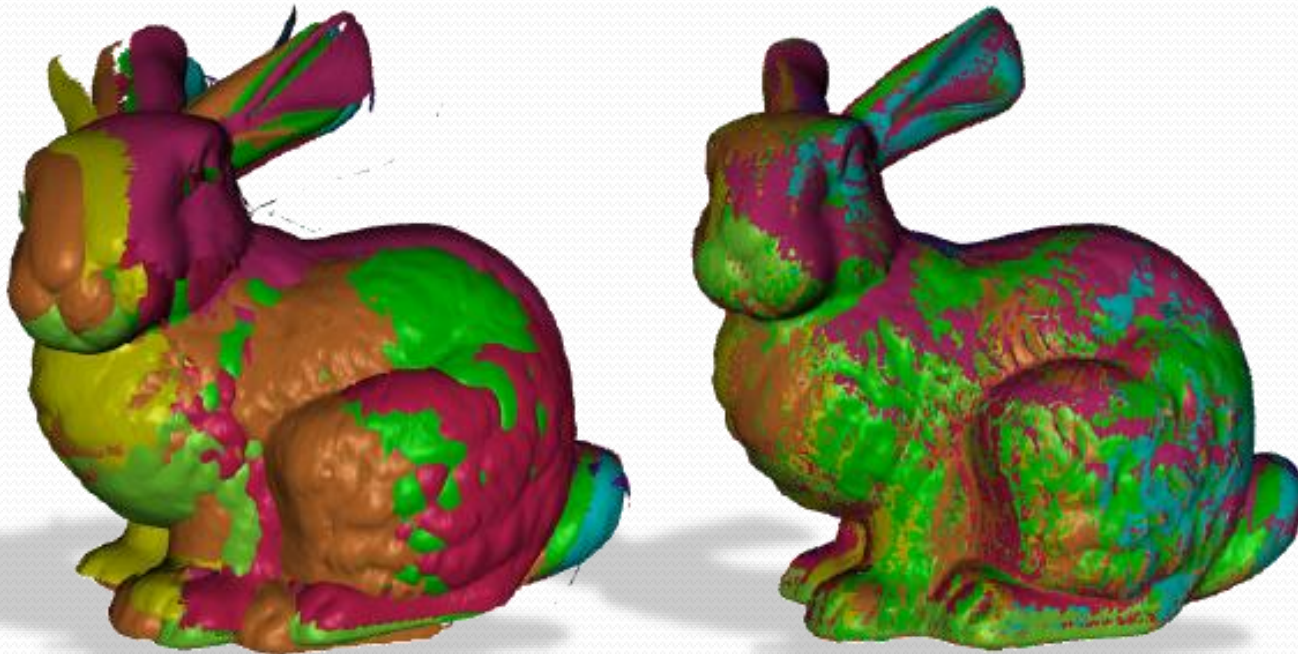
Iterative Closest Point

For a given pair of shapes M and N , **iterate**:

1. For each $x_i \in M$ find its nearest neighbor $y_i \in N$
2. Find the deformation R, t minimizing:

$$\sum_{x_i \in M} \|Rx_i + t - y_i\|$$

Pairwise rigid correspondence



Taxonomy

| | | |
|---------------------------------------|-----|-------------------------------------|
| Local refinement (e.g. ICP) | vs. | Global alignment (search) |
|---------------------------------------|-----|-------------------------------------|

| | | |
|---------------------------------------|-----|--|
| Rigid rotation, translation | vs. | Deformable general deformation |
|---------------------------------------|-----|--|

| | | |
|---------------------------|-----|--------------------------------------|
| Pair two shapes | vs. | Collection multiple shapes |
|---------------------------|-----|--------------------------------------|

Taxonomy

Local
refinement (e.g. ICP)

vs.

Global
alignment (search)

Rigid
rotation, translation

vs.

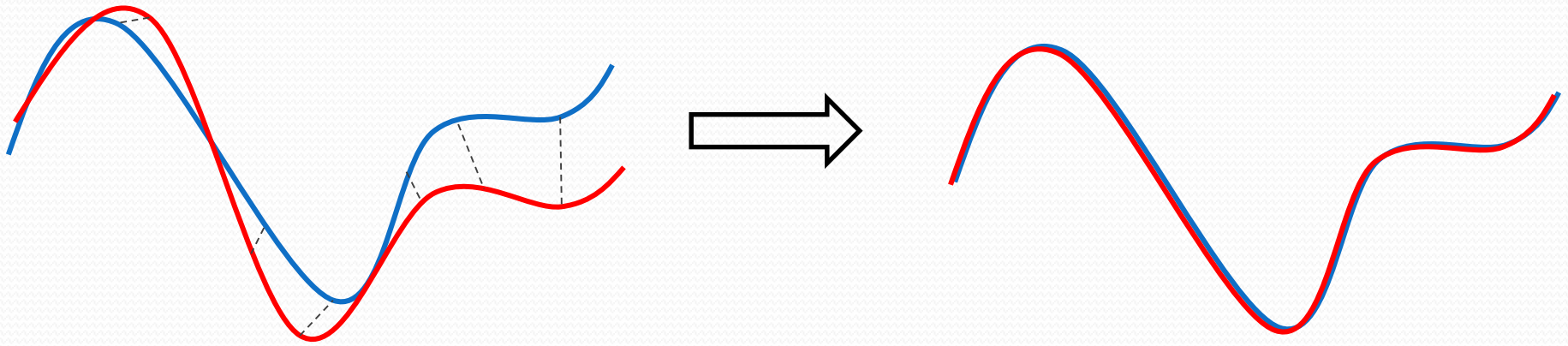
Deformable
general deformation

Pair
two shapes

vs.

Collection
multiple shapes

Pairwise rigid correspondence



Iterative Closest Point

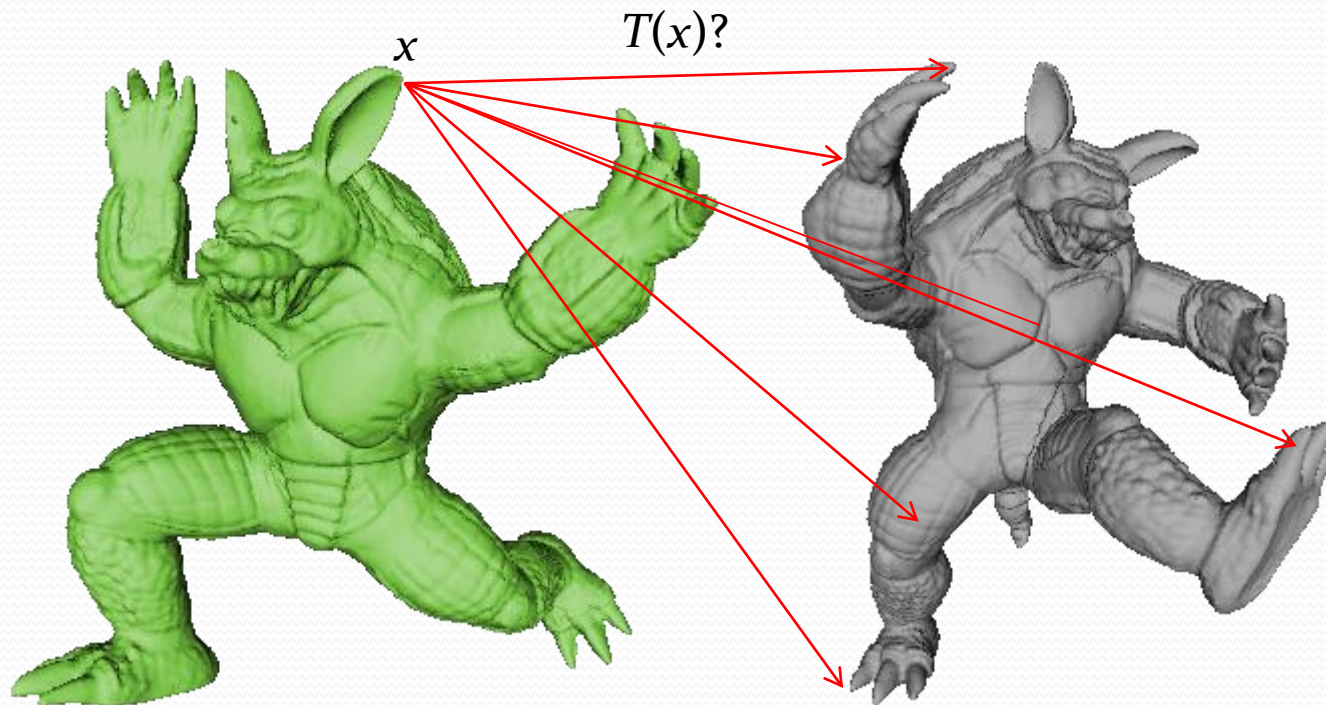
...

...

1. Find the deformation R, t minimizing:

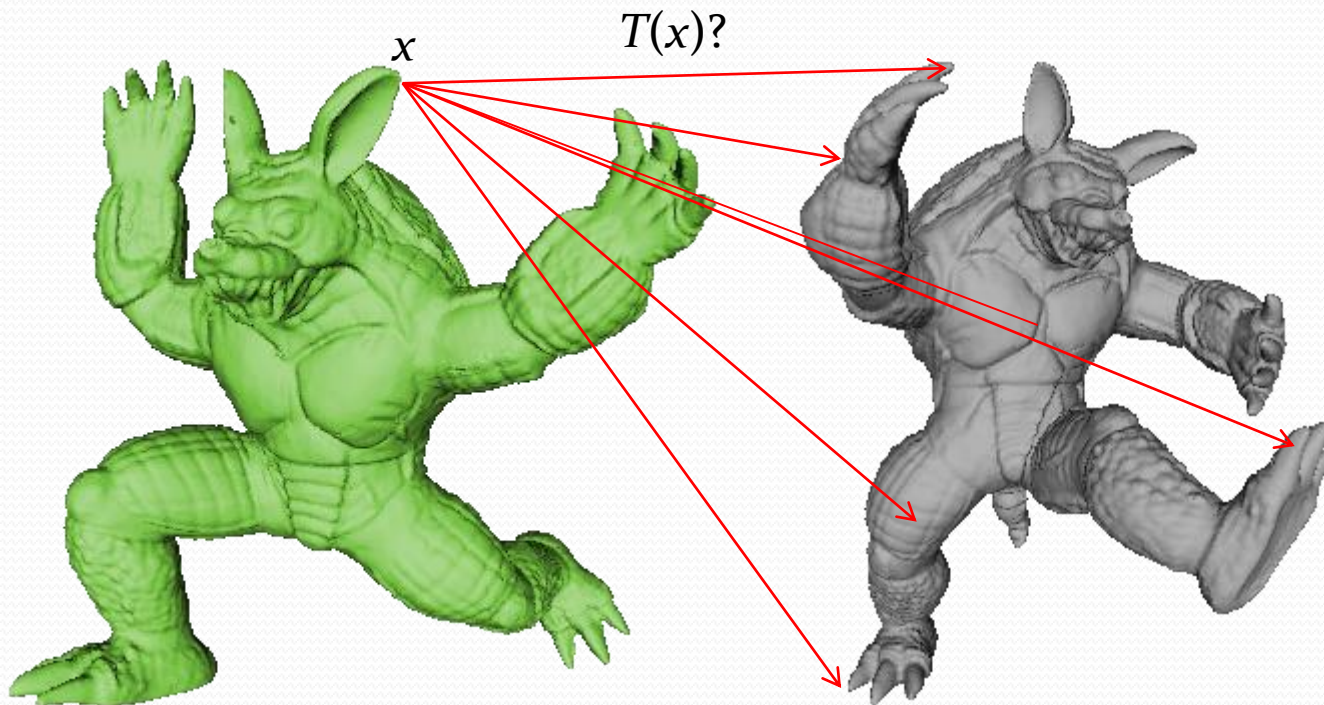
$$\sum_{x_i \in M} \|Rx_i + t - y_i\|$$

Deformable shape matching



- Unlike rigid matching (rotation/translation), there is no *compact representation* to optimize for.

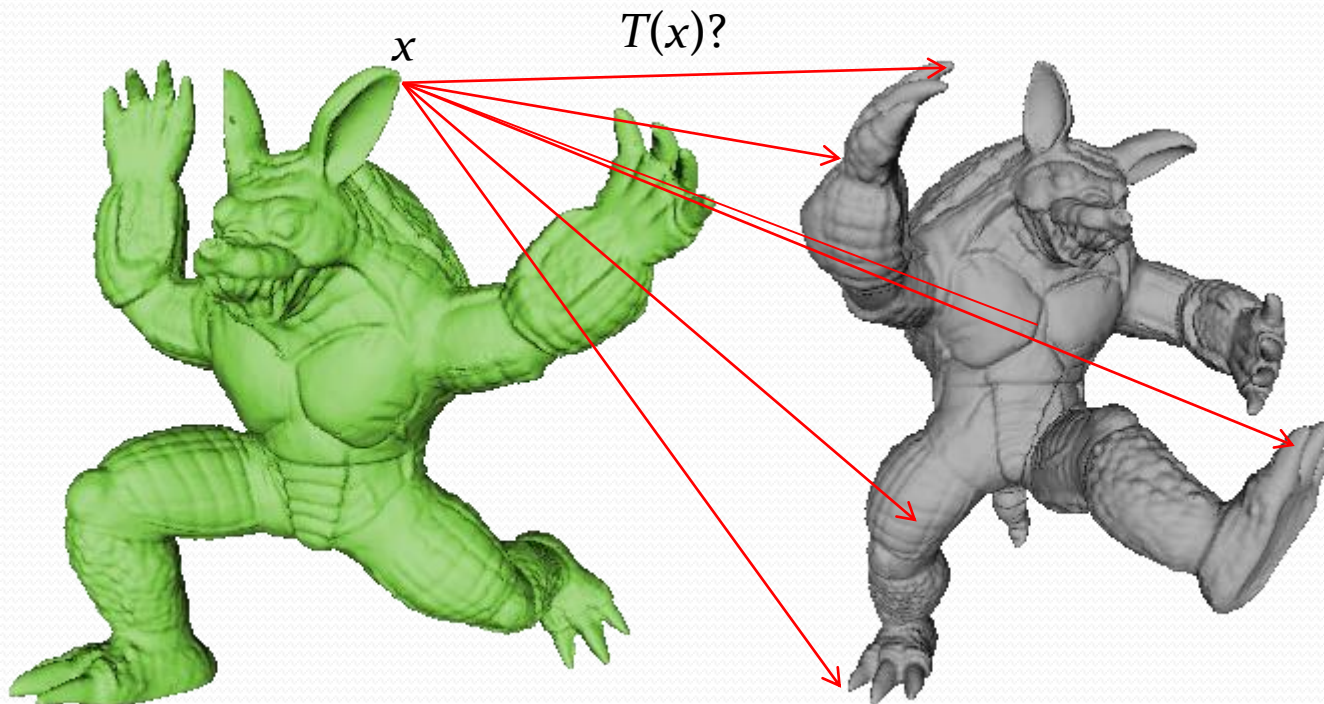
Deformable shape matching



- Instead, directly optimize over all possible point-to-point correspondences.

Signature preservation

$$T_{opt} = \arg \min_{T:M \rightarrow N} \sum_{x_i \in M} \|S(x_i) - S(T(x_i))\|$$



What signature?

