Some Preliminaries

- Material is taken from my machine learning class: https://vision.in.tum.de/teaching/ws2013/ml_ws13
- You can also watch the lectures on youtube: https://vision.in.tum.de/lib/exe/fetch.php?hash=4d9a08&media=http%3A%2F%2Fyoutu.be%2FQZmZFeZxEKI
- I will use OnlineTED in the next slide www.onlineted.de

Boosting

Question

https://www.onlineted.de/vote.php

Reminder: Linear Regression

Given: a set of **basis functions**

$$\phi(\mathbf{x}) = (\phi_0(\mathbf{x}), \phi_1(\mathbf{x}), \ldots, \phi_{M-1}(\mathbf{x})) \qquad \mathbf{x} \in \mathbb{R}^d$$

The goal is to fit a model into the data

$$y(\mathbf{x}, \mathbf{w}) = \mathbf{w}^T \phi(\mathbf{x})$$

To do this, we need to find an error function, e.g.:

$$E(\mathbf{w}) = \frac{1}{2} \sum_{i=1}^{N} (\mathbf{w}^T \phi(\mathbf{x}_i) - t_i)^2$$

To find the optimal parameters, we derived E with respect to $\mathbf{w}$ and set the derivative to zero.

Some Questions

1. Can we do the same for classification?
   As a special case we consider two classes:
   $$t_i \in \{-1, 1\} \quad \forall i = 1, \ldots, N$$
2. Can we use a different (better?) error function?
3. Can we learn the basis functions together with the model parameters?
4. Can we do the learning sequentially, i.e. one basis function after another?

Answer to all questions: Yes, using Boosting!

The Loss Function

Definition: a real-valued function $L(t, y(\mathbf{x}))$, where t is a target value and y is a model, is called a **loss function**.

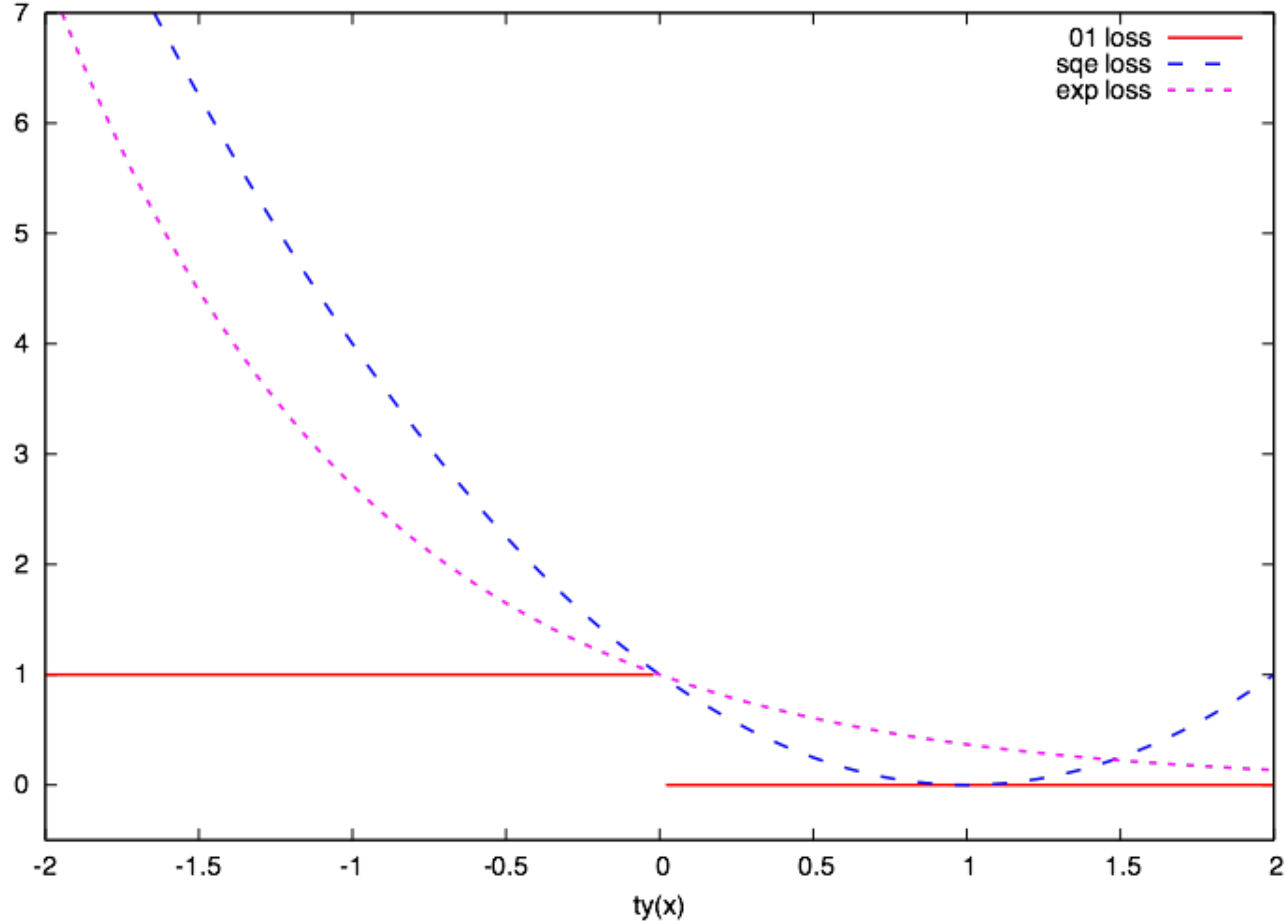
Examples:

