Multiple View Geometry: Exercise Sheet 10

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http://vision.in.tum.de/teaching/ss2014/mvg2014

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Variational Image Smoothing

Part I: Theory

1. What is the goal of image smoothing?

2. Two functionals are given by

$$E_1(u) = \frac{\lambda}{2} \int_\Omega (u(x) - f(x))^2 \, dx + \frac{1}{2} \int_\Omega |\nabla u(x)|^2 \, dx \tag{1}$$

$$E_2(u) = \frac{\lambda}{2} \int_\Omega (u(x) - f(x))^2 \, dx + \int_\Omega |\nabla u(x)| \, dx \tag{2}$$

   where $\Omega \subset \mathbb{R}^2$, $\lambda > 0$ and $f, u : \Omega \to \mathbb{R}$.

   (a) What is the difference between (1) and (2)?
   (b) What is the role of f and u?
   (c) Can you explain the meaning of the additive parts?

3. Compute the gradient descent equations for the two functionals (1) and (2).

   (a) Derive the Euler-Lagrange equation: $\frac{dE}{du} = \frac{\partial \mathcal{L}}{\partial u} - \frac{d}{dx}\frac{\partial \mathcal{L}}{\partial u'}$.
   (b) Compute the Euler-Lagrange equation for the two functionals (1) and (2).
   (c) Determine the update step for u^{t+1} for each functional.

   Hint:
   - Use: $-\frac{dE}{du} = \frac{\partial u}{\partial t} = \frac{u^{t+1} - u^t}{\tau}$, where τ is the stepsize to get the update scheme for u
   - If you need to divide by $|\nabla u|$, use a regularized version of the gradient norm: $|\nabla u|_\varepsilon = \sqrt{u_x^2 + u_y^2 + \varepsilon^2}$ for a small constant $\varepsilon \in \mathbb{R}_{>0}$.

Part II: Practical Exercises

1. Download the Matlab files (mvg_exerciseSheet_10.zip) from the website.

2. Implement the gradient descent for E_1.

3. Implement the gradient descent for E_2.