One possibility: Look for similar textures



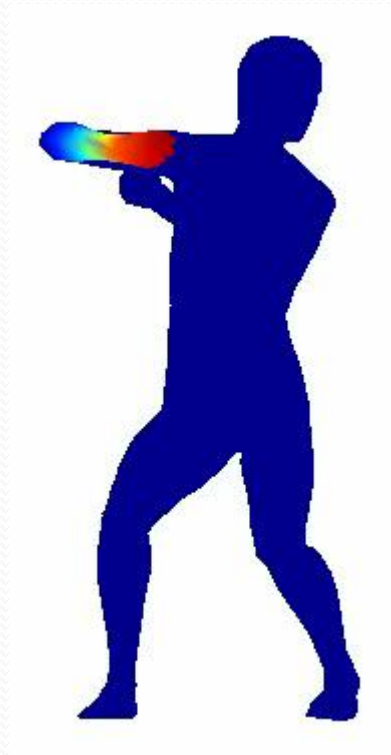
What signature?

Another possibility: Let's look at the geometry!

$$\left(\Delta_X + \frac{\partial}{\partial t}\right) u = 0$$

Heat equation governs the diffusion of heat on manifold X over time

Heat diffusion on manifolds



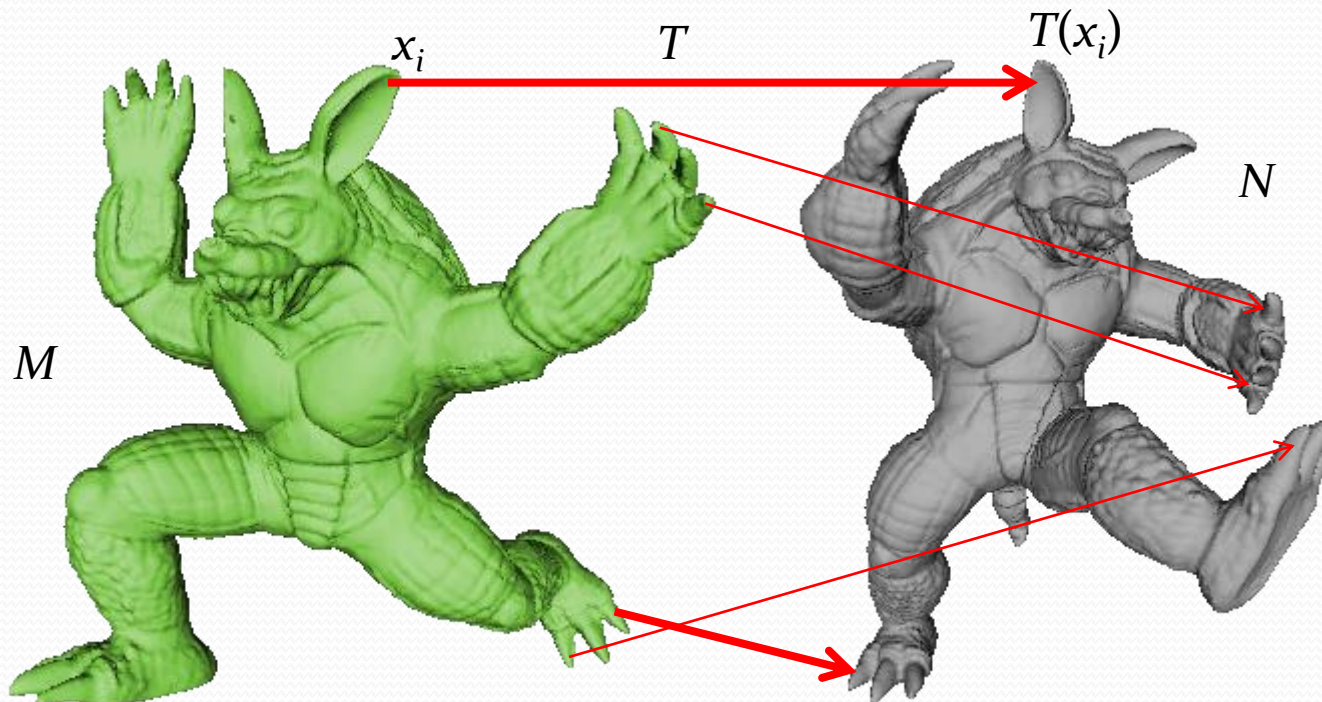
Heat Kernel Signature



Robust to pose variations

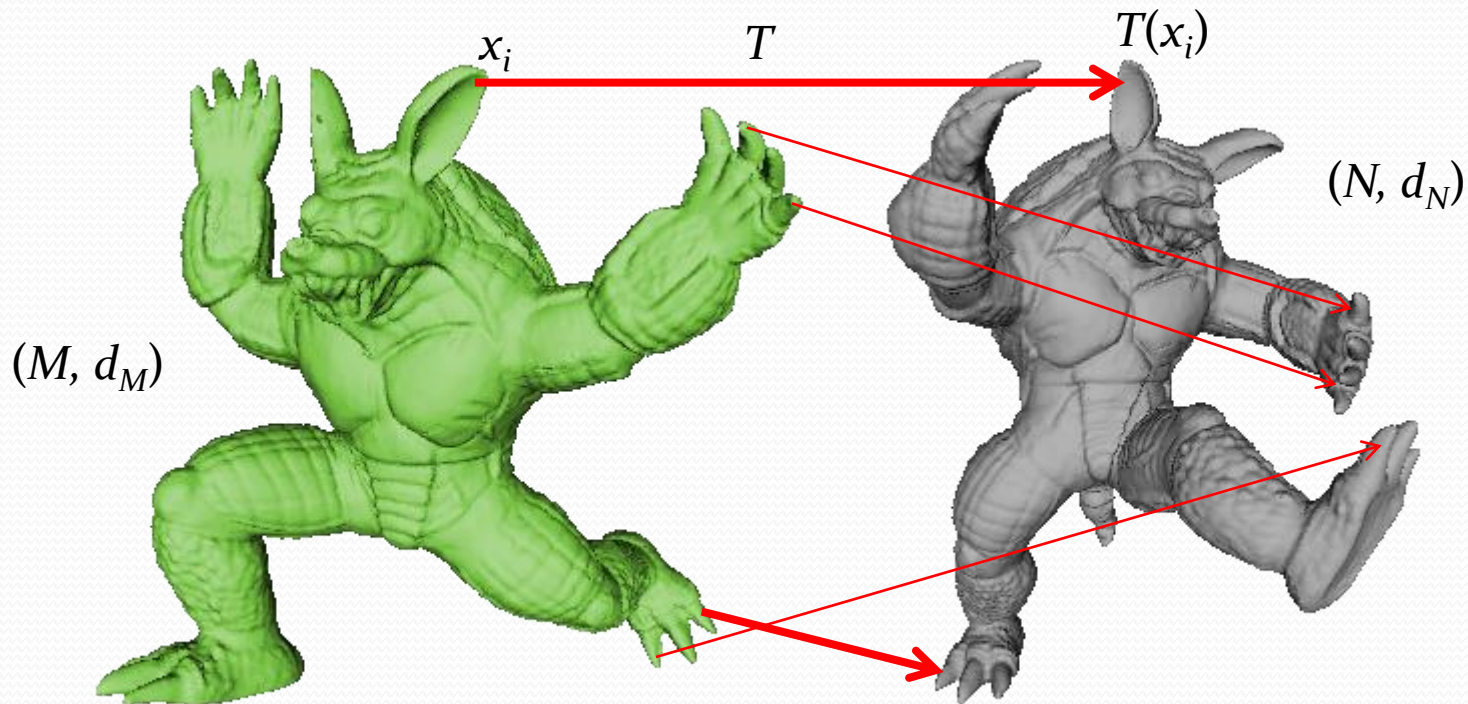
Signature preservation

$$T_{opt} = \arg \min_{T:M \rightarrow N} \sum_{x_i \in M} \|S(x_i) - S(T(x_i))\|$$



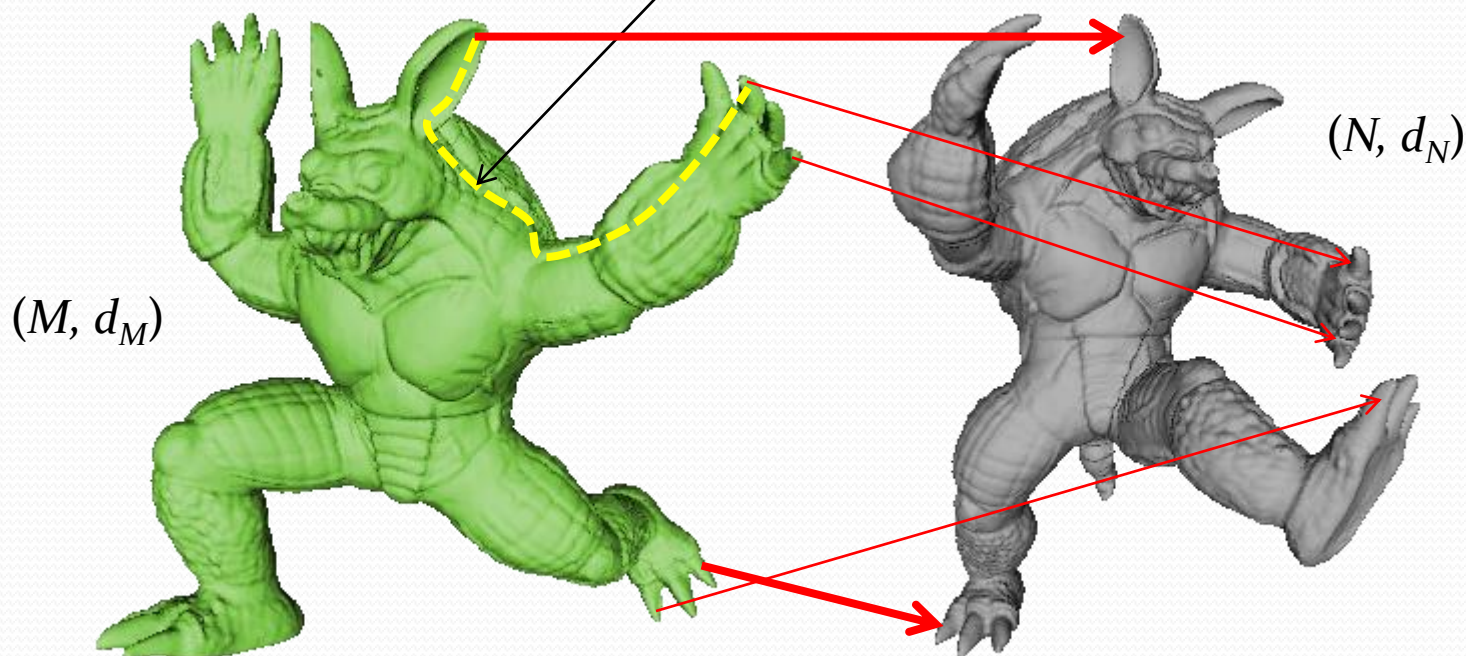
Metric preservation

$$T_{opt} = \arg \min_{T:M \rightarrow N} \sum_{(x_i, x_j) \in M \times M} \|d_M(x_i, x_j) - d_N(T(x_i), T(x_j))\|$$



