Analysis of Three-Dimensional Shapes
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# Exercise Sheet 5

Room: 02.09.023

Tue, 17.06.2014, 13:45-15:15

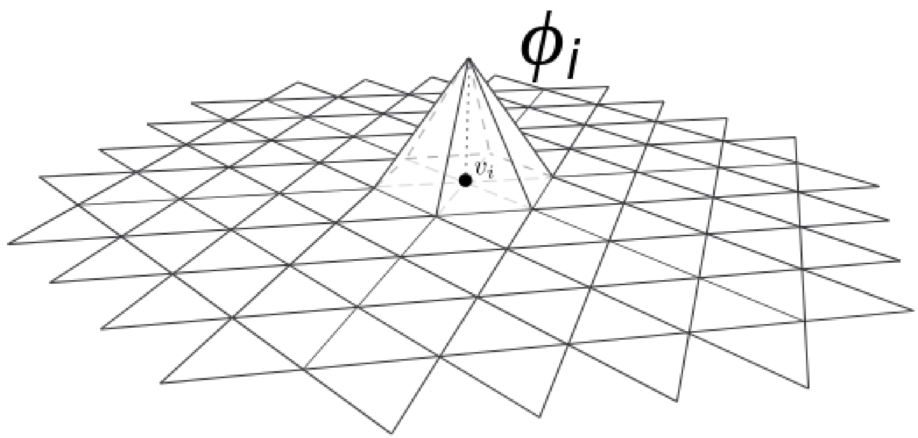
Submission deadline: Mon, 16.06.2014, 23:59 to windheus@in.tum.de

## Mathematics: Laplacian

**Exercise 1** (One Point). *In the lecture we have constructed a piecewise linear function* $f : S \to \mathbb{R}$ *by*

$$f(x) = \sum_i f_i \phi_i(x)$$

*where the basis functions* $\phi_i$ *are linear in each triangle and* $\phi_i(v_j) = \delta_{ij}$*:*




*Calculate the inner products* $-\langle \nabla\phi_i, \nabla\phi_j \rangle_S$ *which are the elements of the stiffness matrix* $C$*.*

**Hint: Use the same technique we used in the lecture to calculate the elements of the *mass matrix* $M$.**

**Exercise 2** (One Point). *In this exercise we investigate the eigenvectors of the Laplace matrix* $L = M^{-1}C$*.*

1. *Show that* $\psi$ *is an eigenvector of* $L$ *with eigenvalue* $\lambda$ *iff is a solution to the generalized eigenvalue problem*

$$\lambda M\psi = \lambda C\psi$$

2. *Show that* $\langle \cdot, \cdot \rangle_M := \langle \cdot, M \cdot \rangle$ *defines an inner product.*

3. *Show that the Laplacian matrix* $L$ *is symmetric with respect to* $\langle \cdot, \cdot \rangle_M$*.*

4. *Let* $\Psi$ *be the matrix with the eigenvectors of* $L$ *as columns:*

$$\Psi = \begin{pmatrix} | & & | \\ \psi_1 & \ldots & \psi_n \\ | & & | \end{pmatrix}$$

*What can you say about* $\Psi^{-1}$*?*

5. *Waht can you say about the eigenvalues of* $L$*?*

# Programming: The Discrete Laplace Operator

**Exercise 3** (Two points)**.** *Download and expand the file* `exercise5.zip` *from the lecture website. Modifiy the files* `cotanmatrix.m`*,* `massmatrix.m`*,* `heatsimulation.m`*, and* `hks.m` *to implement the functions as explained below. You can run the script* `exercise.m` *to test and visualize your solution.*

`cotanmatrix.m` *The function should compute the matrix* $C \in \mathbb{R}^{n \times n}$ *based on the cotangent scheme as defined in the lecture. The triangle mesh is given by matrices* $V \in \mathbb{R}^{n \times 3}$ *and* $F \in \mathbb{N}^{m \times 3}$*, where* $n$ *is the number of vertices and* $m$ *the number of triangles.* $C$ *should be returned in sparse format.*

`massmatrix.m` *The function should compute the matrix* $M \in \mathbb{R}^{n \times n}$ *based on the scheme defined in the lecture. The triangle mesh is given by matrices* $V \in \mathbb{R}^{n \times 3}$ *and* $F \in \mathbb{N}^{m \times 3}$*, where* $n$ *is the number of vertices and* $m$ *the number of triangles.* $M$ *should be returned in sparse format.*

`exercise.m` *Look at the code for the eigen decomposition. You see it is very easy to compute the generalized eigen decomposition* $\lambda M \phi = C \phi$ *by the matlab function* `eigs`*.*

`heatsimulation.m` *Given some initial heat distribution* $u_0 \in \mathbb{R}^n$*, the function simulates the diffusion of heat on the mesh. The function should display the distribution of heat at several given time points* $t_1, \ldots, t_T \in \mathbb{R}_+$*.*

`hks.m` *The function should compute for each point on the mesh the heat kernel signature at time points* $t_1, \ldots, t_T \in \mathbb{R}_+$*.*