Analysis of Three-Dimensional Shapes
(IN2238, TU München, Summer 2015)

Shape Analysis @TUM
(29.06.2015)

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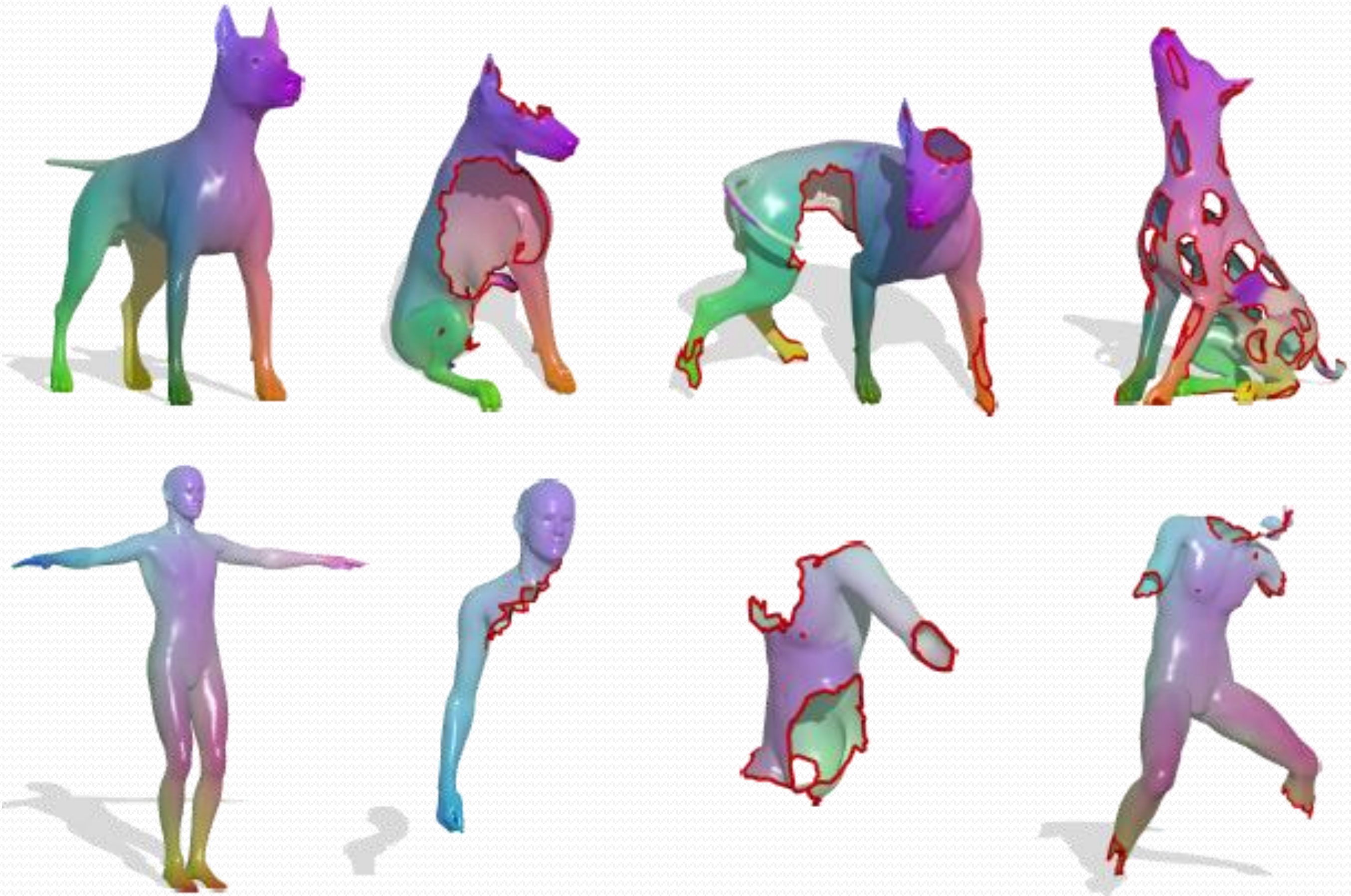
Partial matching

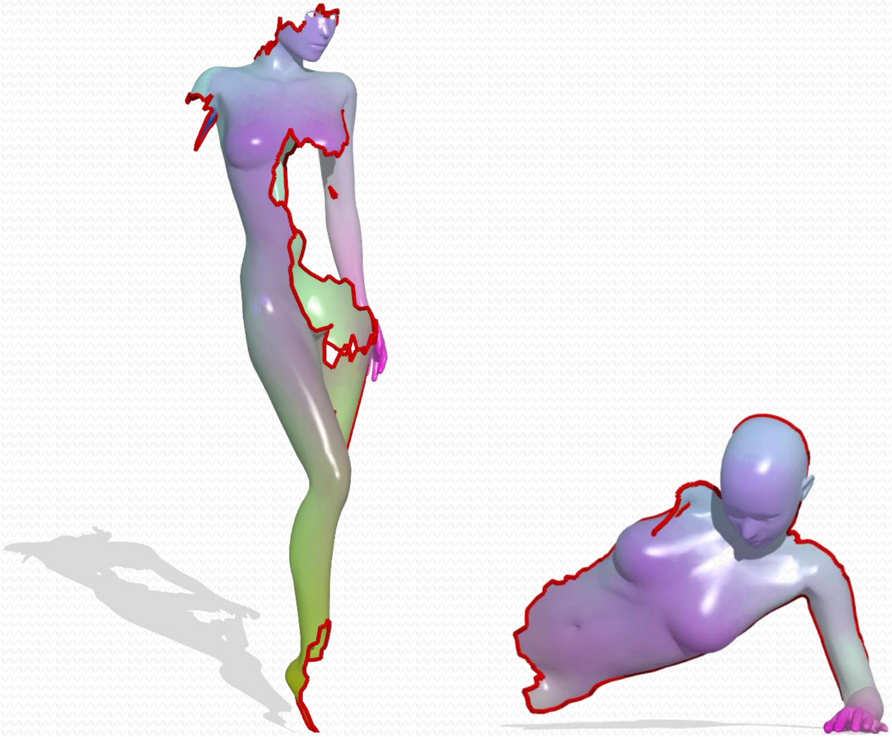


Partial matching

We only considered the "simple" case in which on of the two shapes is full (*i.e.* it can be seen as a template).