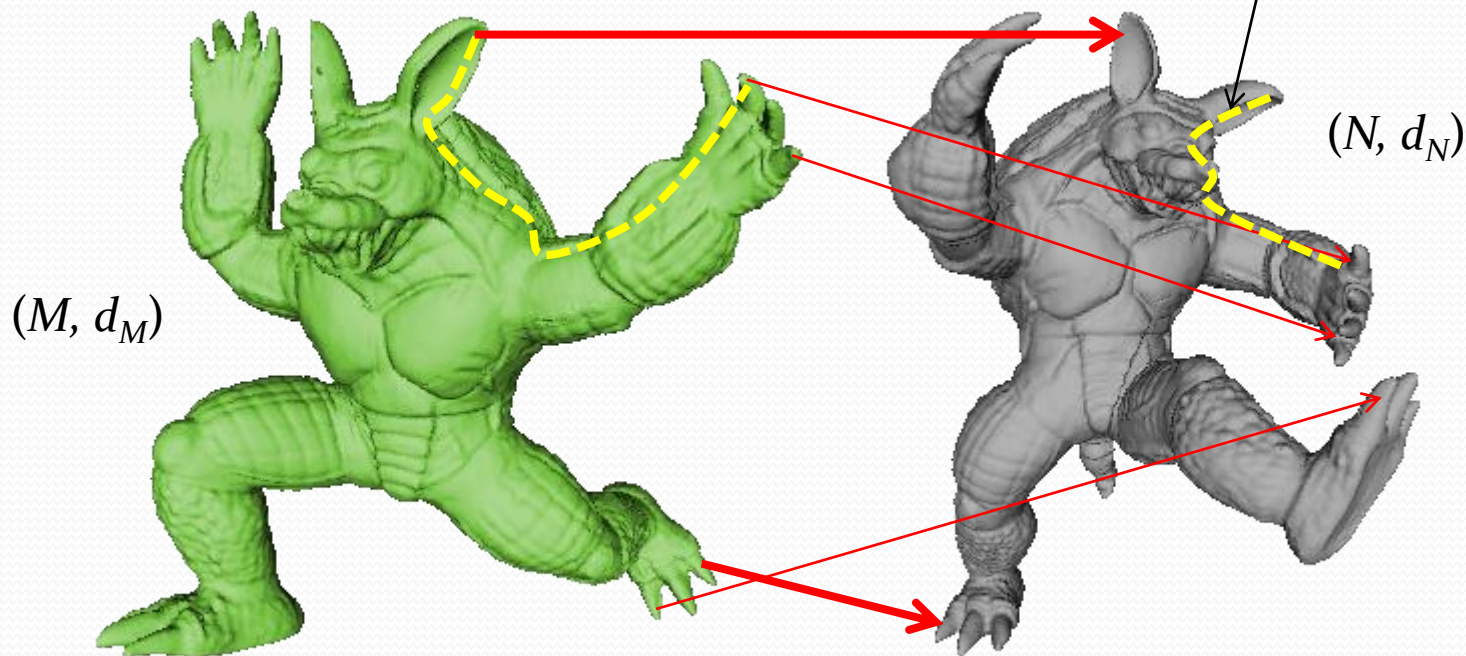
Metric preservation

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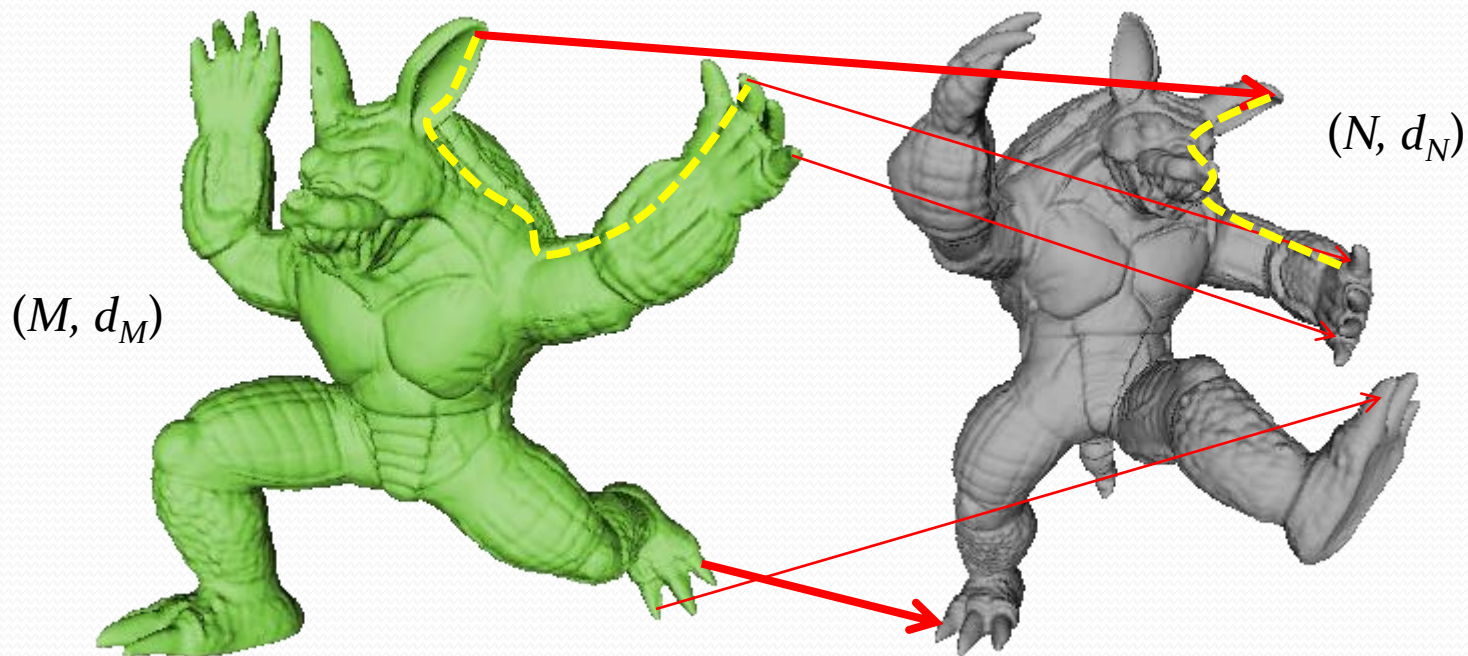
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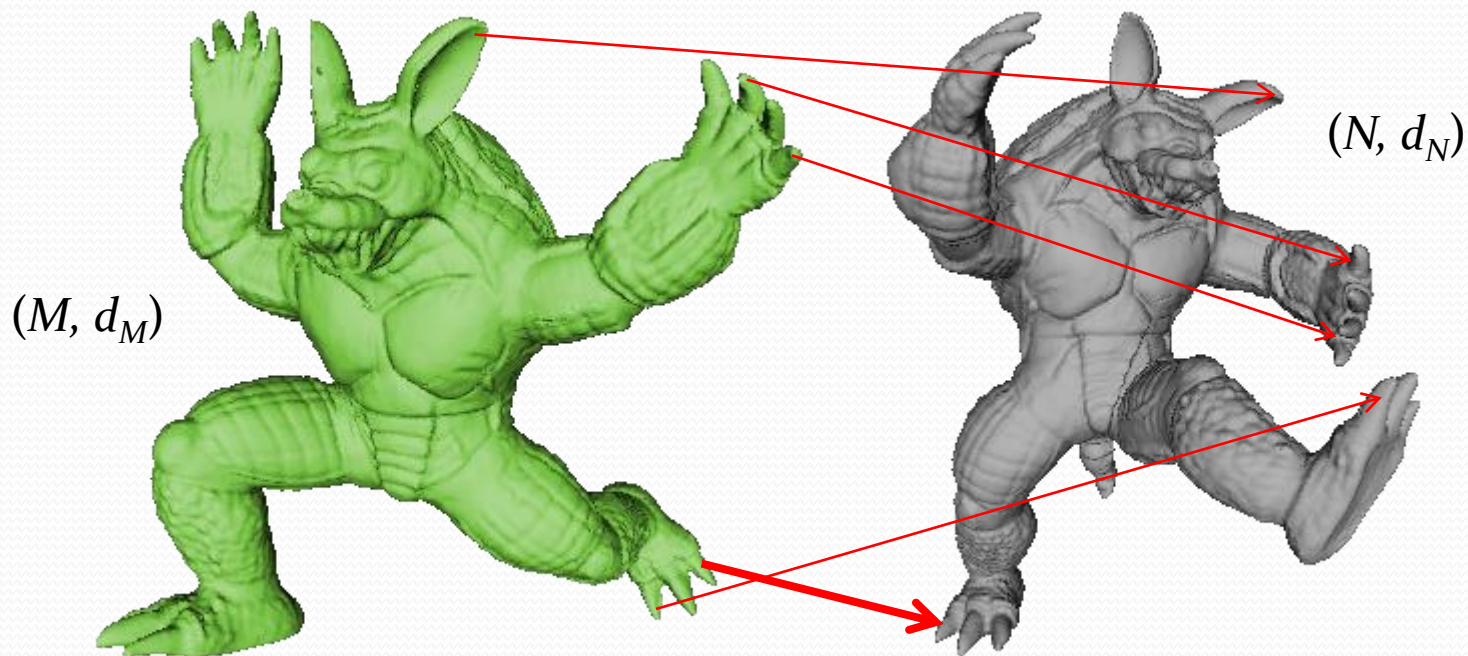
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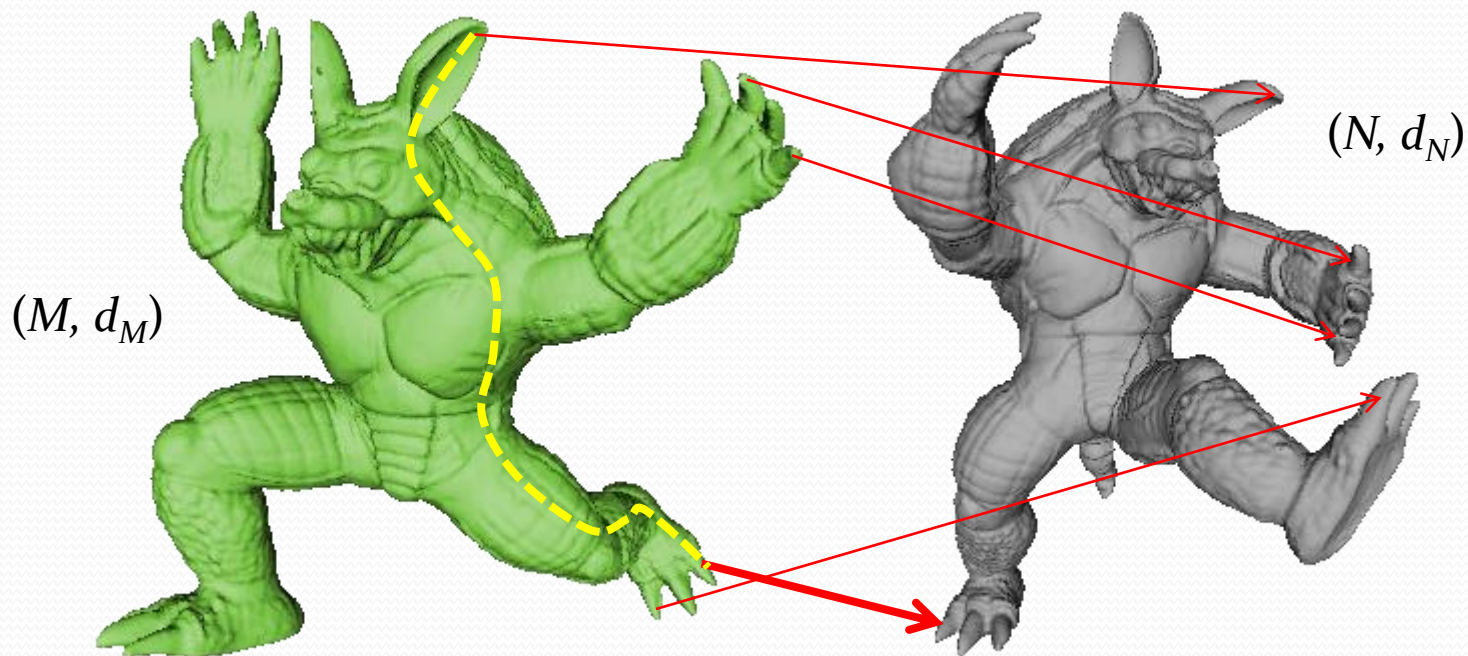
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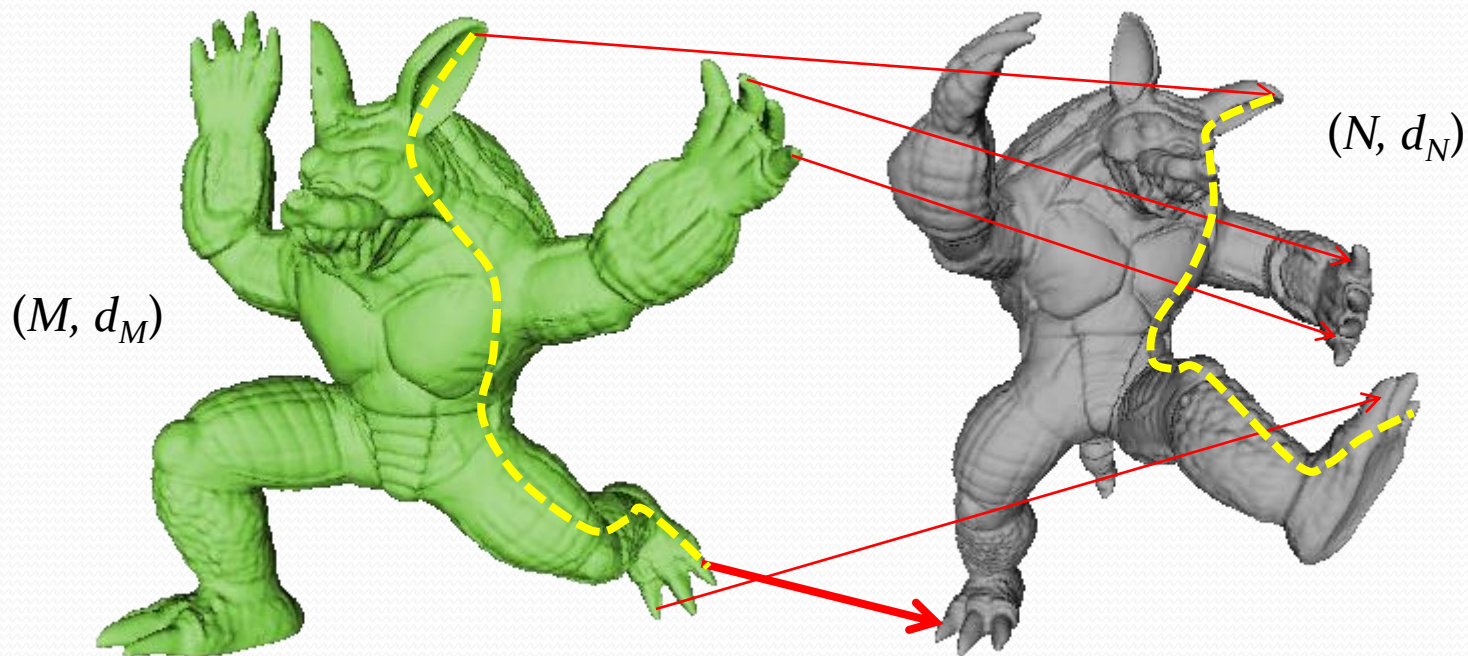
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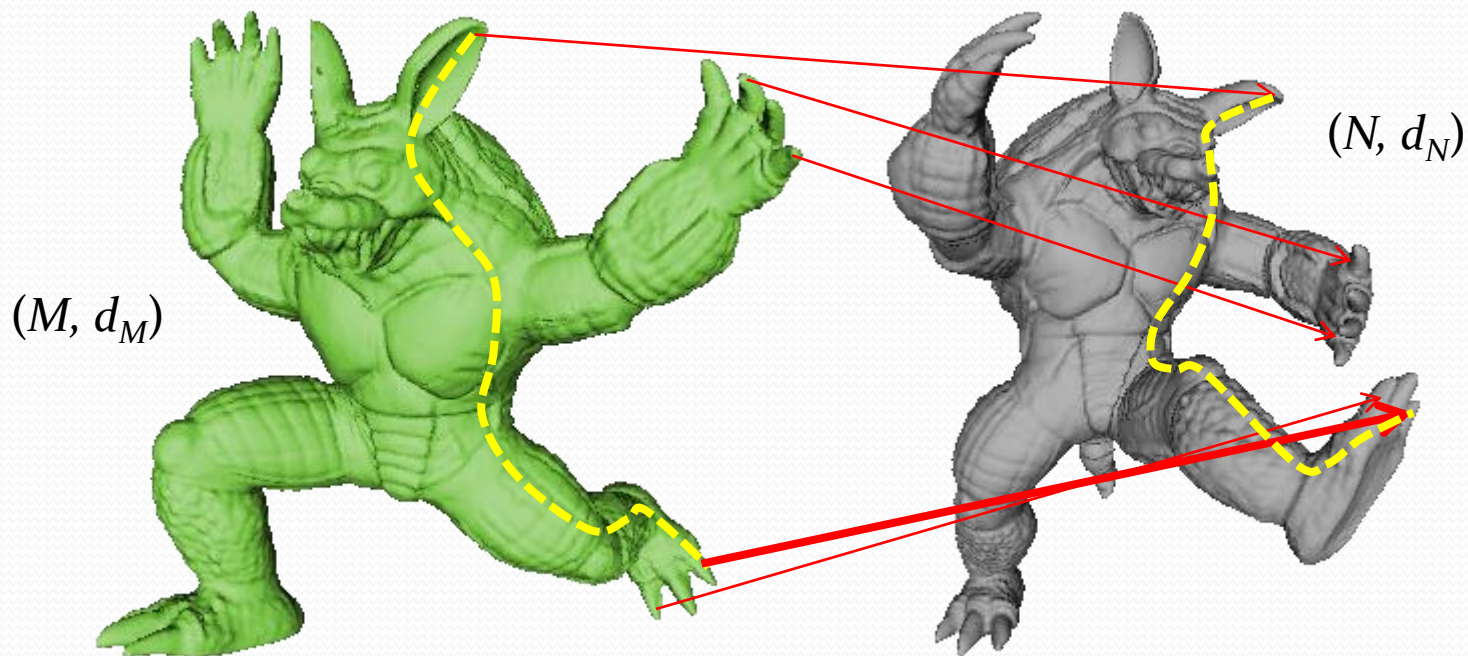
Metric preservation

$$T_{opt} = \arg \min_{T:M \rightarrow N} \sum_{(x_i, x_j) \in M \times M} \|d_M(x_i, x_j) - d_N(T(x_i), T(x_j))\|$$