01-loss: $$L_{01}(t, y(\mathbf{x})) = \begin{cases} 0 & \text{if } t = y(\mathbf{x}) \\ 1 & \text{else} \end{cases}$$

squared error loss: $$L_{sqe}(t, y(\mathbf{x})) = (t - y(\mathbf{x}))^2$$

exponential loss: $$L_{exp}(t, y(\mathbf{x})) = \exp(-ty(\mathbf{x}))$$

Loss Functions



- 01-loss is not differentiable
- squared error loss has only one optimum

Sequential Fitting of Basis Functions

Idea: We start with a basis function $\phi_0(\mathbf{x})$:

$$y_0(\mathbf{x}, w_0) = w_0 \phi_0(\mathbf{x}) \qquad\qquad w_0 = 1$$

Then, at iteration m, we add a new basis function $\phi_m(\mathbf{x})$ to the model:

$$y_m(\mathbf{x}, w_0, \ldots, w_m) = y_{m-1}(\mathbf{x}, w_0, \ldots, w_{m-1}) + w_m \phi_m(\mathbf{x})$$

Two questions need to be answered:

1. How do we find a good new basis function?
2. How can we determine a good value for w_m?

Idea: Minimize the exponential loss function

Minimizing the Exponential Loss

Aim: find w_m and ϕ_m so that

$$(w_m, \phi_m) = \arg\min_{w,\phi} \sum_{i=1}^{N} L(t_i, y_{m-1}(\mathbf{x}_i) + w\phi(\mathbf{x}_i))$$

where $\quad L(t, y) = \exp(-ty)$

Minimizing the Exponential Loss

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where $$L(t, y) = \exp(-ty)$$

Solution: $$\phi_m = \arg\min_{\phi} \sum_{i=1}^{N} v_{i,m} \mathbb{I}(t_i \neq \phi(\mathbf{x}_i))$$

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Solution: $\phi_m = \arg\min_{\phi} \sum_{i=1}^{N} v_{i,m} \mathbb{I}(t_i \neq \phi(\mathbf{x}_i))$

$$w_m = \frac{1}{2} \log \frac{1 - \mathrm{err}_m}{\mathrm{err}_m}$$

Minimizing the Exponential Loss

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where $$L(t, y) = \exp(-ty)$$