What happens **if both of them are incomplete**? In this case, we wouldn't even know how much they overlap!

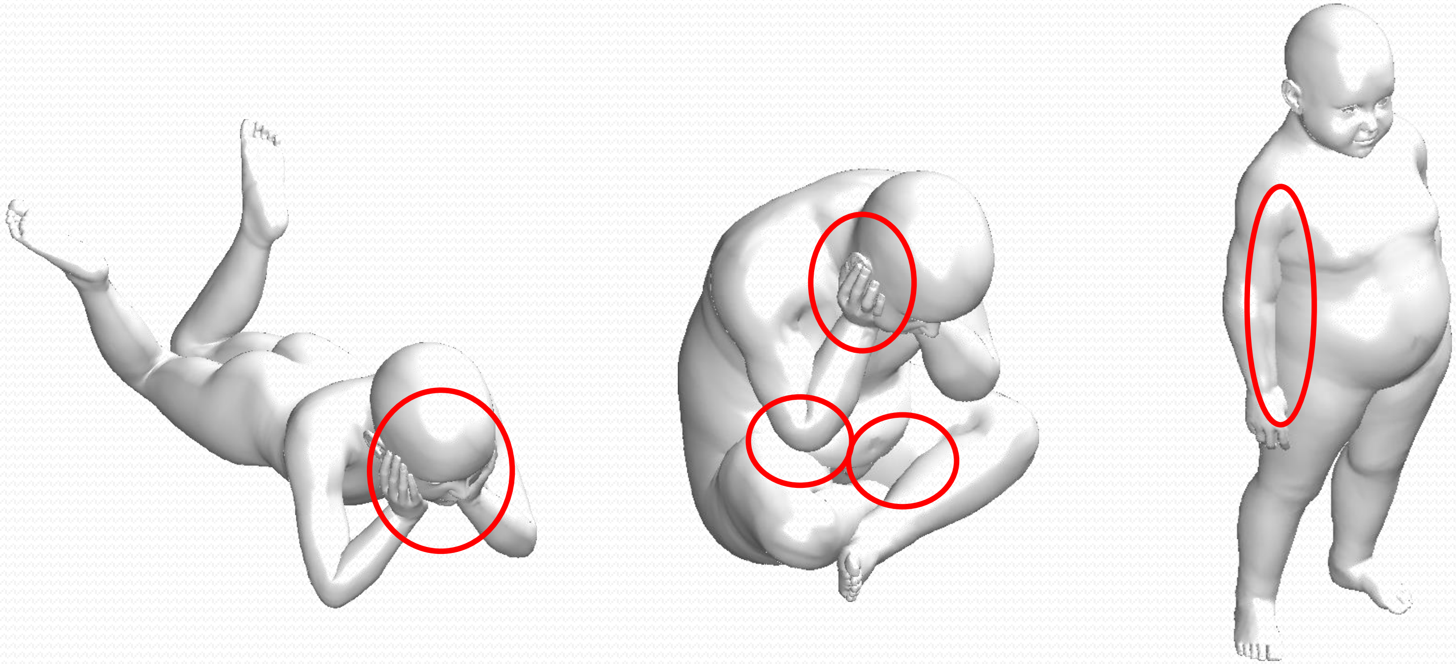
Partial matching



- Unknown overlap
- Topological changes?
- Non-isometric deformations?
- **Clutter?**

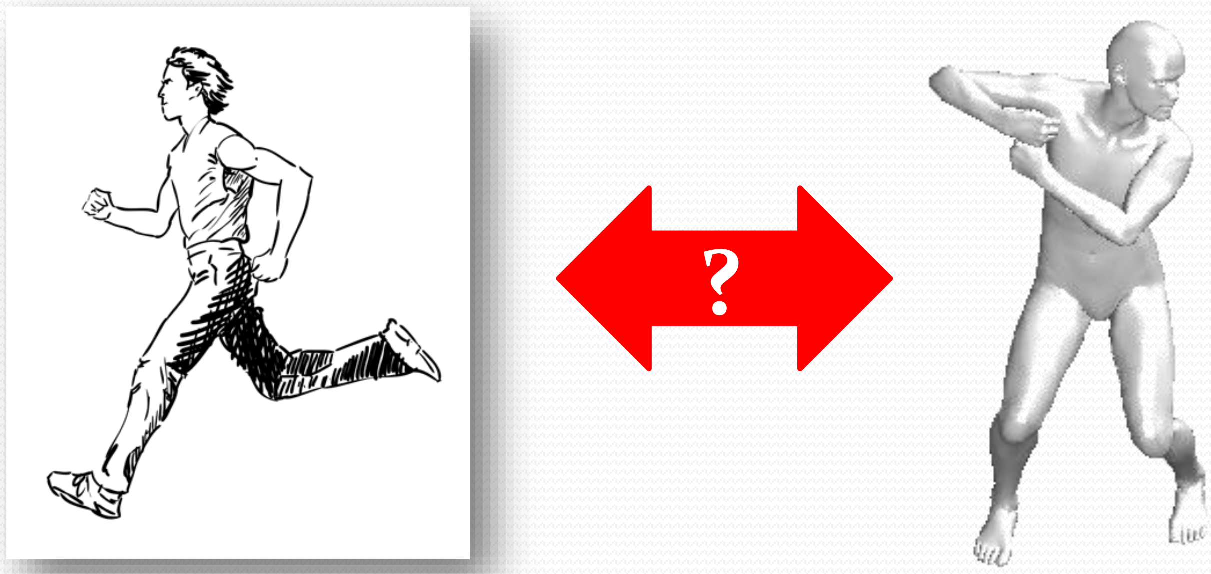
Topological changes

We published a dataset for **deformable shape matching under topological changes**. (http://vision.in.tum.de/data/datasets/kids)



The matching problem in this case is very challenging and there is limited research in this direction. **Can we make the change?**