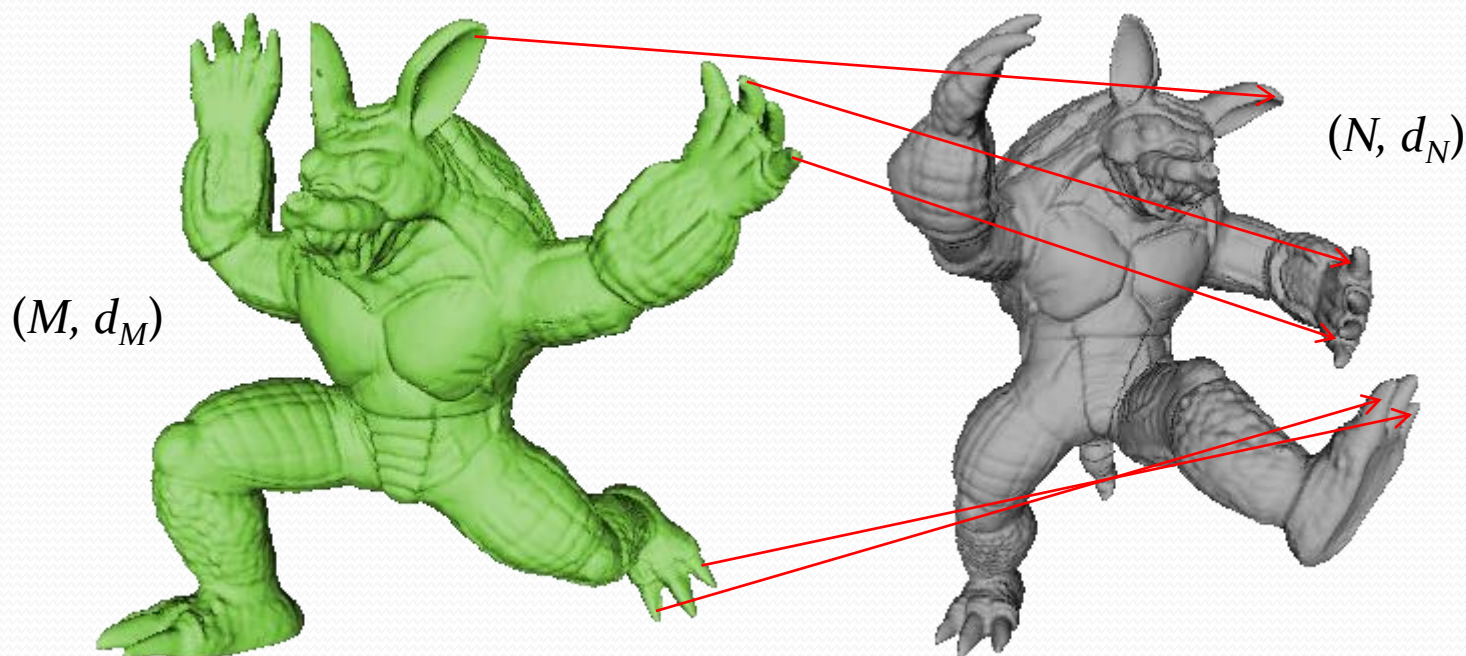
Metric preservation

$$T_{opt} = \arg \min_{T:M \rightarrow N} \sum_{(x_i, x_j) \in M \times M} \|d_M(x_i, x_j) - d_N(T(x_i), T(x_j))\|$$

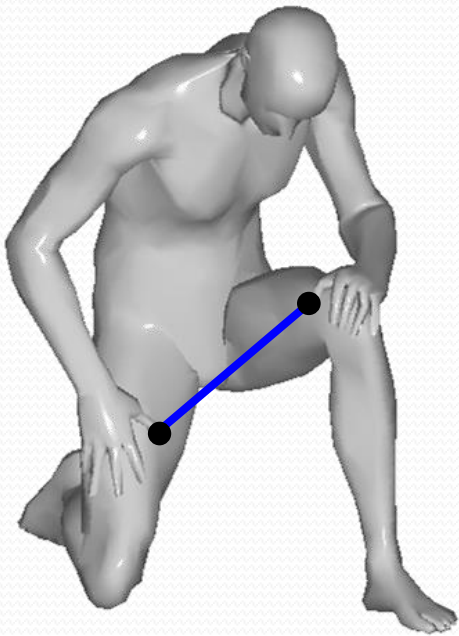


Metric preservation

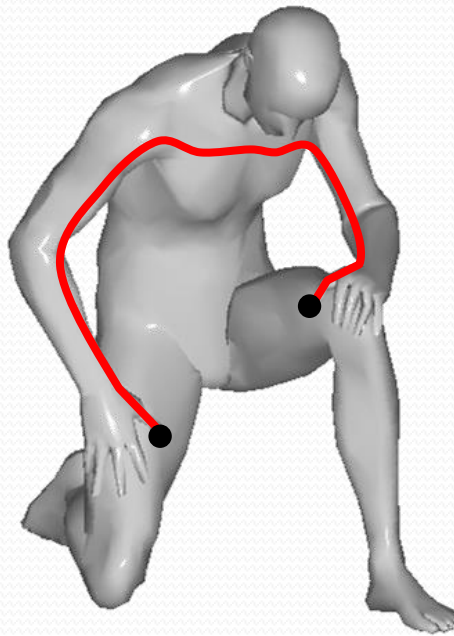
$$T_{opt} = \arg \min_{T:M \rightarrow N} \sum_{(x_i, x_j) \in M \times M} \|d_M(x_i, x_j) - d_N(T(x_i), T(x_j))\|$$



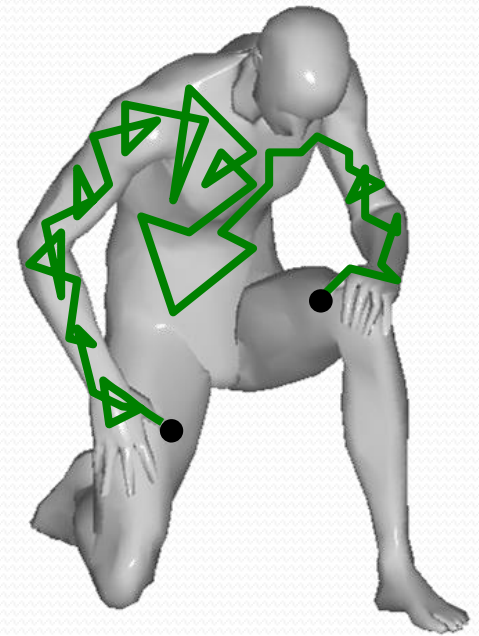
Examples of metrics



Euclidean



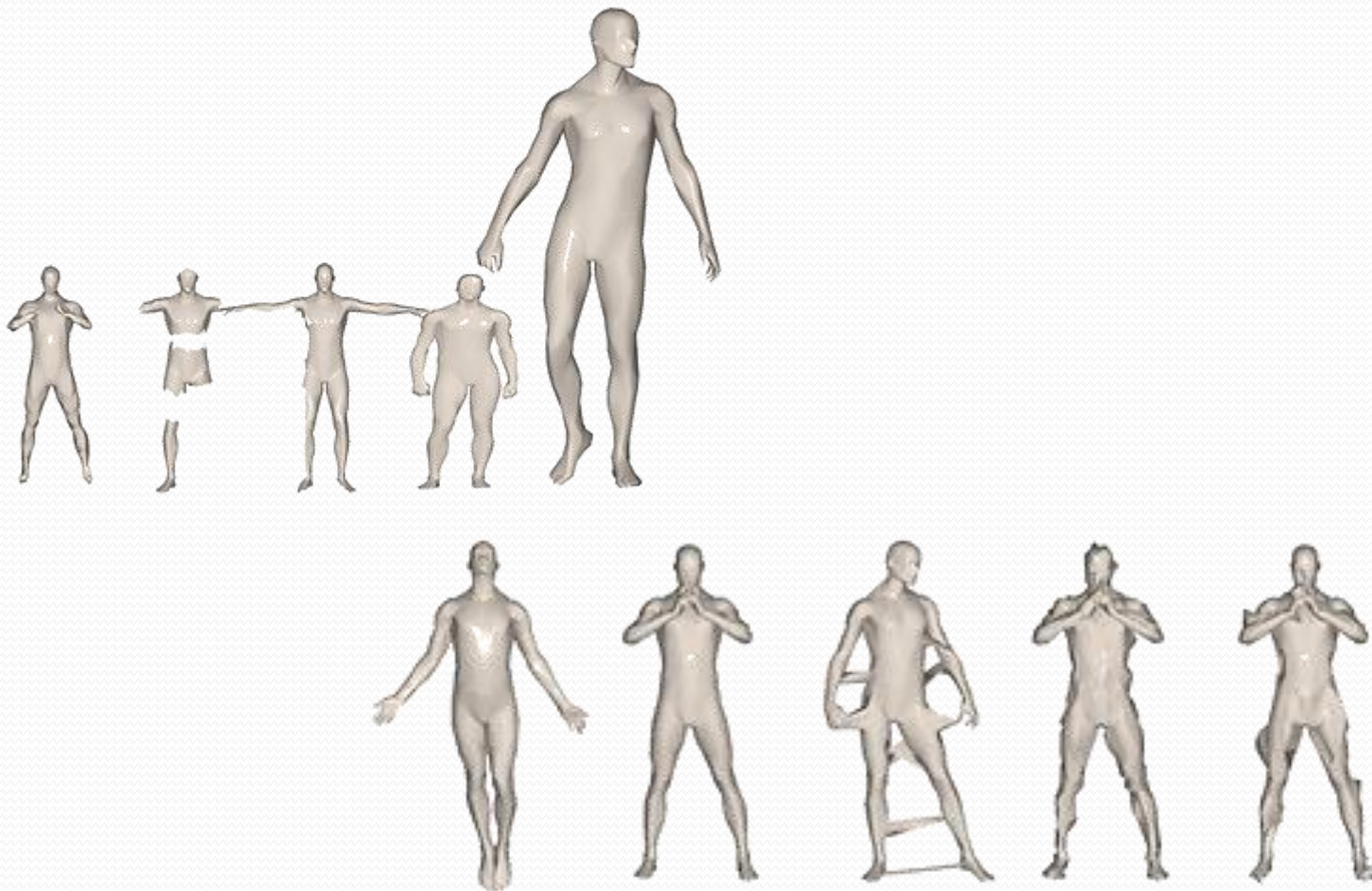
Geodesic



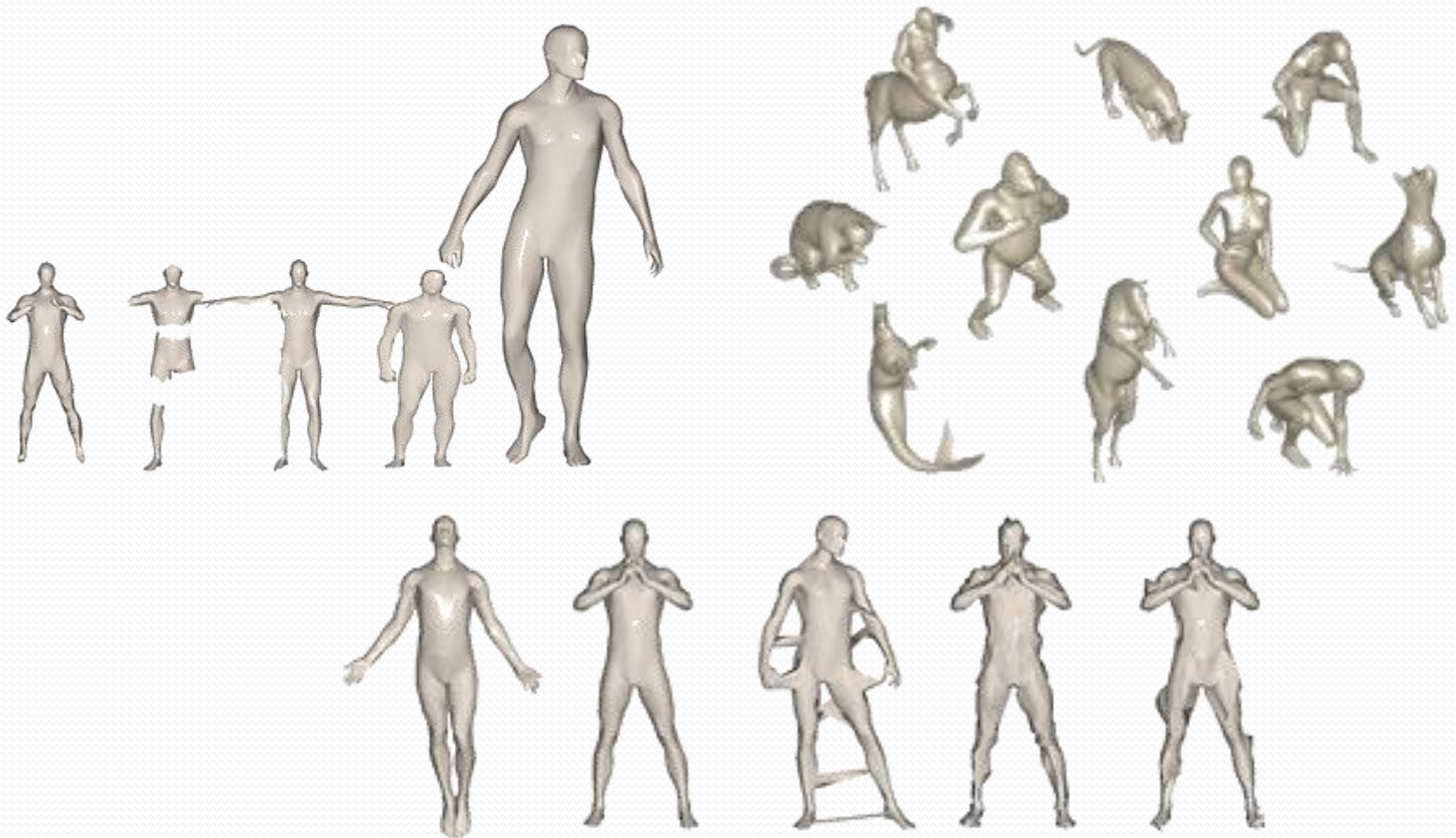
Diffusion



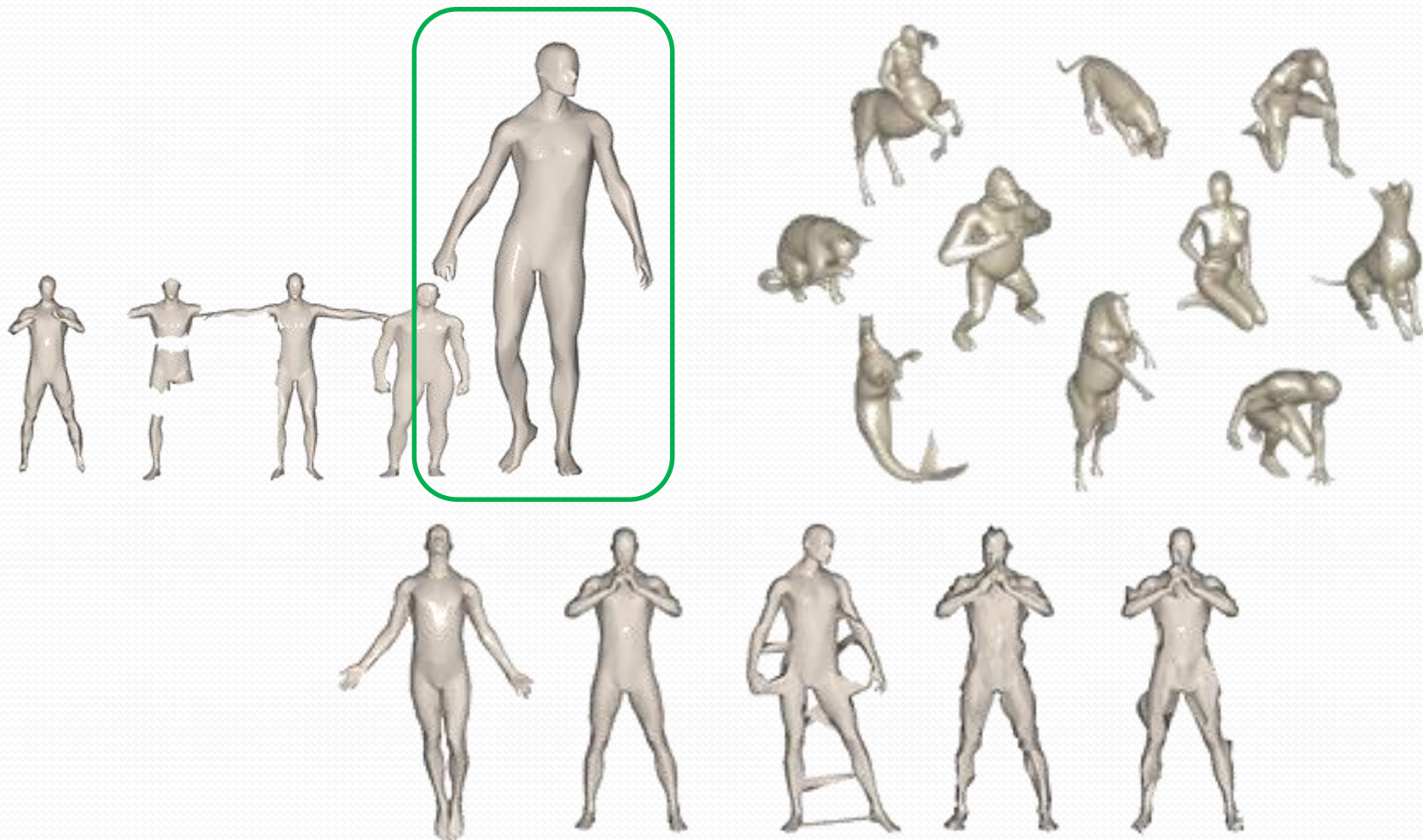
Invariance to what?