Solution: $$\phi_m = \arg\min_{\phi} \sum_{i=1}^{N} v_{i,m} \mathbb{I}(t_i \neq \phi(\mathbf{x}_i))$$

$$w_m = \frac{1}{2} \log \frac{1 - \mathrm{err}_m}{\mathrm{err}_m} \qquad v_{i,m+1} = v_{i,m} \exp(2 w_m \mathbb{I}(t_i \neq \phi_m(\mathbf{x}_i)))$$

The AdaBoost Algorithm

1. For $i = 1, \ldots, N$: $v_i \leftarrow 1/N$
2. For $m = 1, \ldots, M$

   Fit a classifier ("basis function") ϕ_m that minimizes

$$\sum_{i=1}^{N} v_i \mathbb{I}(t_i \neq \phi_m(\mathbf{x}_i))$$

   Compute $\mathrm{err}_m = \frac{\sum_{i=1}^{N} v_i \mathbb{I}(t_i \neq \phi_m(\mathbf{x}_i))}{\sum_{i=1}^{N} v_i}$ and $\alpha_m = \log \frac{1 - \mathrm{err}_m}{\mathrm{err}_m}$

   Update the weights: $v_i \leftarrow v_i \exp(\alpha_m \mathbb{I}(t_i \neq \phi_m(\mathbf{x}_i)))$
3. Use the resulting classifier:

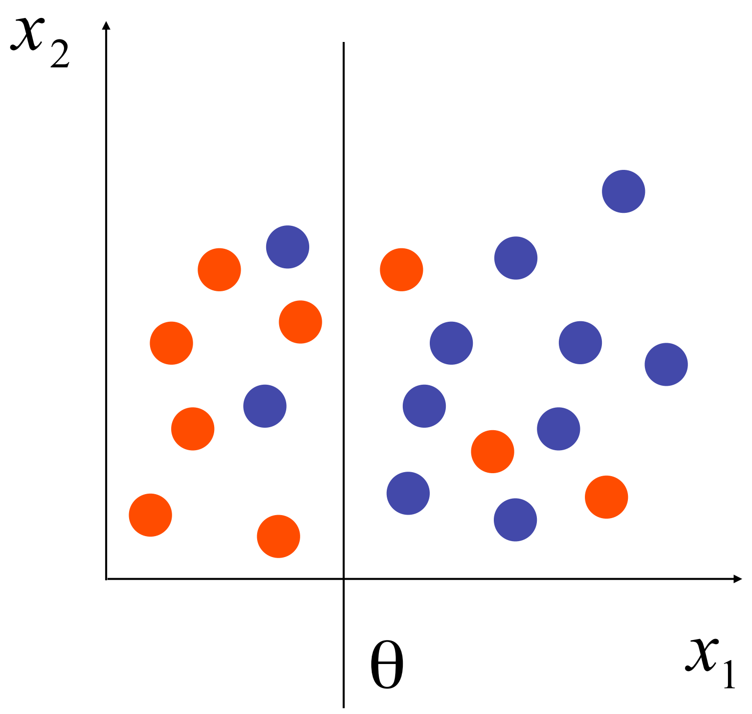
$$y(\mathbf{x}) = \mathrm{sgn} \sum_{m=1}^{M} \alpha_m \phi_m(\mathbf{x})$$

The “Basis Functions”

- Can be any classifier that can deal with weighted data
- Most importantly: if these “base classifiers” provide a training error that is at most as bad as a random classifier would give (i.e. it is a **weak** classifier), then AdaBoost can return an arbitrarily small training error (i.e. AdaBoost is a strong classifier)
- Many possibilities for weak classifiers exist, e.g.:
  - Decision stumps
  - Decision trees

Decision Stumps

Decision Stumps are a kind of very simple weak classifiers.



