



Multiple View Geometry: Exercise Sheet 8

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Exercise: June 16th, 2014

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Denote by $[R, T]$ the relative coordinate transformation between two camera views. Then the associated essential matrix is denoted by $F = \hat{T}R$. Show that the following holds for the epipoles e_1 and e_2 :

$$e_2 \sim T \quad \text{and} \quad e_1 \sim R^\top e_2$$

where \sim means equivalence in the sense of homogeneous coordinates.

Hint: Consider the relations between the epipoles and the fundamental matrix, i.e.

$$F e_1 = 0 \quad \text{and} \quad e_2^\top F = 0. \quad (\text{compare Exercise Sheet 6})$$

2. Suppose p_1, p_2 are two points on the line L . Let x_1, x_2 be the images of the points p_1, p_2 , respectively, and let l be the coimage of the line L .

Furthermore suppose L_1, L_2 are two lines intersecting in the point p . Let x be the image of the point p and let l_1, l_2 be the coimages of the lines L_1, L_2 , respectively.

Draw a picture and convince yourself of the following relationships:

- (a) Show that

$$l \sim \hat{x}_1 x_2, \quad x \sim \hat{l}_1 l_2,$$

- (b) Show that for some $r, s, u, v \in \mathbb{R}^3$,

$$l_1 \sim \hat{x} u, \quad l_2 \sim \hat{x} v, \quad x_1 \sim \hat{l} r, \quad x_2 \sim \hat{l} s$$

where \sim means equivalence in the sense of homogeneous coordinates.

3. Let x_1 and x_2 be two image points with projection matrices Π_1, Π_2 . Show that the rank constraint

$$\text{rank} \begin{pmatrix} \hat{x}_1 \Pi_1 \\ \hat{x}_2 \Pi_2 \end{pmatrix} \leq 3$$

ensures that x_1 and x_2 are images (/ projections) of the same three-dimensional point X .

4. Suppose two projection matrices $\Pi = [R, T]$ and $\Pi' = [R', T'] \in \mathbb{R}^{3 \times 4}$ are related by a common transformation H of the form

$$H = \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} \in \mathbb{R}^{4 \times 4} \quad \text{where } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

That is, $[R, T]H \sim [R', T']$ are equal up to scale.

Show that Π and Π' give the same essential matrices ($E = \hat{T}R$ and $E' = \hat{T}'R'$) up to a scale factor.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Download the package `mvg_exerciseSheet_08.zip` from the website. Extract the images `batinria0.pgm` and `batinria1.pgm`. Their corresponding camera calibration matrices can be found in the file `calibration.txt`.
2. Show the two images with matlab and select a point in the first image. You can use the command `[x, y]=ginput(n)` to retrieve the image coordinates of a mouse click.
3. Think about where the corresponding epipolar line l_2 in the second image could be.
4. Now compute the epipolar line $l_2 = Fx_1$ in the second image corresponding to the point x_1 in the first image. To this end you will need to compute the fundamental matrix F between the two images.
(Use the calibration data from the file `calibration.txt`.)
5. Test your program for different points x_1 . What do you observe?
6. *Bonus: Determine the best matching point on the epipolar line via normalized cross correlation.*