

## Weekly Exercises 9

Room: 02.09.023

Wed, 24.06.2015, 14:15-15:45

Submission deadline: Tue, 23.06.2015, 23:59 to windheus@in.tum.de

Please send in only Latex-PDF. If you have hand-written solutions, please hand them in during the lecture.

### Shape Differences

Let  $\mathcal{M}, \mathcal{N}$  be two shapes, define  $h_a^{\mathcal{M}}(f, g) = \int_{\mathcal{M}} f(p)g(p)dp$ ,  $h_a^{\mathcal{N}}(f, g) = \int_{\mathcal{N}} f(p)g(p)dp$ . Discretize the shapes' Laplace-Beltrami operators using the usual finite element method resulting in mass matrices  $M_{\mathcal{M}}, M_{\mathcal{N}}$  and stiffness matrices  $W_{\mathcal{M}}, W_{\mathcal{N}}$ , such that  $\Phi_{\mathcal{M}}, \Phi_{\mathcal{N}}$  are the eigen functions (written as columns of the matrices) and  $\Lambda_{\mathcal{M}}, \Lambda_{\mathcal{N}}$  are their eigen values (written as diagonal matrices). Recall that  $\Phi_{\mathcal{M}}$  is orthonormal with respect to  $M_{\mathcal{M}}$ , i.e.  $\Phi_{\mathcal{M}}^{\top} M_{\mathcal{M}} \Phi_{\mathcal{M}} = \text{Id}$ .

- Exercise 1** (One Point). 1. Let  $F$  be a linear transformation (functional map) between  $L^2(\mathcal{M})$  and  $L^2(\mathcal{N})$ , represented by matrix  $C$  mapping functions in the eigen basis  $\Phi_{\mathcal{M}}$  onto functions in the eigen basis  $\Phi_{\mathcal{N}}$ . The Riesz representation theorem tells us we can write  $h_a^{\mathcal{N}}(F(f), F(g)) = h_a^{\mathcal{M}}(f, D(g))$ , for any functions  $f, g \in L^2(\mathcal{M})$ , where  $D : L^2(\mathcal{M}) \rightarrow L^2(\mathcal{M})$  is a linear map. Show that  $D$  is self adjoint, i.e.  $h_a^{\mathcal{M}}(f, D(g)) = h_a^{\mathcal{M}}(D(f), g)$  for all  $f, g$ .
2. Now assume  $F$  is area preserving and  $D$  is represented by matrix  $V$  in the eigen basis  $\Phi_{\mathcal{M}}$ , show that  $V = C^{\top} C$ .
3. Define  $h_c^{\mathcal{M}}(f, g) = \int_{\mathcal{M}} \langle \nabla f(p), \nabla g(p) \rangle dp$ ,  $h_c^{\mathcal{N}}(f, g) = \int_{\mathcal{N}} \langle \nabla f(p), \nabla g(p) \rangle dp$ . Again let  $F$  be a linear transformation (functional map) between  $L^2(\mathcal{M})$  and  $L^2(\mathcal{N})$ , represented by matrix  $C$  mapping functions in the eigen basis  $\Phi_{\mathcal{M}}$  onto functions in the eigen basis  $\Phi_{\mathcal{N}}$ . Let  $f, g \in L^2(\mathcal{M})$ , such that  $f = \sum_i \phi_i \alpha_i$ ,  $g = \sum_i \phi_i \beta_i$ . Show that  $h_c^{\mathcal{N}}(F(f), F(g)) = \alpha^{\top} R \beta$ , where  $R = M_{\mathcal{M}}^{-1} C^{\top} M_{\mathcal{N}} C^{\top}$ .
4. In case of  $h_c$ , show that if  $F$  is area preserving,  $R = \Lambda_{\mathcal{M}}^{-1} C^{\top} \Lambda_{\mathcal{N}} C^{\top}$ .

### Programming: Shape Differences

**Exercise 2** (Two points). Download and expand the file `ex9.zip` from the lecture website. Modify the file `findAnalogousShape.m` to implement the function as explained below. You need one data set of shapes to test implementation. Choose and download one data set from [http://people.csail.mit.edu/drdaniel/mesh\\_](http://people.csail.mit.edu/drdaniel/mesh_)

animation/index.html (one of the meshes.tgz files linked on the page). The meshes in those data sets are registered. Corresponding vertices on two meshes have the same index on both meshes. You can run the script `exercise.m` to test and visualize your solutions after you specified the correct path to the shape data set.

function [meshIndex] = findAnaloguousShape(M1, M2, M3, meshFileNames, k) accepts three meshes M1, M2, M3 and a cell array of filenames meshFileNames. k is the number of eigen functions. It returns the index of the mesh that relates best to M3 as M2 relates to M1. Use the formula shown on page 22 from the lecture on *shape differences* to quantify shape analogy.