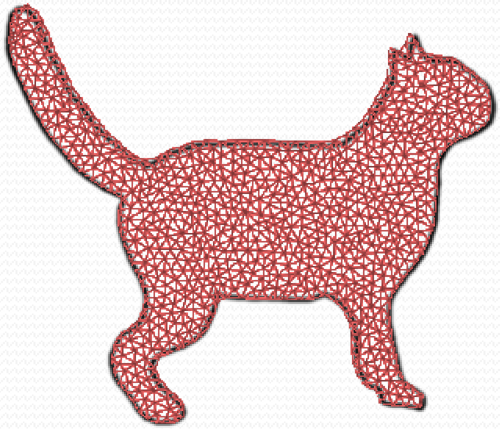
2D-to-3D Matching

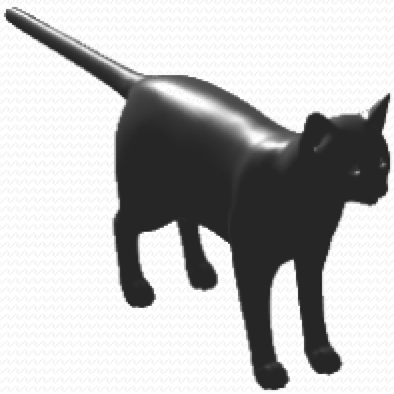


- Are they **similar**?
- Can we find a **point-to-point correspondence**?
- Can we find a **functional correspondence**?
- Can we **transfer the deformation** from one shape to the other?

2D-to-3D Matching

It turns out that we can indeed establish a functional correspondence between the drawing and the 3D model, if we take some simplifying assumptions.

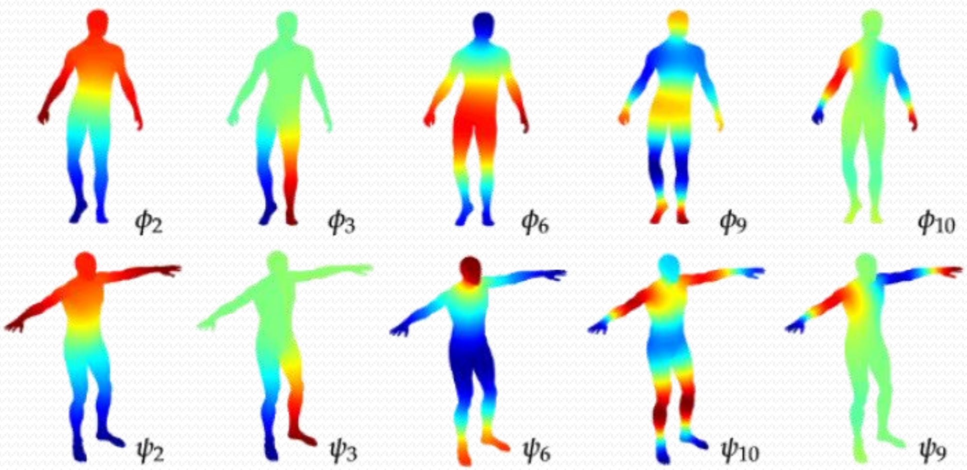




2D outline ⇨ flat 3D shape (2D manifold with boundary)

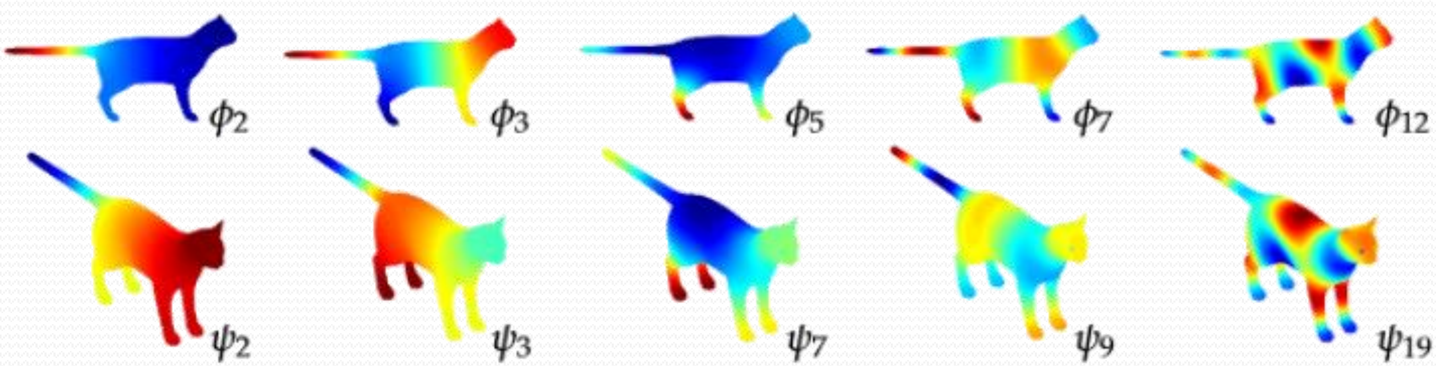
full 3D shape

2D-to-3D Matching



Surprisingly, we observed a similar behavior to the 3D-to-3D partial matching case!

What information is the **boundary** giving us?



2D-to-3D Matching

We applied a similar approach to the one we used for partial matching:

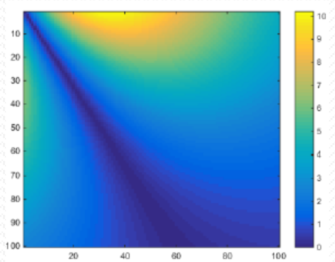
$$\min_C \|CA - B\|_F^2 + \mu\rho(C)$$

where we considered the following **convex** regularizers $\rho(C)$:

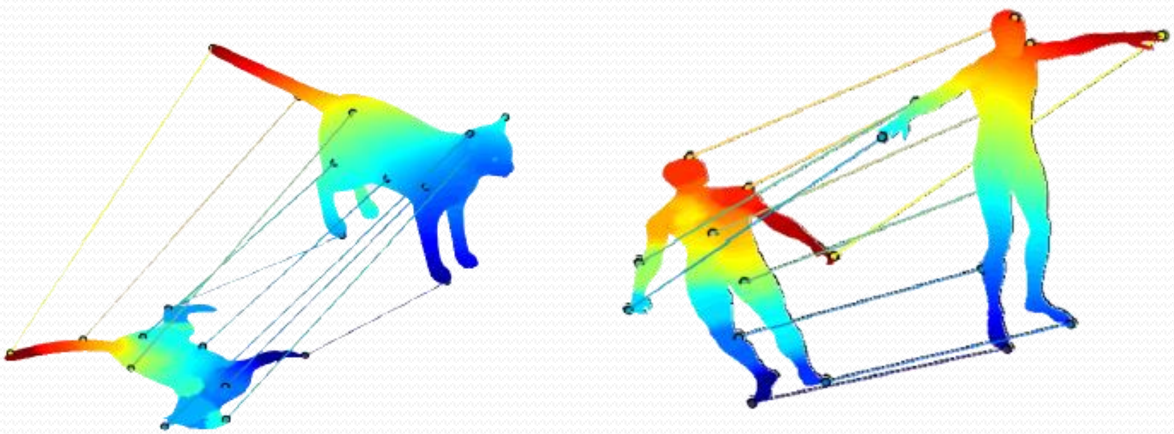
$$\|W \circ C\|_F^2 \qquad \|W \circ C\|_1 \qquad \|C\|_*$$

slanted-diagonal mask

low-rank functional map
(due to partiality)

$W =$ 

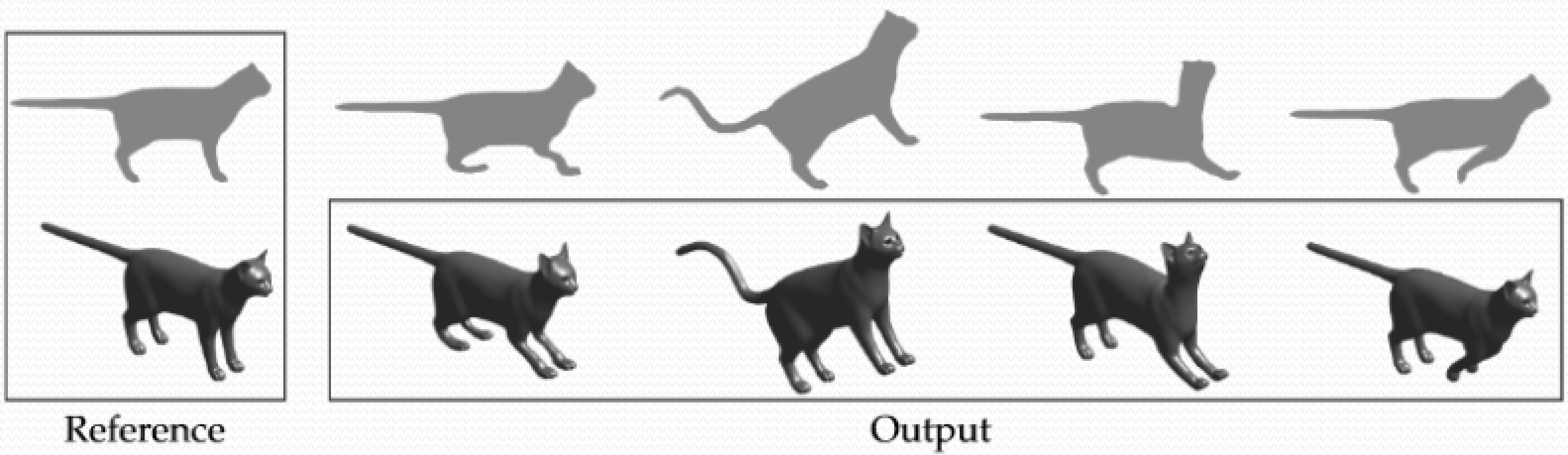
2D-to-3D Matching



Master's Thesis:

Aneta Stevanovic

"Dense 2D-to-3D Shape Matching via Low-Rank Functional Correspondence"



From functional to point-to-point

We have seen how to convert a given functional map $C \in \mathbb{R}^{k \times k}$ back to a point-to-point map $T : M \to N$.

The approach boils down to a nearest-neighbor search in $\mathbb{R}^k$. For each point $y \in N$, we look for the point $x \in M$ minimizing:

$$\min_{x \in M} \|C\Phi^T \delta_x - \Psi^T \delta_y\|_2$$

However, the resulting point-to-point map can (and will) be **one-to-many**.

In addition, if the two shapes are only **approximately** isometric, the *L2* distance might not be the best choice!

From functional to point-to-point

Let us take another look at the conversion process from point-to-point to functional:

$$C = \Psi^T P \Phi$$

We said that we can regard the functional map representation as simply a change of basis (from standard basis to Laplacian eigenbasis).

However, the moment we **truncate the basis** from $n = 10,000$ to $k = 50$ basis functions, there is a huge loss of information!

$$C = \Psi^T P \Phi$$

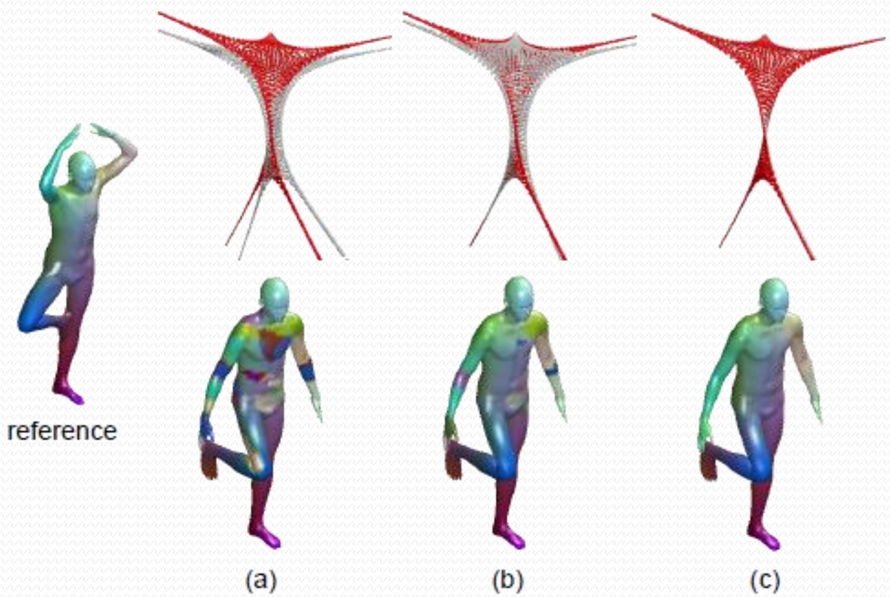
rank at most 50 rank at most 10,000

From functional to point-to-point

Thus, the inverse conversion is **ill-posed and highly underdetermined**:

$$P = \Psi C \Phi^T$$

We proposed to rephrase the point-to-point map recovery problem in a probabilistic setting. Simply put, we perform a non-rigid alignment in $\mathbb{R}^k$:



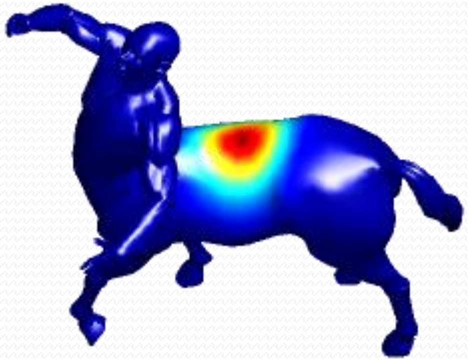
Anisotropic Laplacian

We defined the Laplacian on a manifold as the linear operator:

$$\Delta f = \mathrm{div}(\nabla f)$$

From the Laplacian we derived several interesting quantities such as the **heat kernel**, diffusion distance, GPS, and functional maps.

The heat kernel is an **intrinsic** quantity of the surface, and it is **isotropic**: the diffusion process does not depend on any particular direction, in other words **heat diffuses equally in all directions**.



Anisotropic Laplacian

Can we diffuse heat **along a particular direction** of our choice?

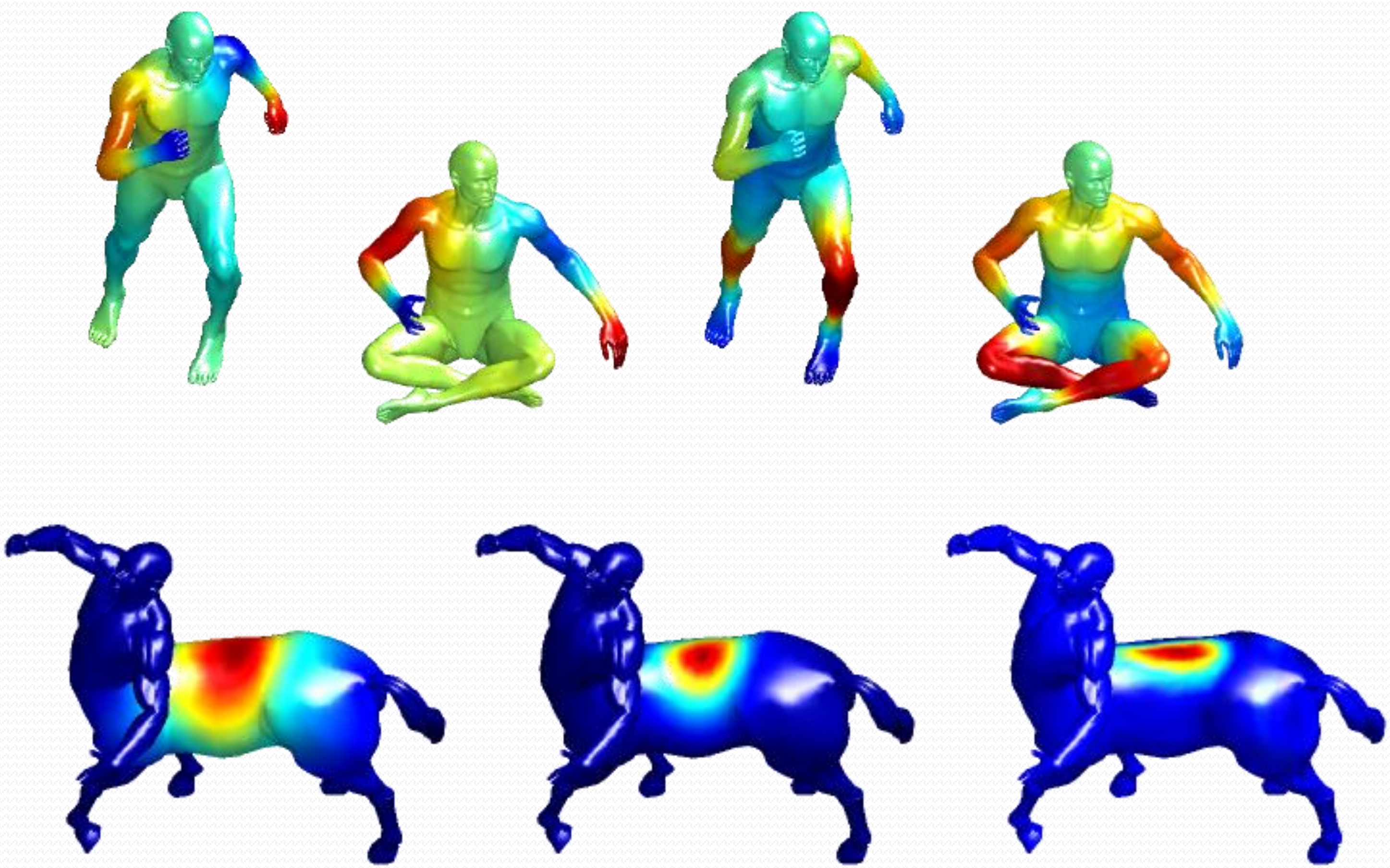
We did so by considering the following operator, which we called **anisotropic Laplace-Beltrami operator**:

$$\Delta f = \mathrm{div}(D(\nabla f))$$

where *D* is a diagonal matrix promoting faster diffusion along the directions of principal curvature.