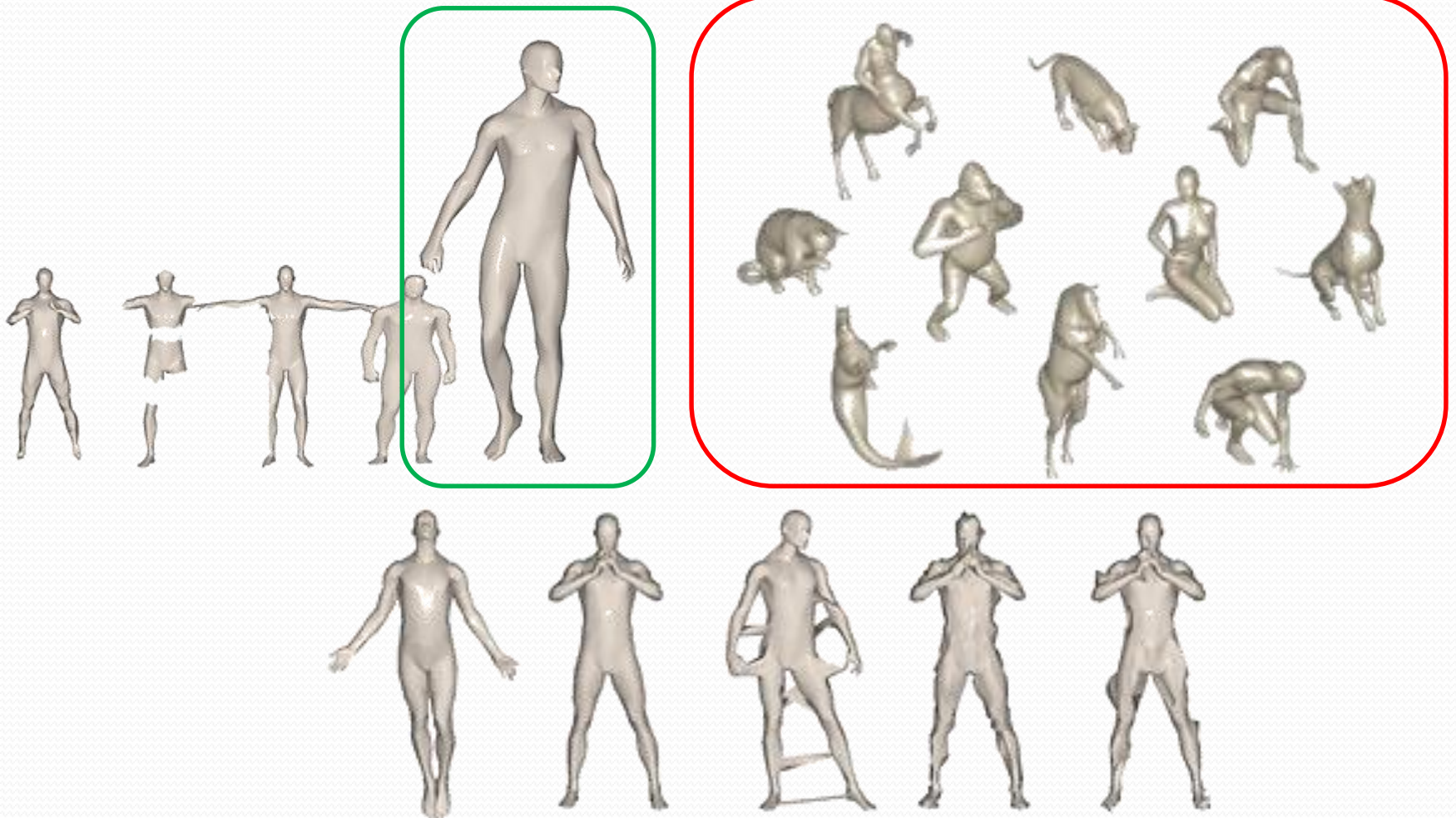
Invariance to what?



Invariance to what?

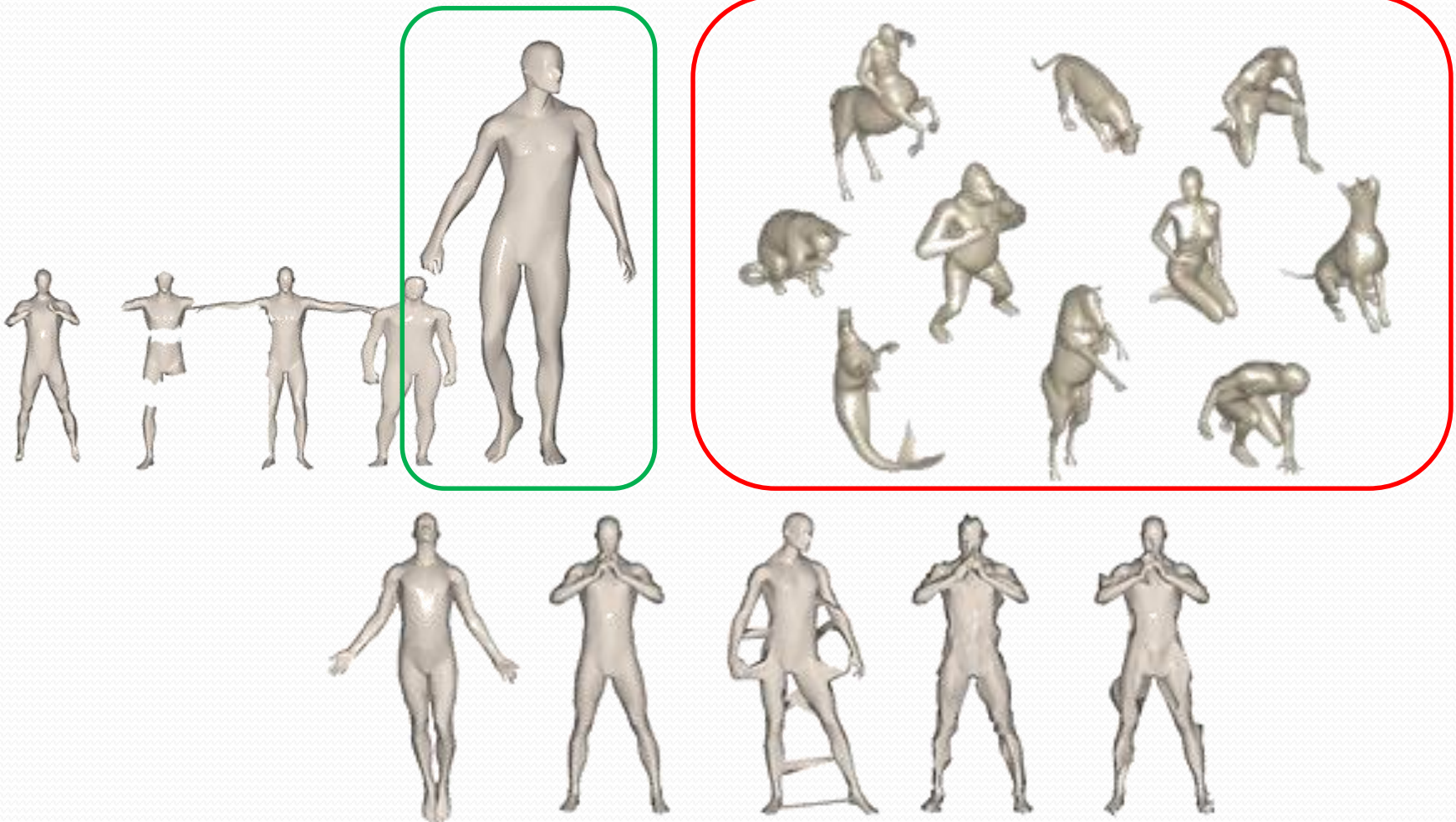


Invariance to what?



Invariance to what?

Shapes belong to other classes!





Inter-class matching, or...

Inter-class matching, or...

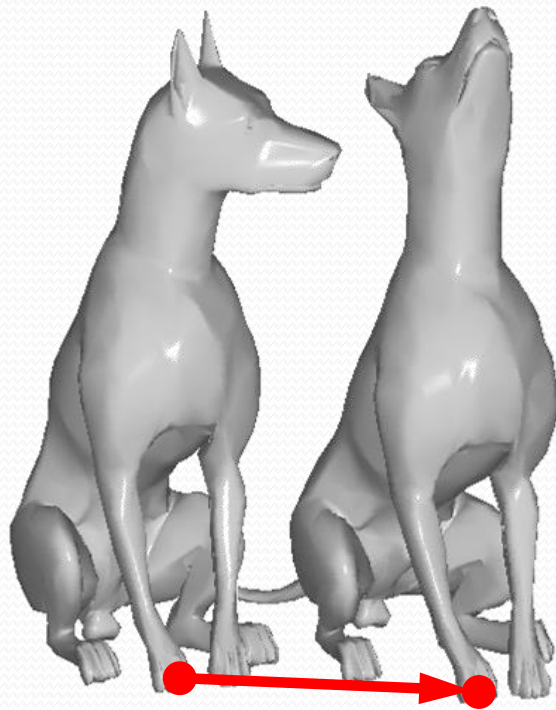
Matching a shark to a tornado



Inter-class matching, or...

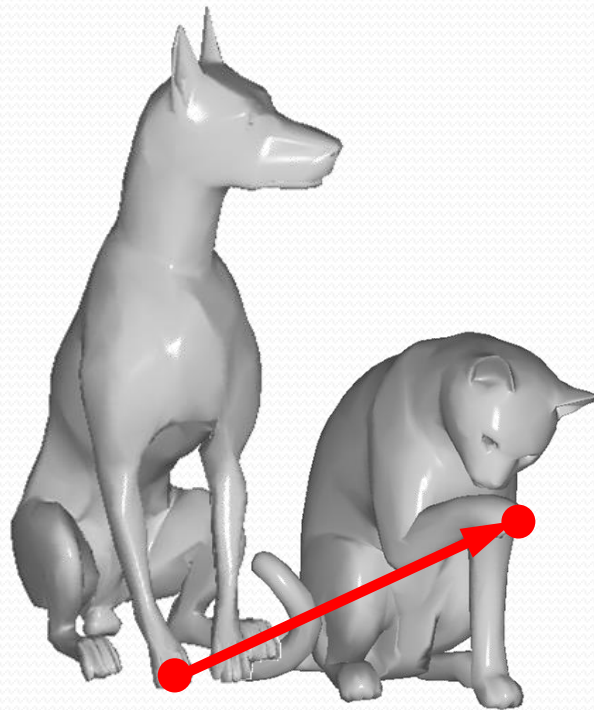
Matching a shark to a tornado





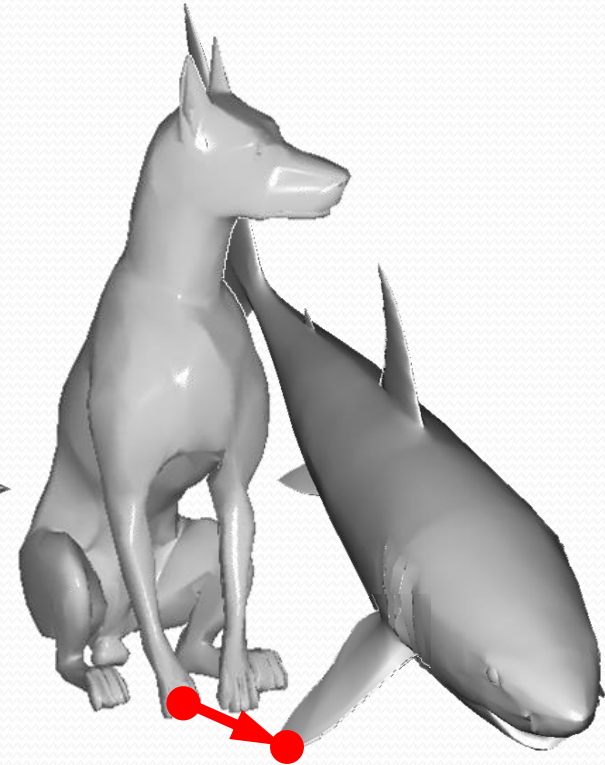
Geometric

“accurate”



Semantic

“makes sense”

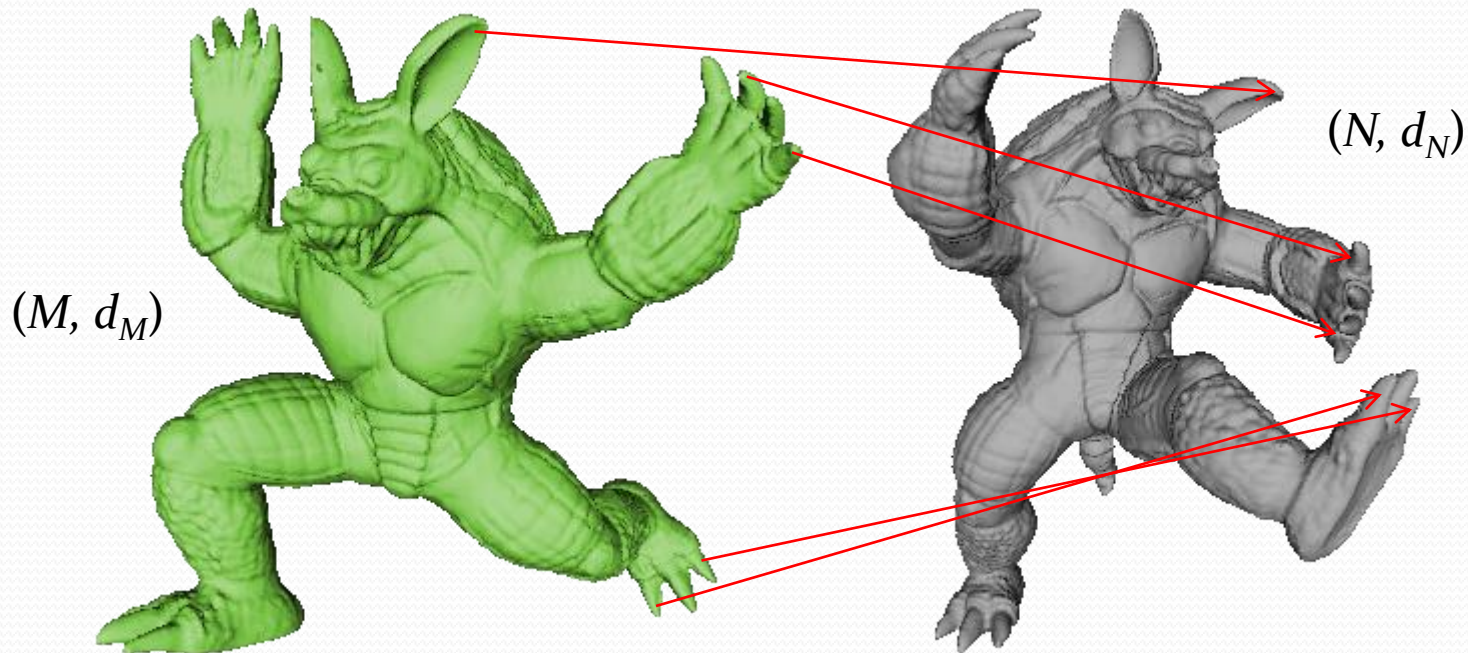


Aesthetic

“beautiful”