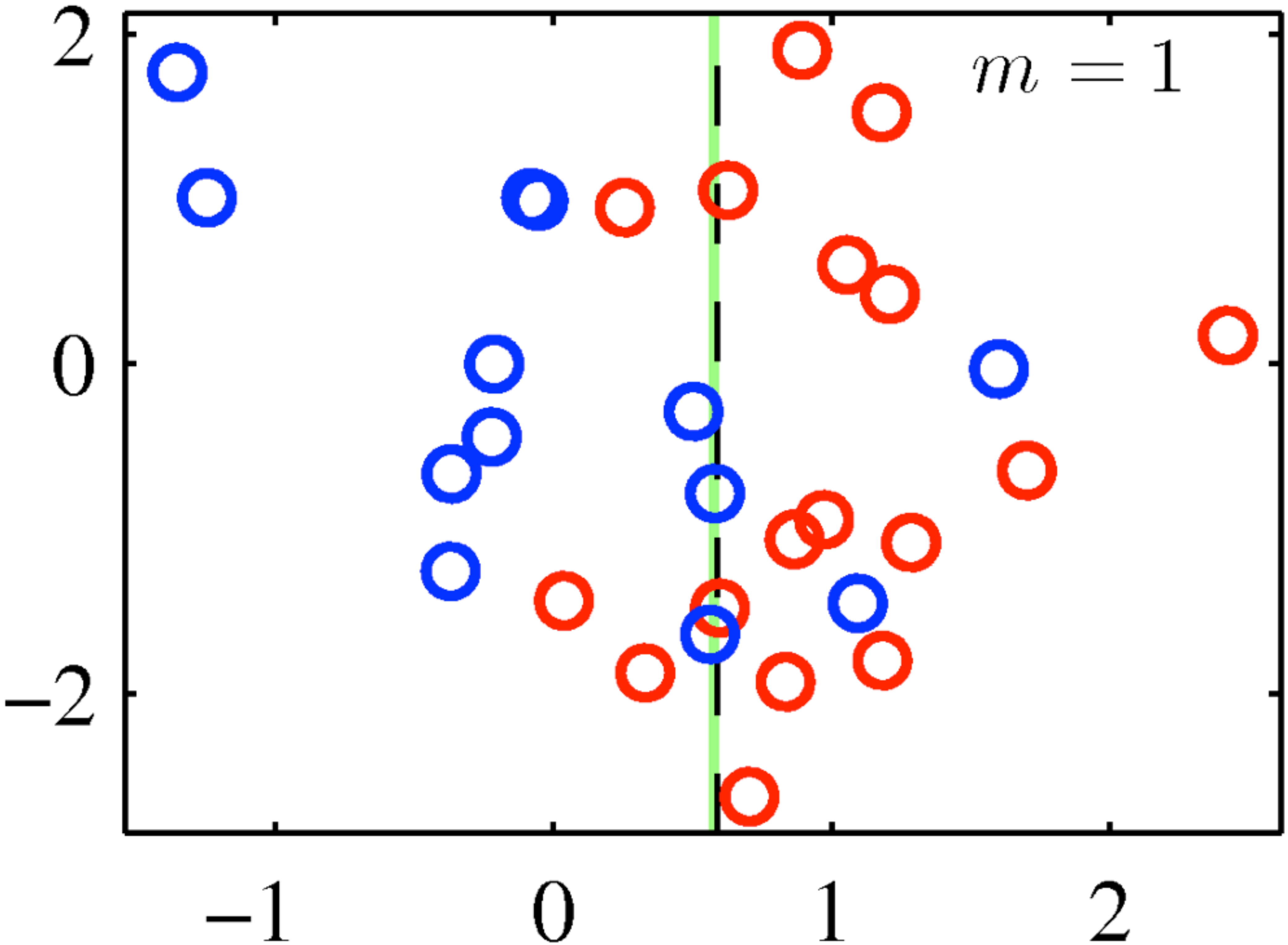
Goal: Find an axis-aligned hyperplane that minimizes the class. error

This can be done for each feature (i.e. for each dimension in feature space)