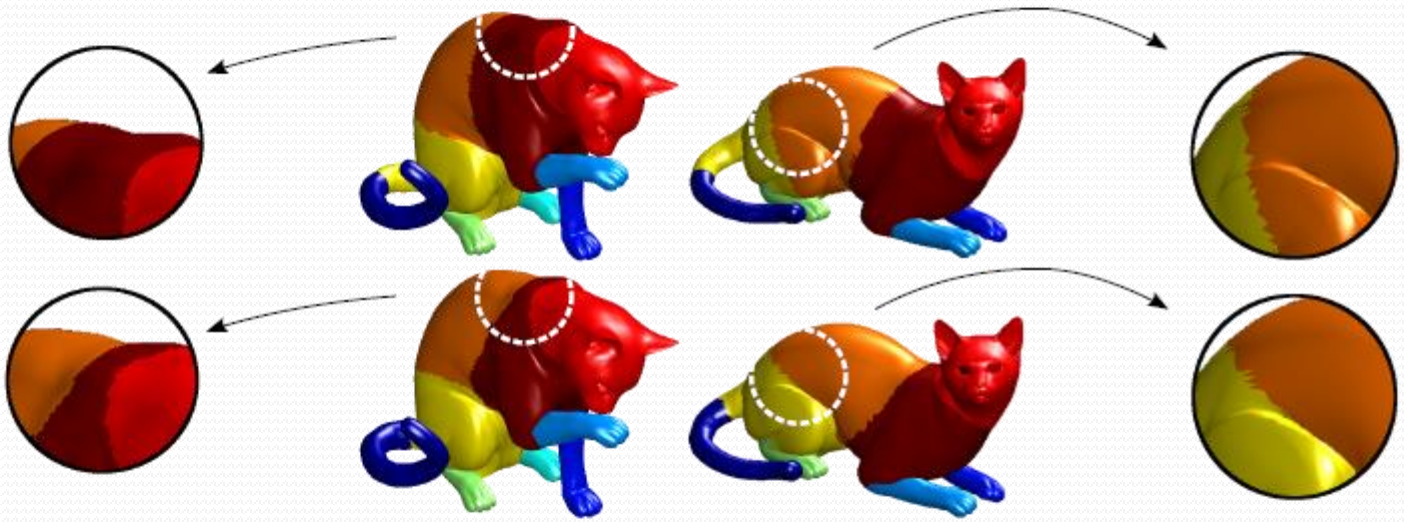
The resulting operator is still **linear**, and still admits an eigen-decomposition with **real eigenvalues** and **orthonormal eigenfunctions**.

Anisotropic Laplacian



Anisotropic Laplacian

Shape segmentation can take advantage of the anisotropy:



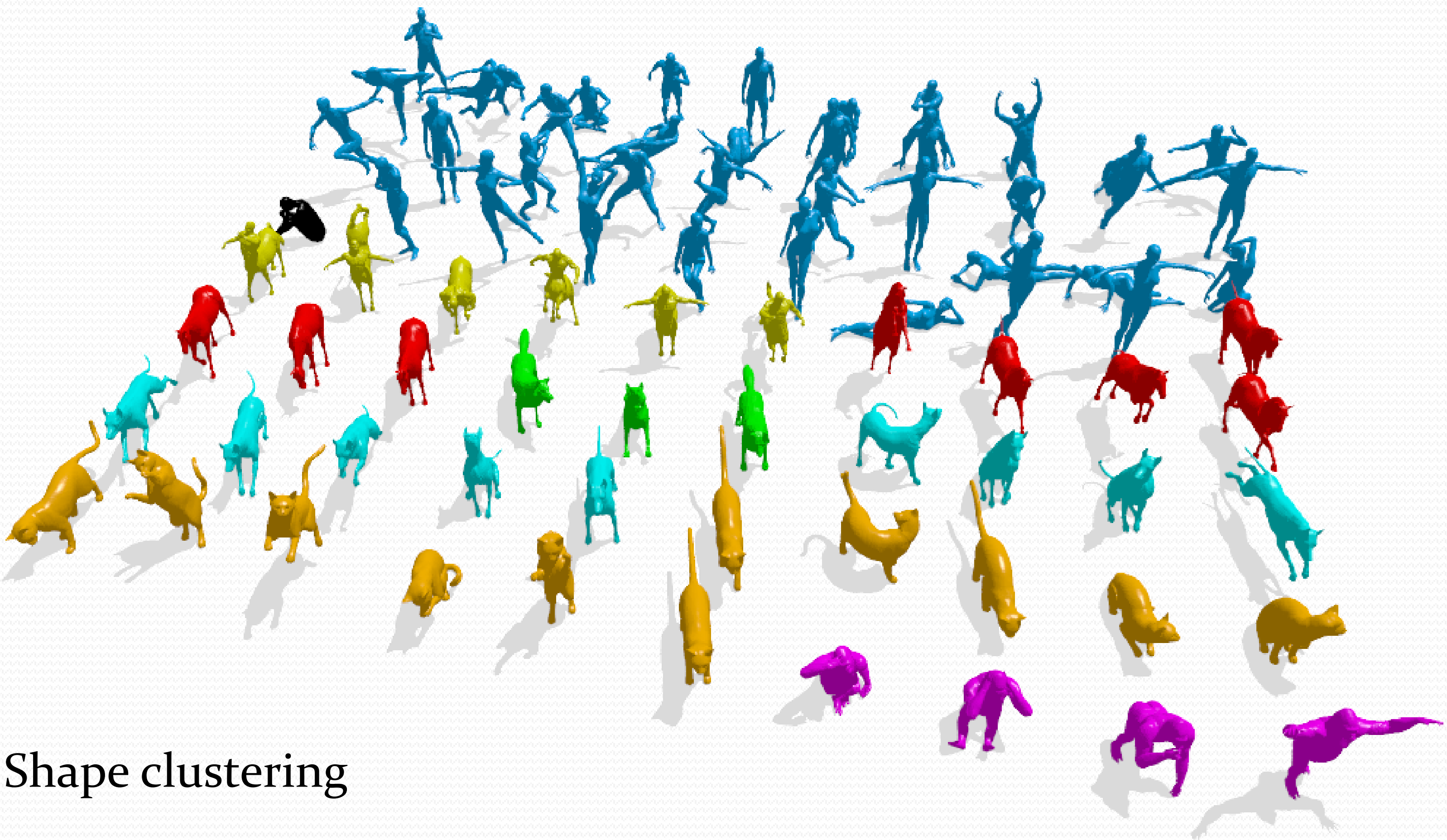
Open question: **What else can we do with this operator?**

Multi-way matching

As **3D shape collections** become increasingly available, there is a growing interest into analyzing them.