Gromov-Hausdorff distance

$$T_{opt} = \arg \min_{T:M \rightarrow N} \sum_{(x_i, x_j) \in M \times M} \|d_M(x_i, x_j) - d_N(T(x_i), T(x_j))\|$$



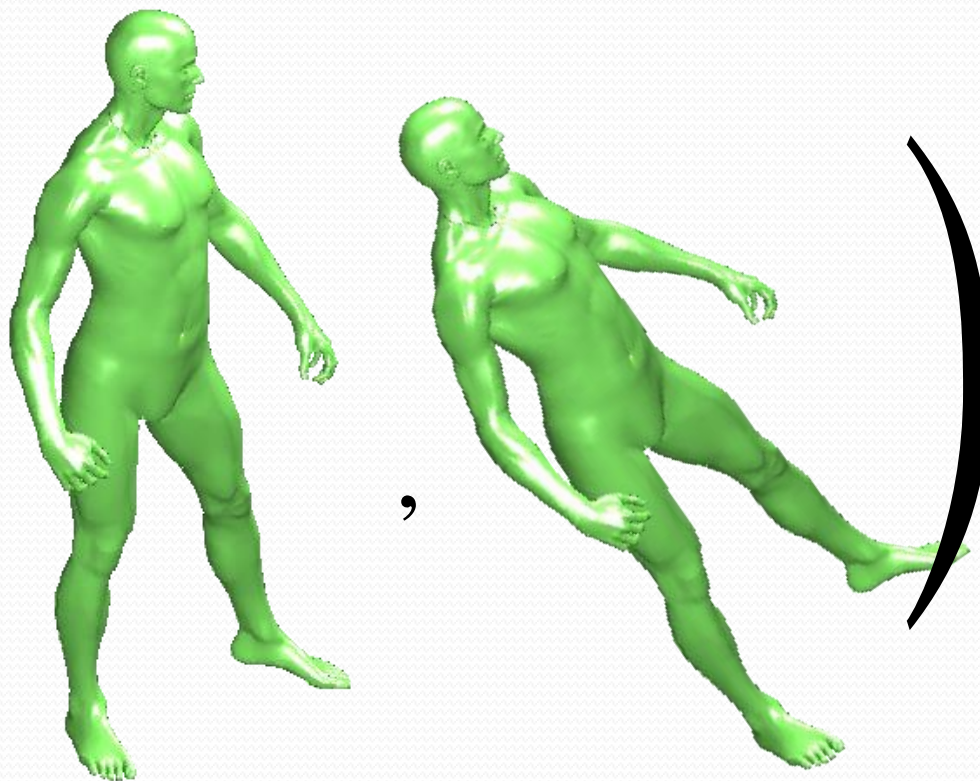
Gromov-Hausdorff distance

- Minimizing the worst-case distortion of the metric caused by the correspondence T is given by:

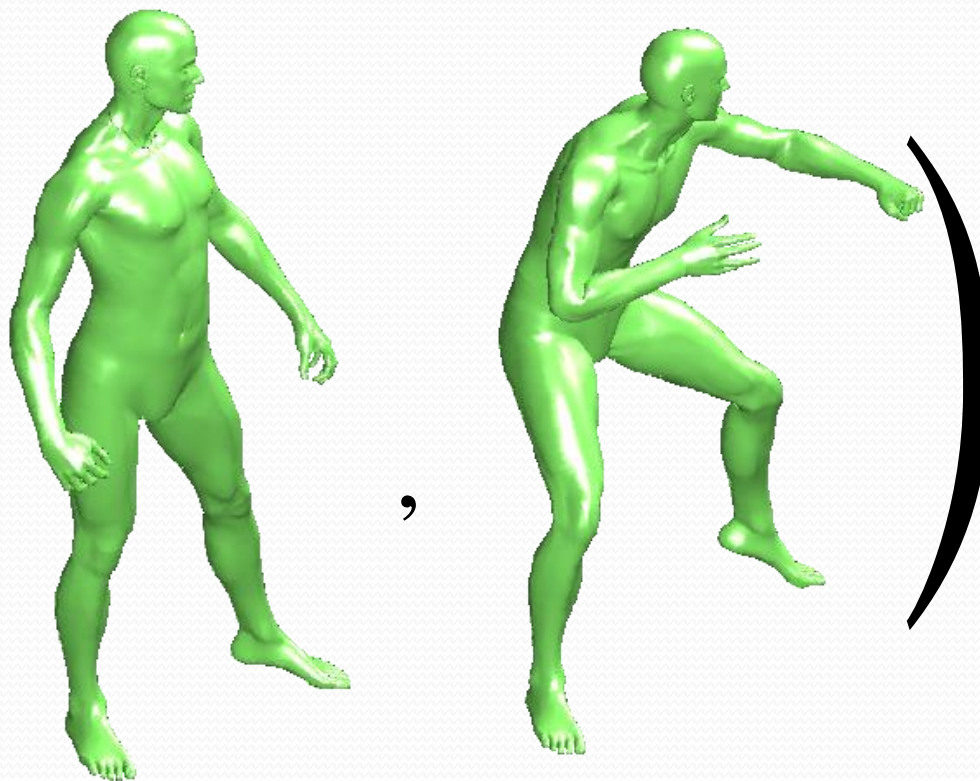
$$D_{\mathcal{GH}}(M, N) = \min_{T: M \rightarrow N} \max_{(x_i, x_j) \in M \times M} \|d_M(x_i, x_j) - d_N(T(x_i), T(x_j))\|$$

This is a *true* distance among shapes 😊

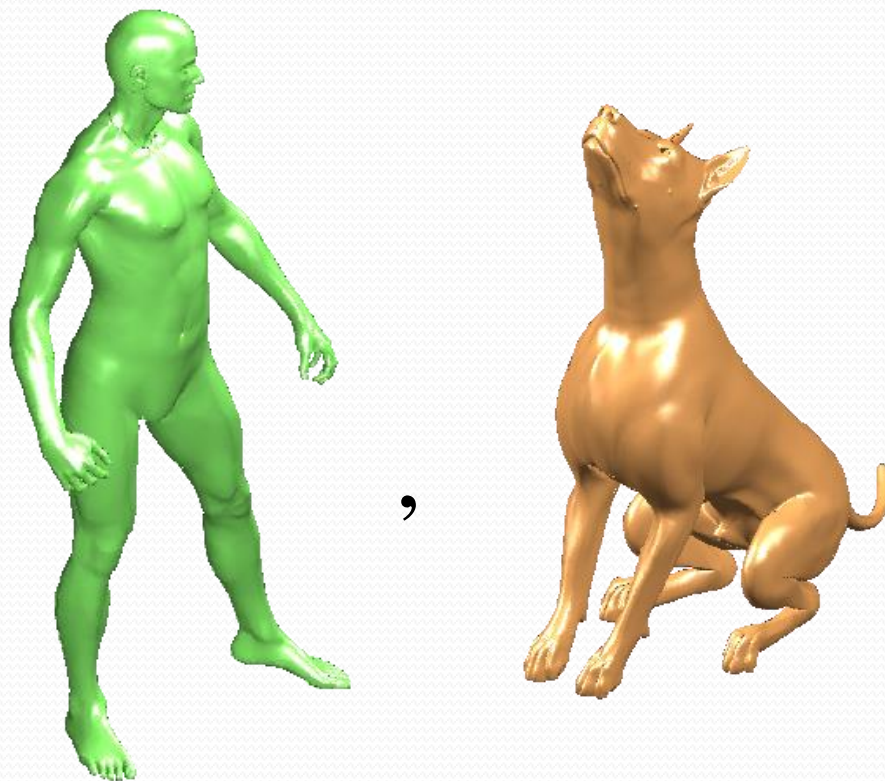
Gromov-Hausdorff distance

$$D_{\mathcal{GH}} \left(\text{standing figure}, \text{reclining figure} \right) = 0$$
The image shows two 3D renderings of a human figure, colored in a vibrant green. The figure on the left is in a standing pose, facing slightly to the right. The figure on the right is in a reclining pose, lying on its side with one leg bent and the other extended. Both figures are rendered with a smooth, metallic-looking texture. They are positioned between large black parentheses that are part of a mathematical expression. A comma is placed between the two figures, and the entire expression is followed by an equals sign and a zero.

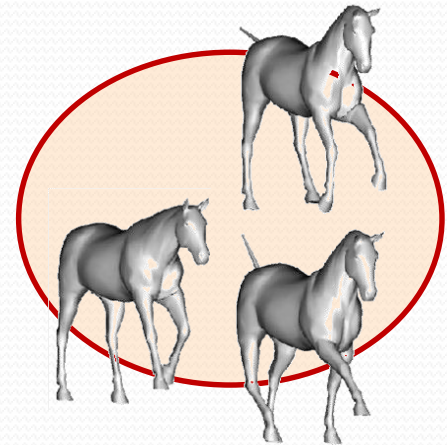
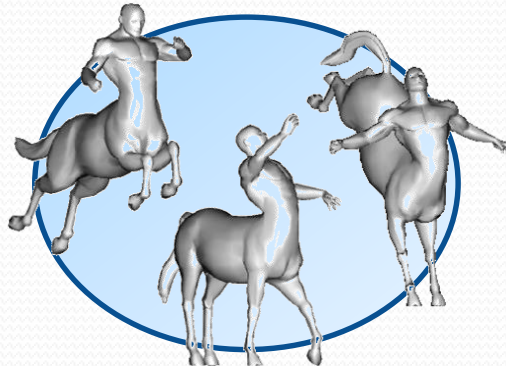
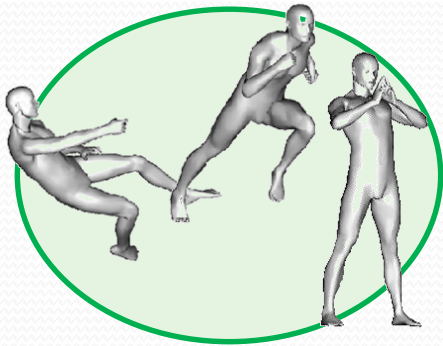
Gromov-Hausdorff distance

$$D_{\mathcal{GH}} \left(\text{standing figure}, \text{crouching figure} \right) = 0.82$$
The image shows two 3D models of a human figure, rendered in a solid blue color. The figure on the left is in a standing pose, facing slightly to the right. The figure on the right is in a crouching pose, also facing slightly to the right. Both figures are positioned between two large, black, curved parentheses that are part of the Gromov-Hausdorff distance formula. A comma is placed between the two figures, indicating they are the two sets being compared.

Gromov-Hausdorff distance

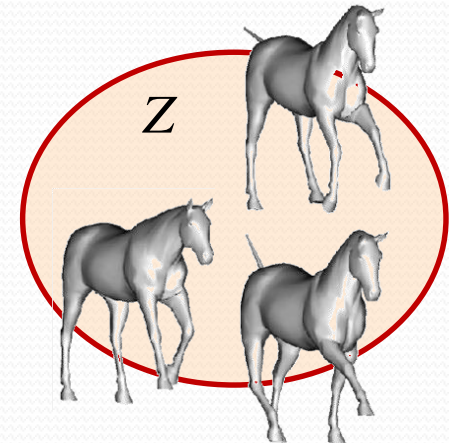
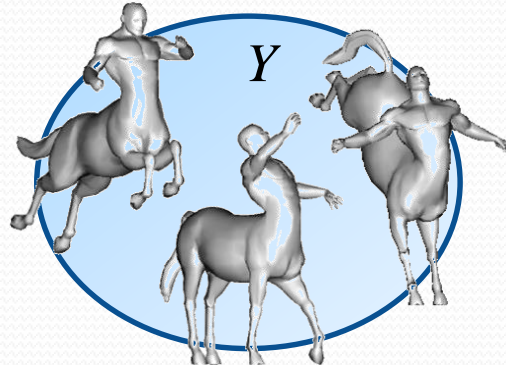
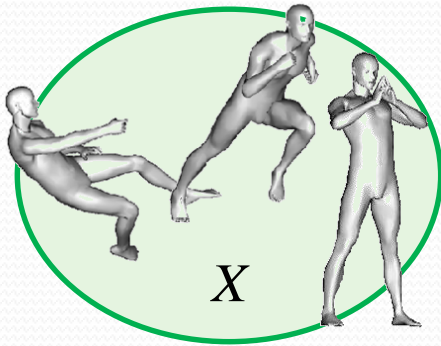
$$D_{\mathcal{GH}} \left(\text{Human}, \text{Dog} \right) = 10^3$$
A 3D rendering of a green human figure and a brown dog figure, illustrating the Gromov-Hausdorff distance between two shapes. The human figure is on the left, and the dog figure is on the right. They are separated by a comma, and the entire pair is enclosed in large parentheses. The equation is set against a white background with a blue wavy border at the top.

Space of shapes



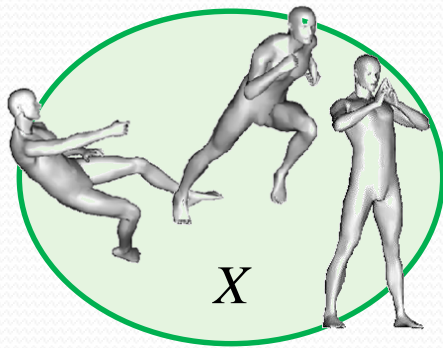
Is there something like a “space of shapes”?

