Multiple View Geometry: Solution Exercise Sheet 8

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Part I: Theory

1.

$$\begin{aligned}
& e_2^\top F = 0 \qquad \text{(Exercise Sheet 5, Nr. 3)} \\
\Rightarrow\ & F^\top e_2 = 0 \\
\Rightarrow\ & (\hat{T}R)^\top e_2 = 0 \\
\Rightarrow\ & R^\top \hat{T}^\top e_2 = 0 \\
\Rightarrow\ & R^\top(-T \times e_2) = 0 \qquad \text{(because } \hat{T} \text{ is skew-symmetric)} \\
\Rightarrow\ & -T \times e_2 = 0 \\
\Rightarrow\ & e_2 \sim T
\end{aligned}$$

$$\begin{aligned}
& F e_1 = 0 \\
\Rightarrow\ & \hat{T} R e_1 = 0 \\
\Rightarrow\ & T \times R e_1 = 0 \\
\Rightarrow\ & T \sim R e_1 \\
\Rightarrow\ & e_1 \sim R^\top T \qquad \text{(because } R^\top R = I\text{)} \\
\Rightarrow\ & e_1 \sim R^\top e_2 \qquad \text{(because } T \sim e_2\text{)}
\end{aligned}$$

2. (a) l is coimage of L, and therefore l is normal vector to the plane that is determined by the camera position and L.

$$\Rightarrow \begin{array}{l} l^T x_1 = 0 \\ l^T x_2 = 0. \end{array}$$

$$\Rightarrow l \sim x_1 \times x_2 = \hat{x_1} x_2.$$

l_1 and l_2 are normal vectors to the planes through camera position and L_1, L_2 respectively.

$$\Rightarrow \begin{array}{l} l_1^T x = 0 \\ l_2^T x = 0 \end{array}$$

$$\Rightarrow x \sim l_1 \times l_2 = \hat{l_1} l_2.$$

(b) i. $l_1 \sim \hat{x} u$:
x is in the preimage of L_1. $\Rightarrow l_1^\top x = 0$.
$\exists$ point $u \neq p$ in L_1. $\Rightarrow l_1^\top u = 0$
$\Rightarrow l_1 \sim \hat{x} u$.

ii. $l_2 \sim \hat{x} v$: analog to i.

iii. $x_1 \sim \hat{l} r$:
x_1 is in the preimage of L. $\Rightarrow x_1^\top l = 0$
$\exists$ a line L' through p_1 with coimage $r \neq l$. $\Rightarrow x_1^\top r = 0$.
$\Rightarrow x_1 \sim \hat{l} r$.

iv. $x_2 \sim \hat{l} s$: analog to iii.

3. rank $\begin{pmatrix} \hat{x_1}\Pi_1 \\ \hat{x_2}\Pi_2 \end{pmatrix} \leqq 3$

$\Rightarrow \exists X \in \mathbb{R}^4 \backslash \{0\}$ with $\begin{pmatrix} \hat{x_1}\Pi_1 \\ \hat{x_2}\Pi_2 \end{pmatrix} X = 0.$

$\Rightarrow \hat{x_1}\Pi_1 X = 0 \;\; \wedge \;\; \hat{x_2}\Pi_2 X = 0,$

$\Rightarrow x_1 \times \Pi_1 X = 0 \;\; \wedge \;\; x_2 \times \Pi_2 X = 0.$

$\Rightarrow x_1$ and $\Pi_1 X$ are linearly dependent; and x_2 and $\Pi_2 X$ are linearly dependent.

$\Rightarrow \exists \lambda_1, \lambda_2 \in \mathbb{R}$ with $\Pi_1 X = \lambda_1 x_1 \;\; \wedge \;\; \Pi_2 X = \lambda_2 x_2$

$\Rightarrow x_1$ and x_2 are projections of X.

4. $\exists\, \lambda \in \mathbb{R} : [R', T'] = \lambda\, [R, T] H = \lambda\, [R, T] \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} = \lambda\, [R + T v^\top, T v_4]$

$$\begin{aligned} E' &= \hat{T}' R' \\ &= (\widehat{\lambda v_4 T}) \cdot (\lambda (R + T v^\top)) \\ &= \lambda^2 v_4 \hat{T} (R + T v^\top) \\ &= \lambda^2 v_4 \hat{T} R + \lambda^2 v_4 \underbrace{\hat{T} T}_{=0} v^\top \\ &= \lambda^2 v_4 \hat{T} R \\ &= \lambda^2 v_4 E \quad \text{with } \lambda^2 v_4 \in \mathbb{R} \end{aligned}$$

$\Rightarrow \;\; E' \sim E$