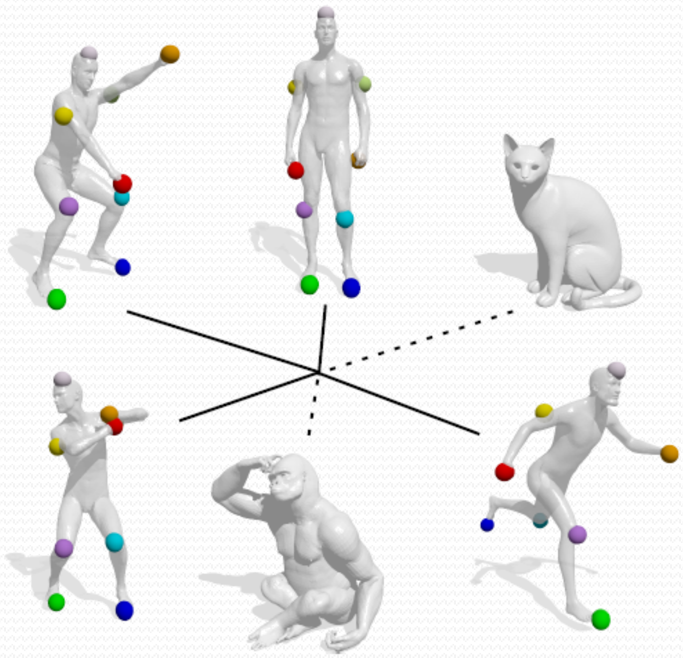
A good amount of research is being devoted to the problem of **jointly matching multiple shapes**.