Space of shapes

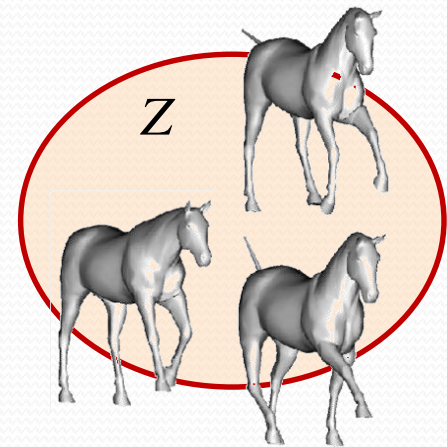
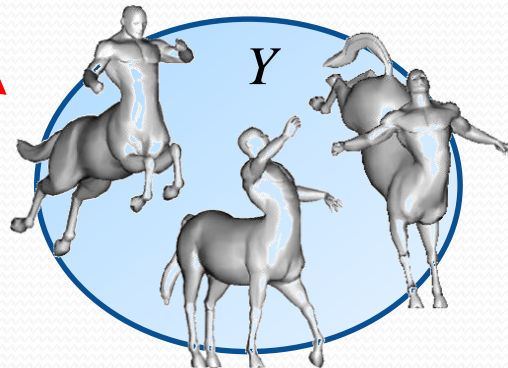


Is there something like a “space of shapes”? **Yes!**

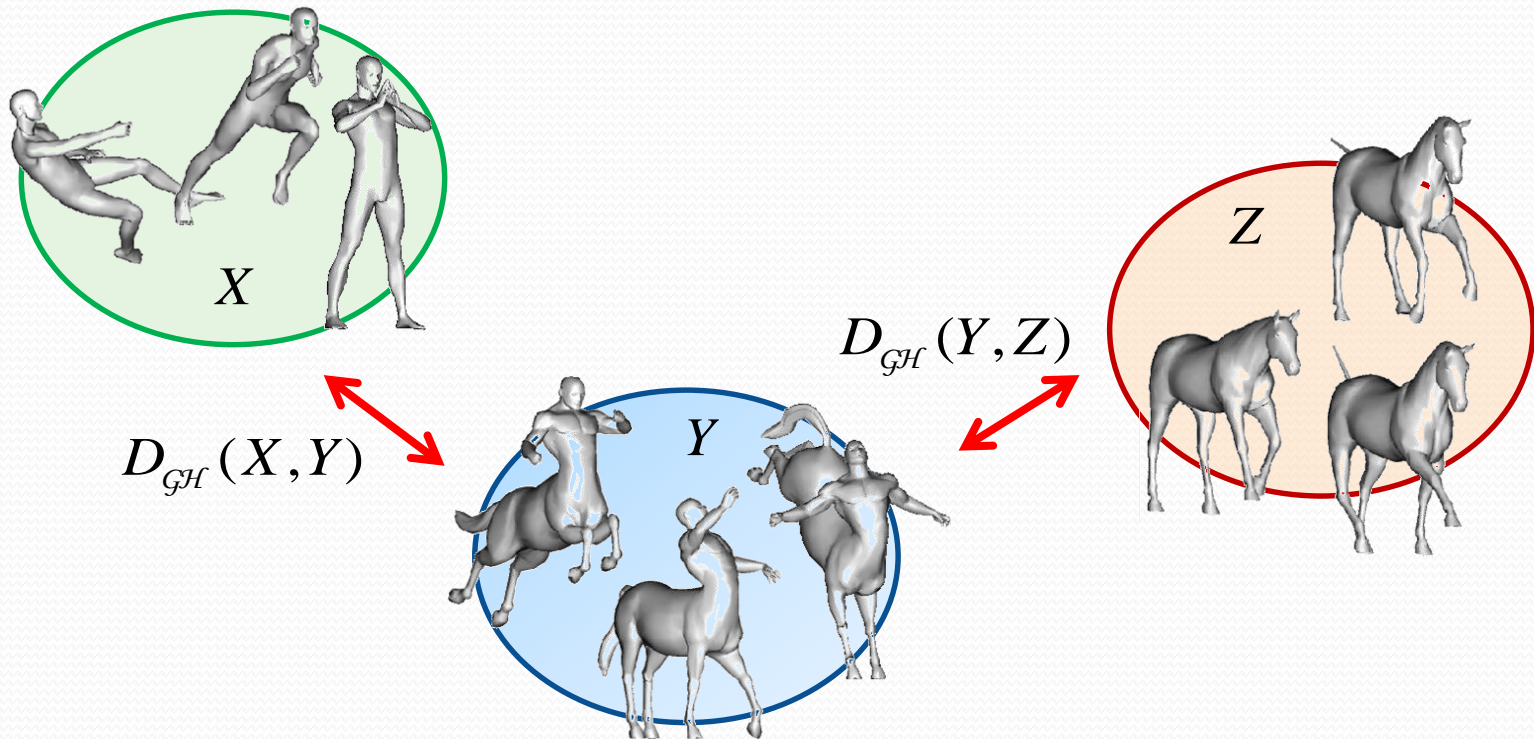
Space of shapes



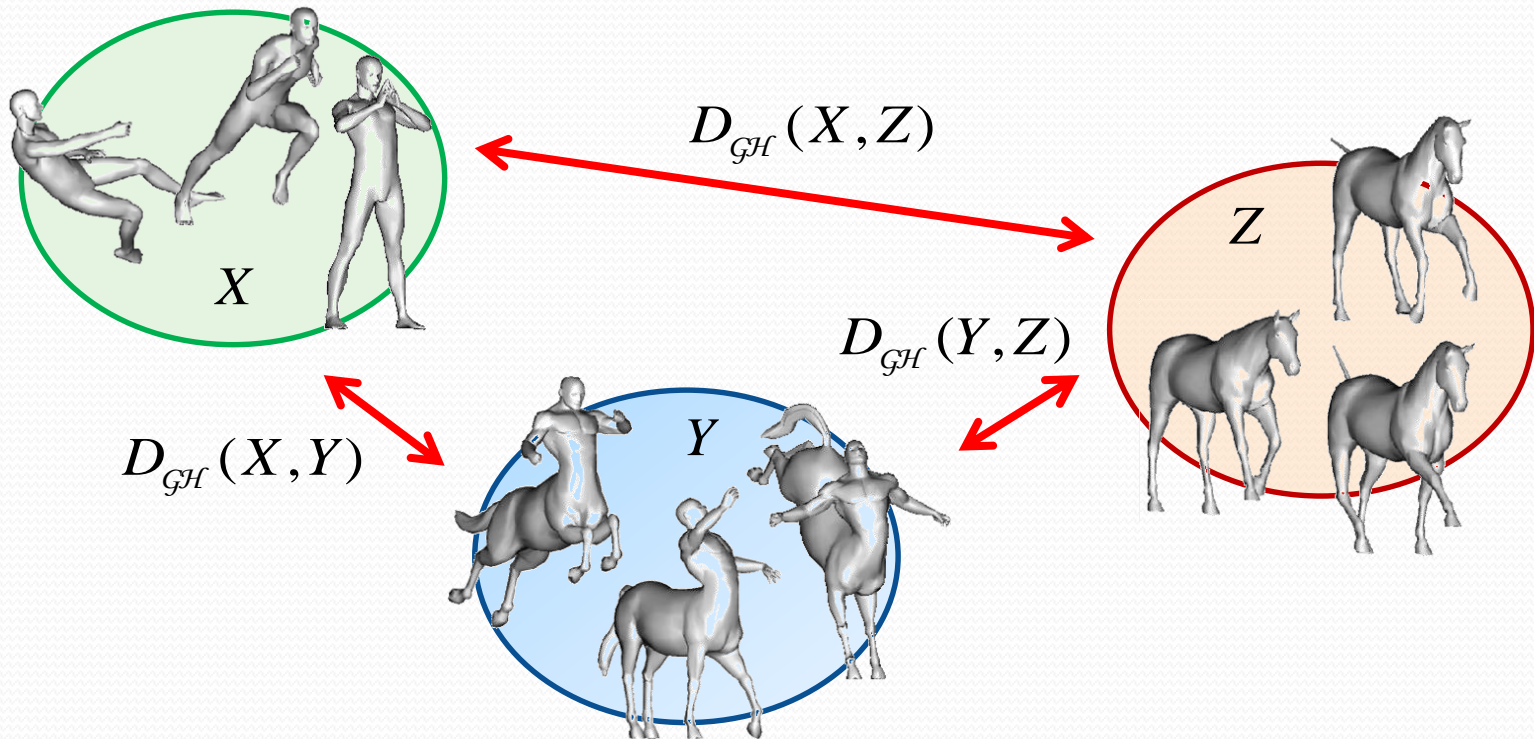
$D_{\mathcal{GH}}(X, Y)$



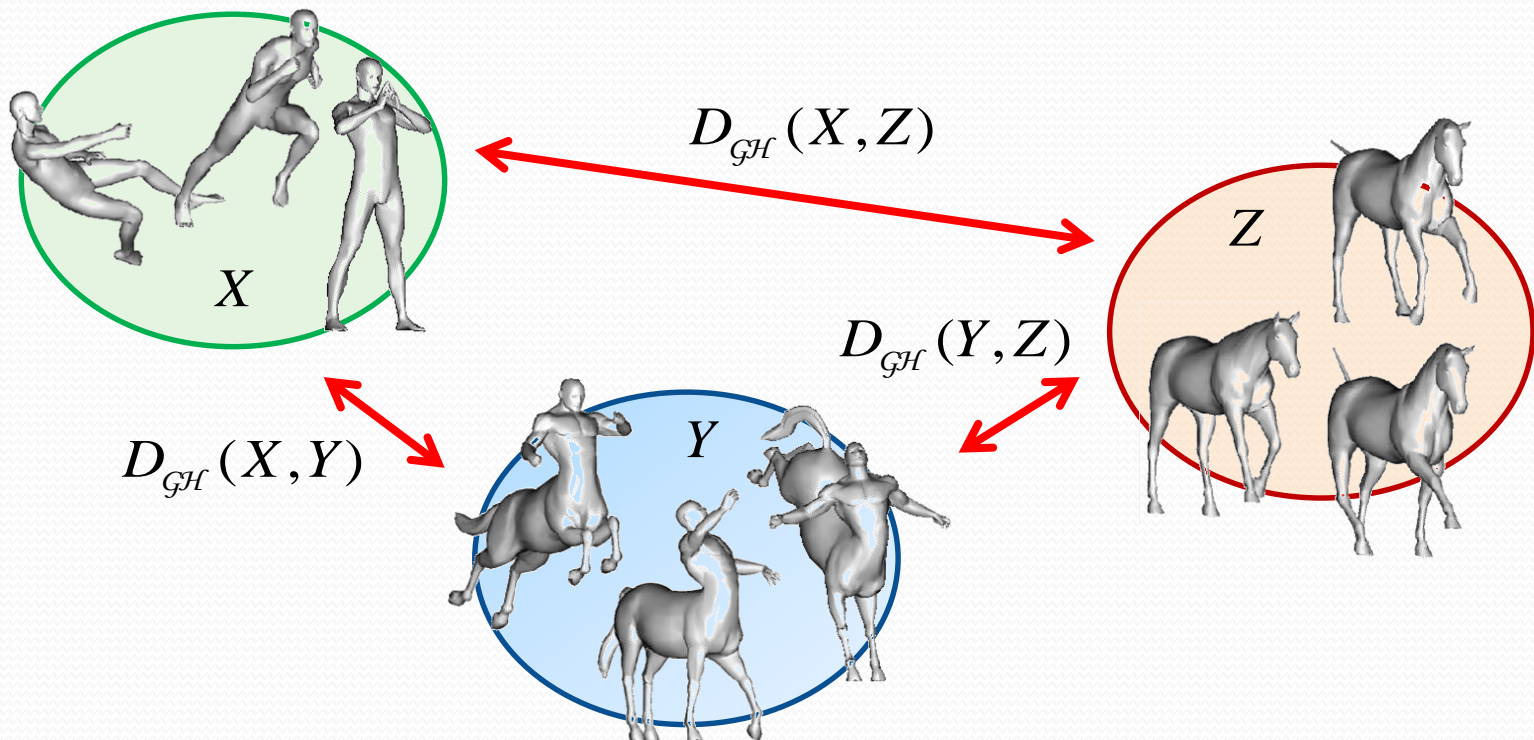
Space of shapes



Space of shapes



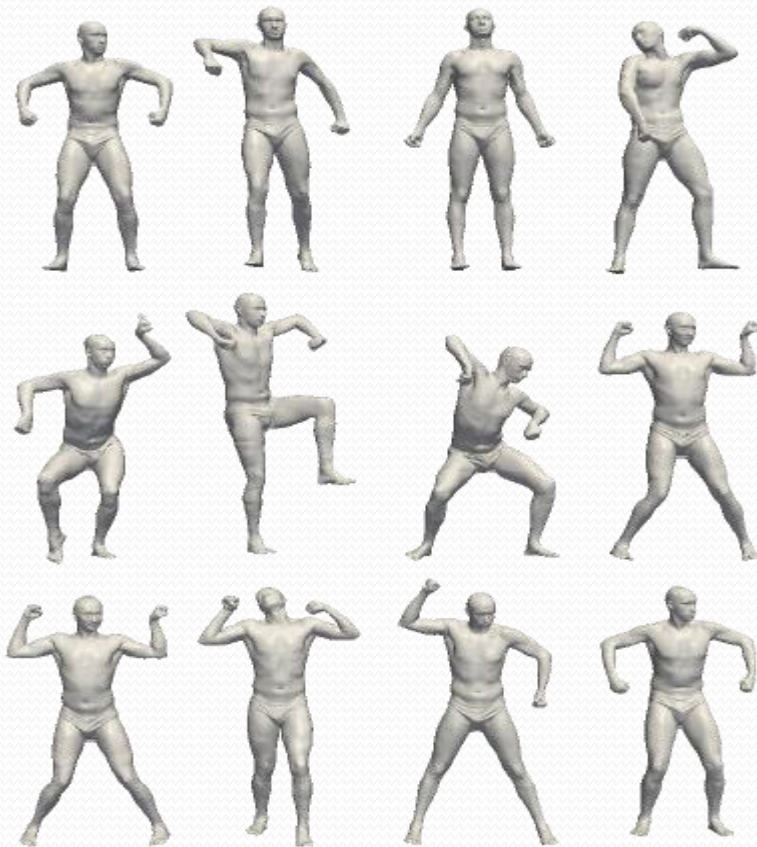
Space of shapes



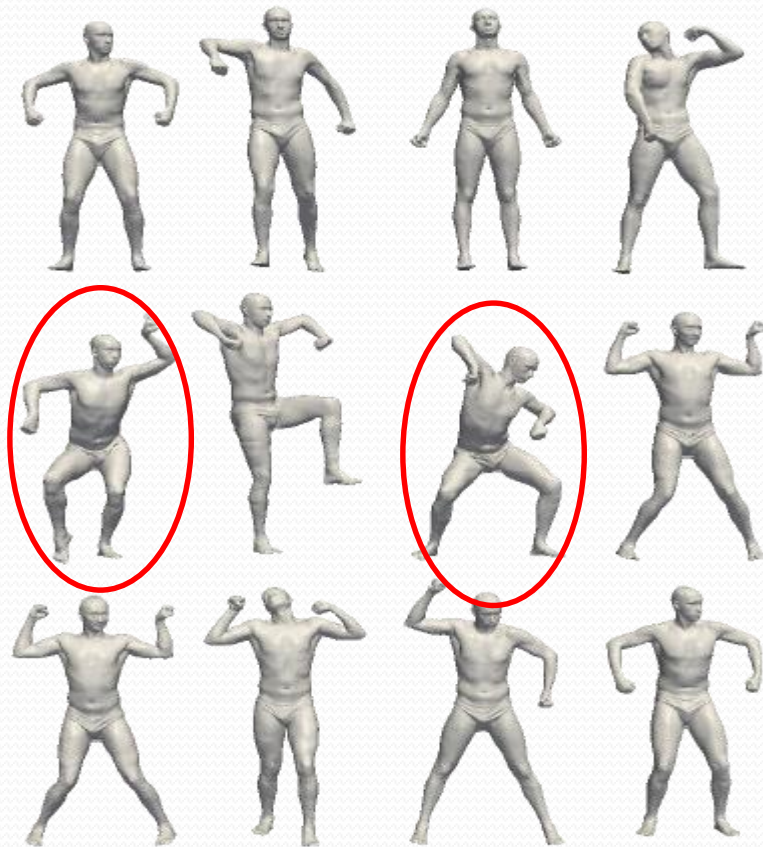
Triangle inequality: $D_{\mathcal{GH}}(X, Y) + D_{\mathcal{GH}}(Y, Z) \geq D_{\mathcal{GH}}(X, Z)$

Beyond two shapes

- Let us consider an entire **collection** of shapes

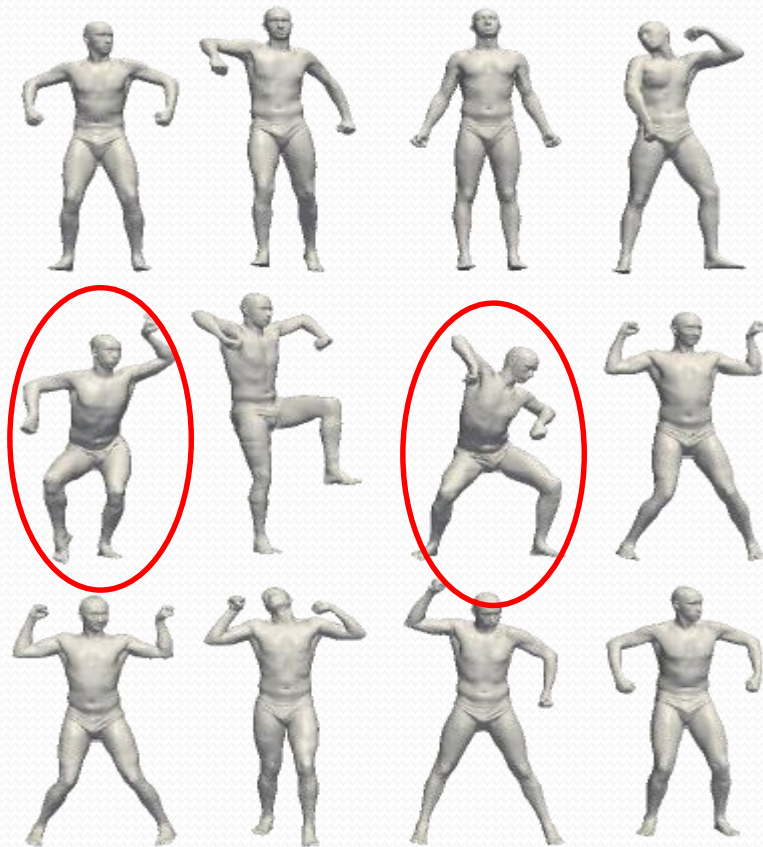


Beyond two shapes



Difficult to match!