Multi-way matching

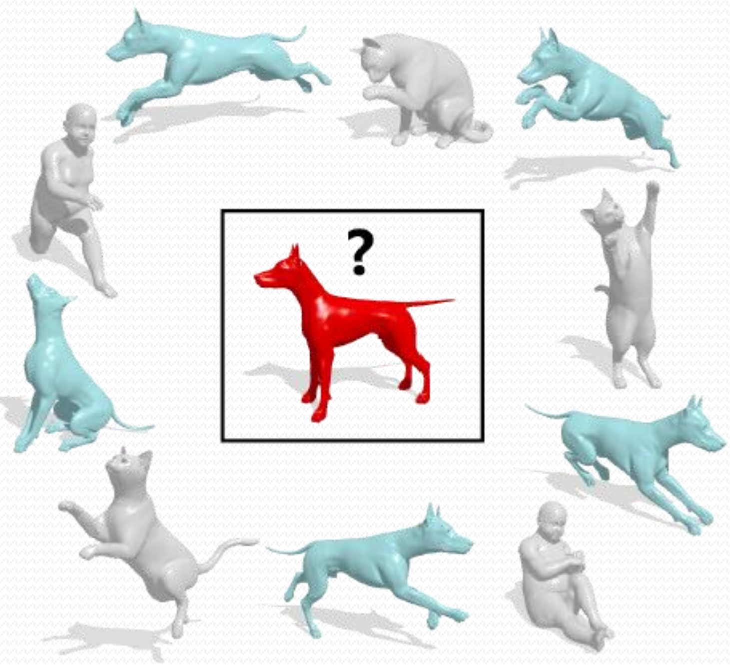


Shape clustering

Multi-way matching



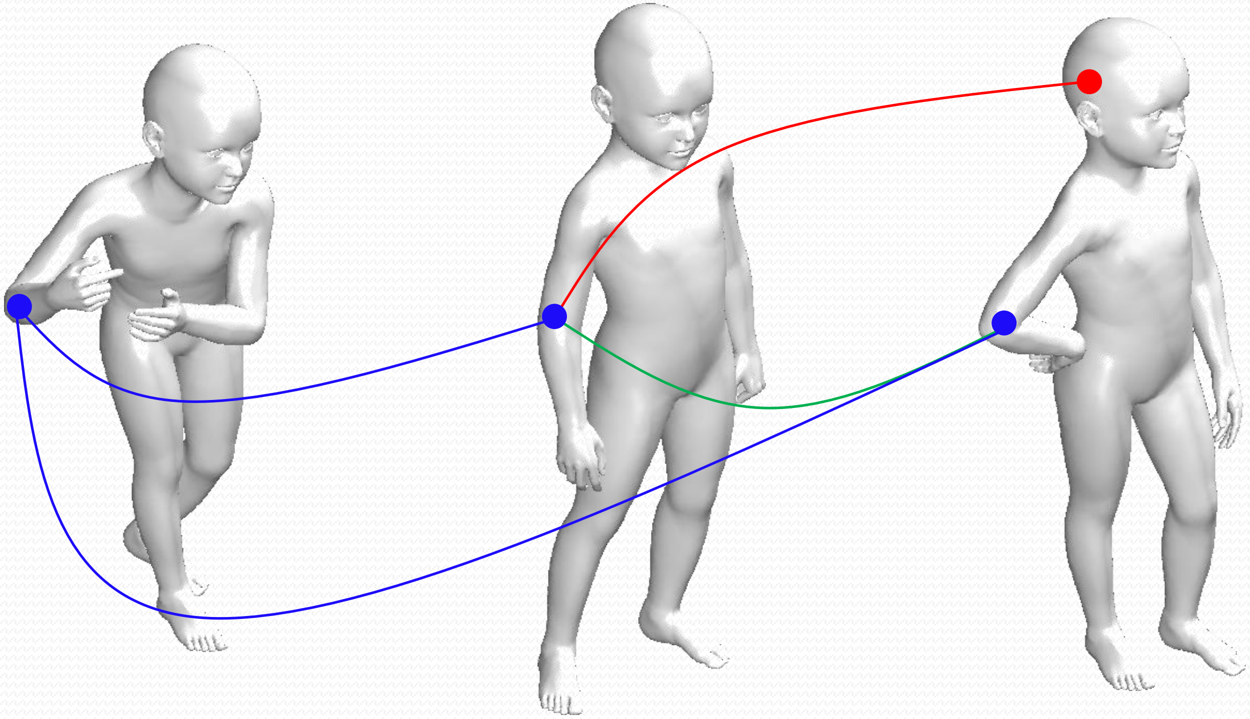
Point-to-point maps



Shape retrieval

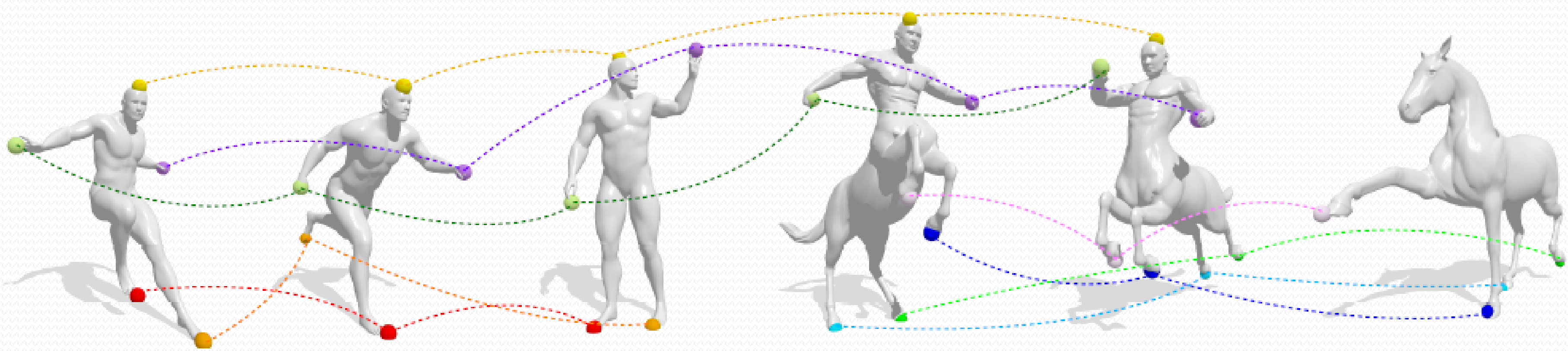
Multi-way matching

One of the key requirements is that the extracted correspondence should be "transitive", or **cycle-consistent**:



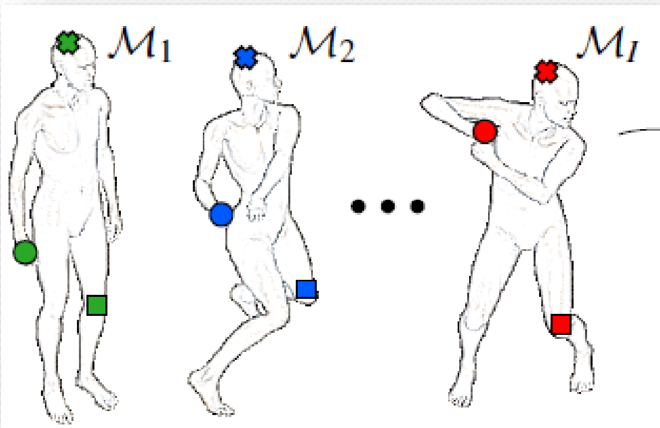
Multi-way matching

We extended the notion of **Gromov-Hausdorff distance** to multiple metric spaces, and provided an efficient algorithm to find **cycle-consistent** matches which are **robust to partiality and outliers**.

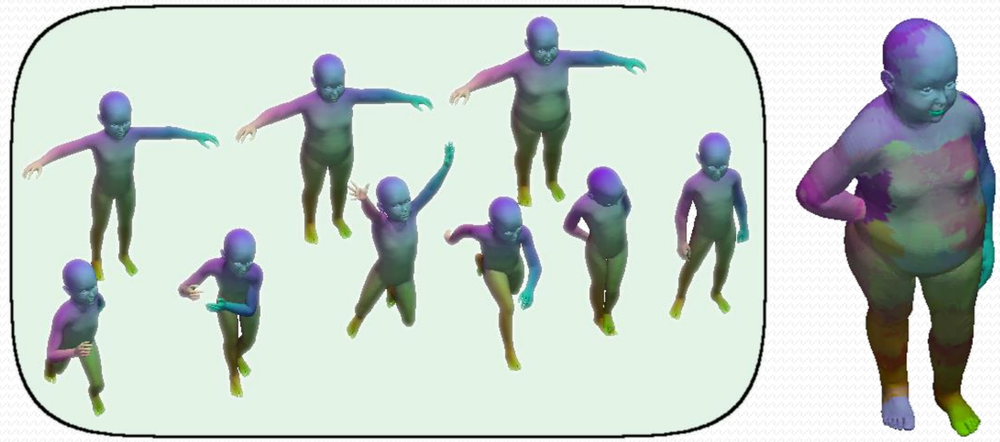


Open question: **Can we make it dense?**

Machine Learning and Shape Analysis



Optimal intrinsic descriptors



Example-based dense matching

What else?

BS/MS Thesis

We offer theses for BS/MS students in Computer Science and Mathematics.
See: http://vision.in.tum.de/members/rodola/teaching

The group is currently composed of
2 post-docs, **2 PhD students**, and **2 Master’s students**.

The coolest topics we are currently pursuing are kept secret to the human eye. If you want to find out more, come visit us!

References

- *Partial Functional Correspondence*. E. Rodolà et al. 2015. http://arxiv.org/abs/1506.05274
- *Point-wise Map Recovery and Refinement from Functional Correspondence*. E. Rodolà et al. 2015. http://arxiv.org/abs/1506.05603
- KIDS: http://vision.in.tum.de/data/datasets/kids
- *Dense 2D-to-3D Shape Matching via Low-Rank Functional Correspondence*. A. Števanovic 2015. http://vision.in.tum.de/_media/members/rodola/teaching/aneta_stevanovic_master_thesis.pdf
- *Anisotropic Laplace-Beltrami Operators for Shape Analysis*. M. Andreux et al. 2014. http://vision.in.tum.de/_media/spezial/bib/andreux-nordia14.pdf
- *Consistent Partial Matching of Shape Collections via Sparse Modeling*. L. Cosmo et al. 2015. (submitted to CGF)