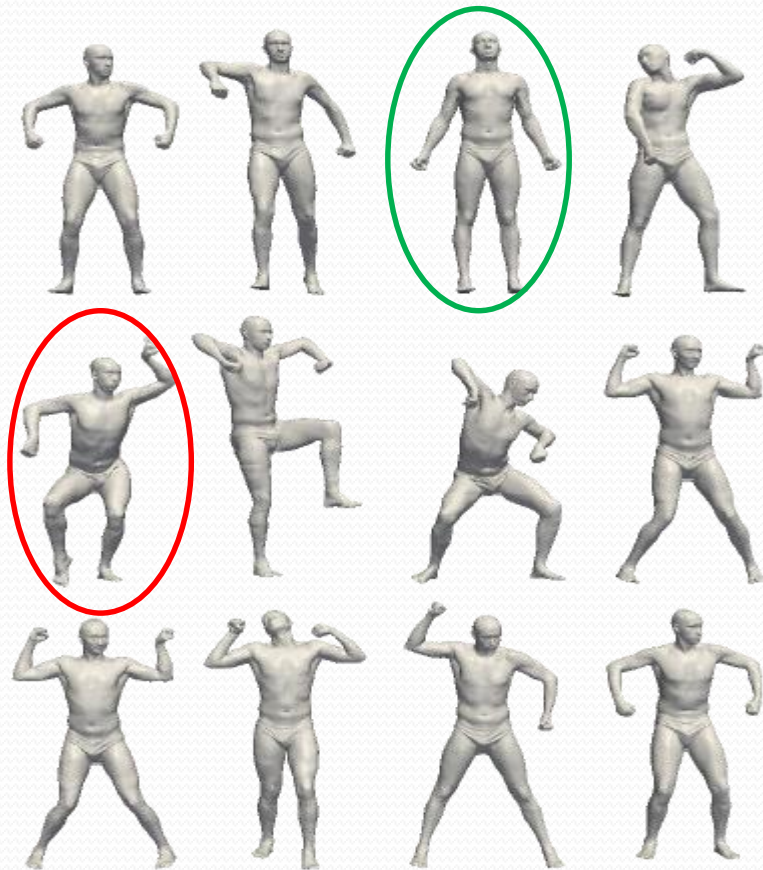
Beyond two shapes



Difficult to match!

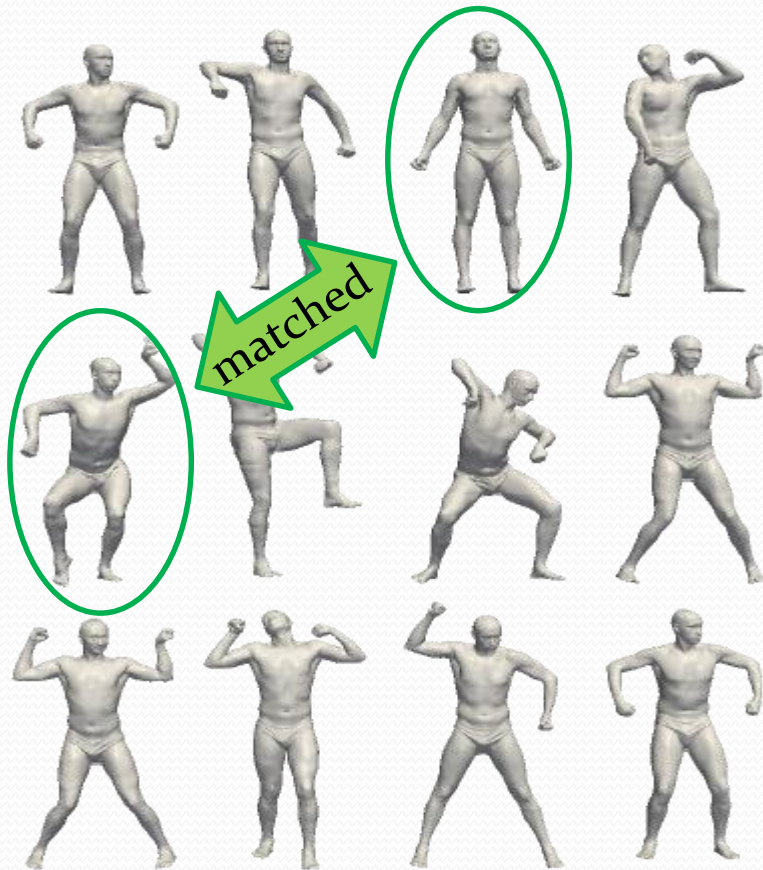
Can we use additional information to produce better correspondences?

Beyond two shapes

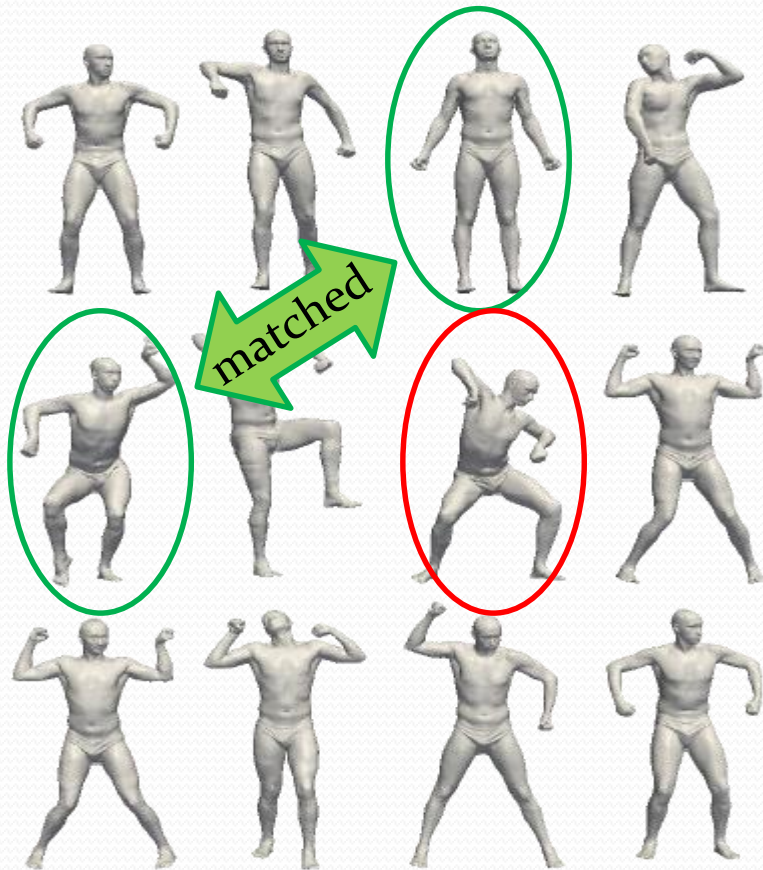


Easier to match!

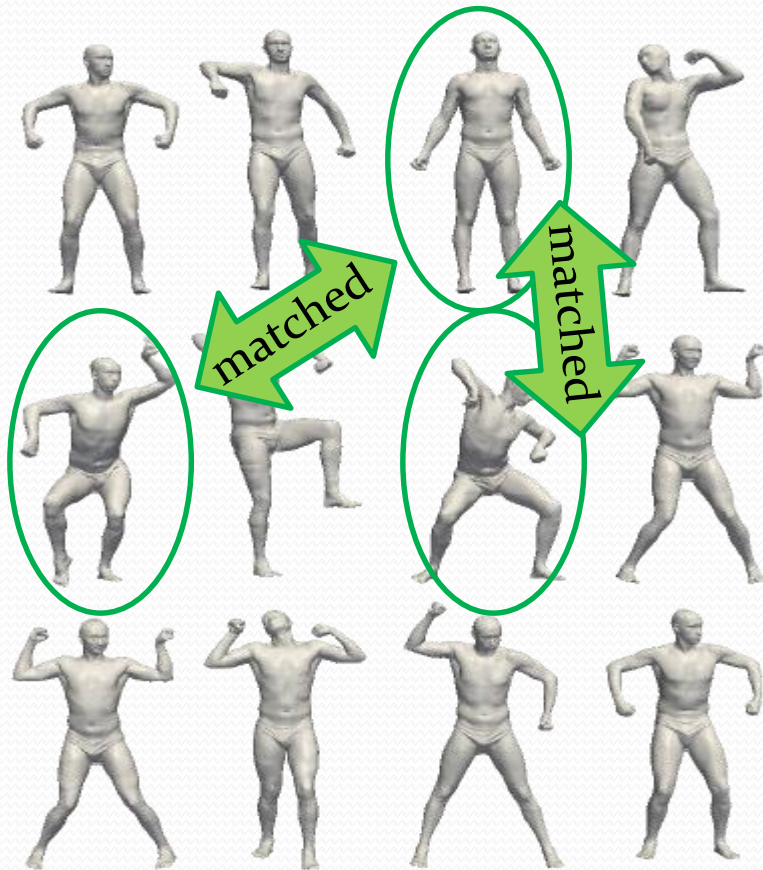
Beyond two shapes



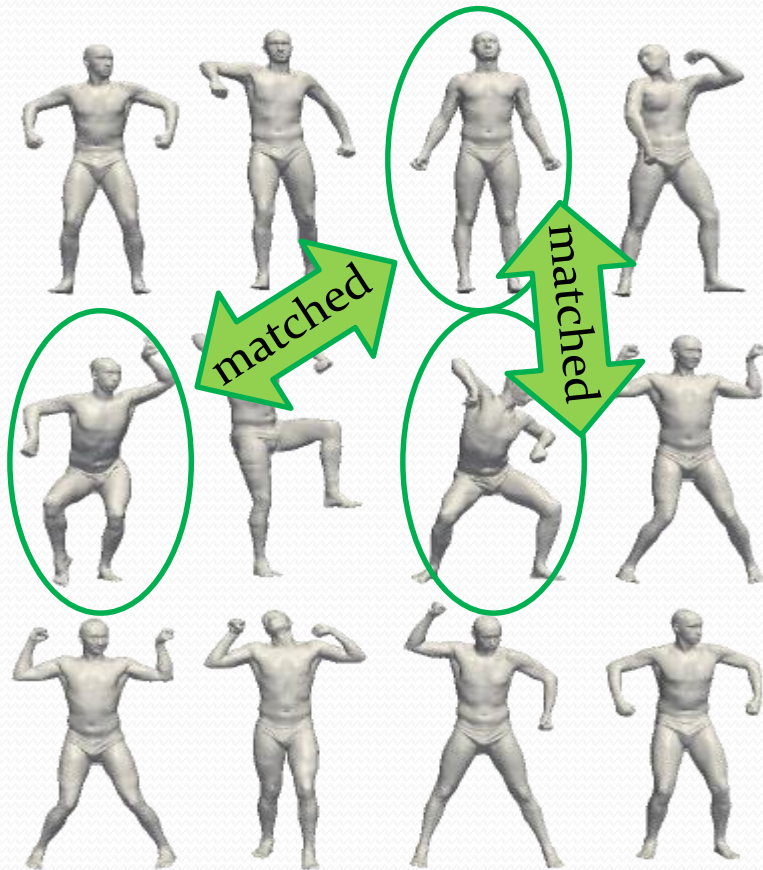
Beyond two shapes



Beyond two shapes

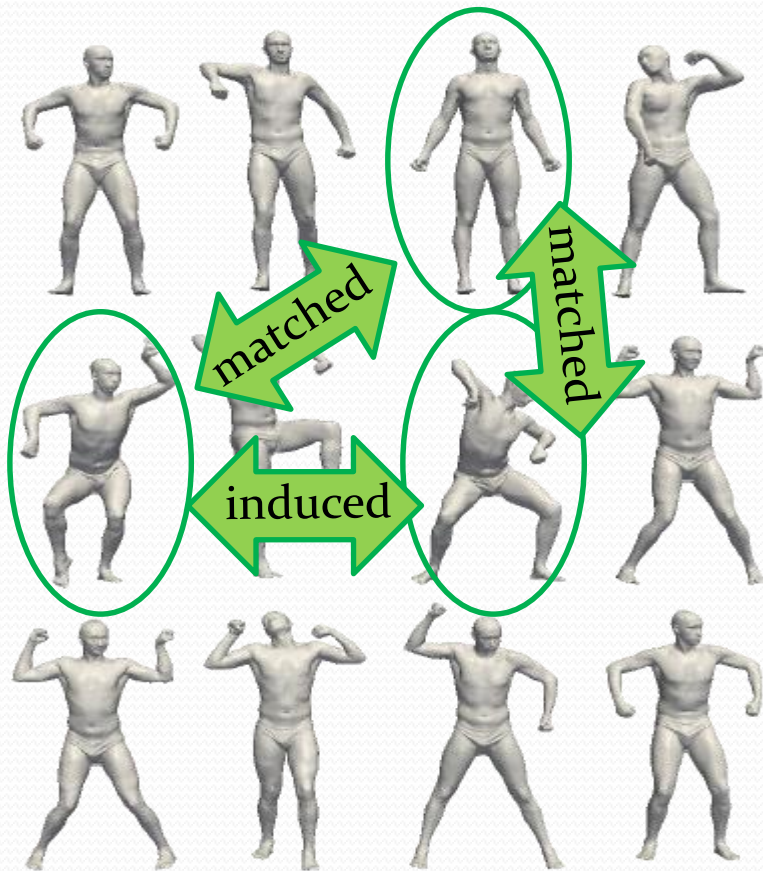


Beyond two shapes



A correspondence can now be induced by transitivity or “triangle consistency”

Beyond two shapes



A correspondence can now be induced by transitivity or “triangle consistency”

Suggested readings

- *Numerical geometry of non-rigid shapes*. Chapter 1 – Introduction.