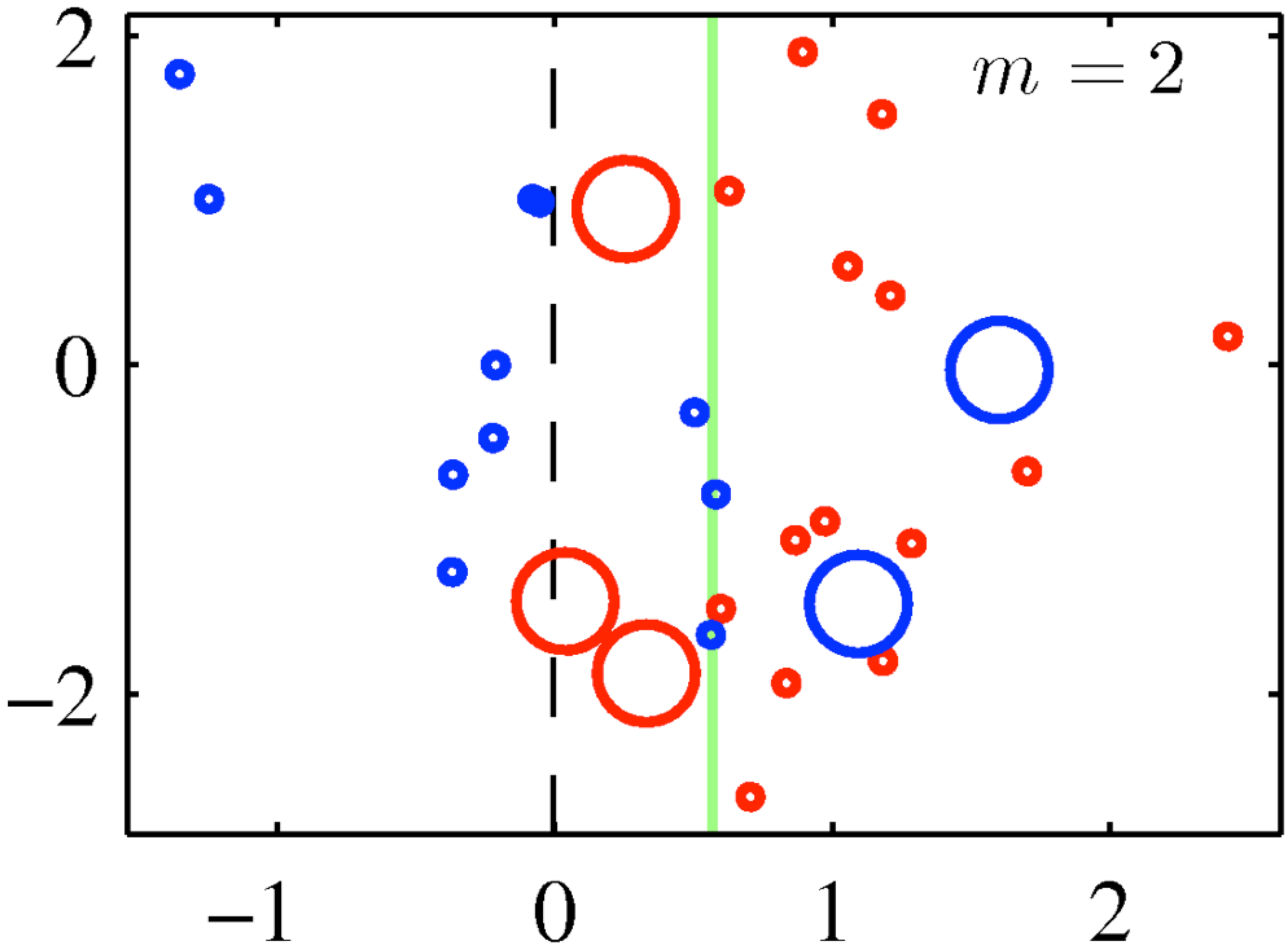
It can be shown that the classif. error is always better than 0.5 (random guessing)

Idea: apply many weak classifiers, where each is trained on the misclassified examples of the previous.

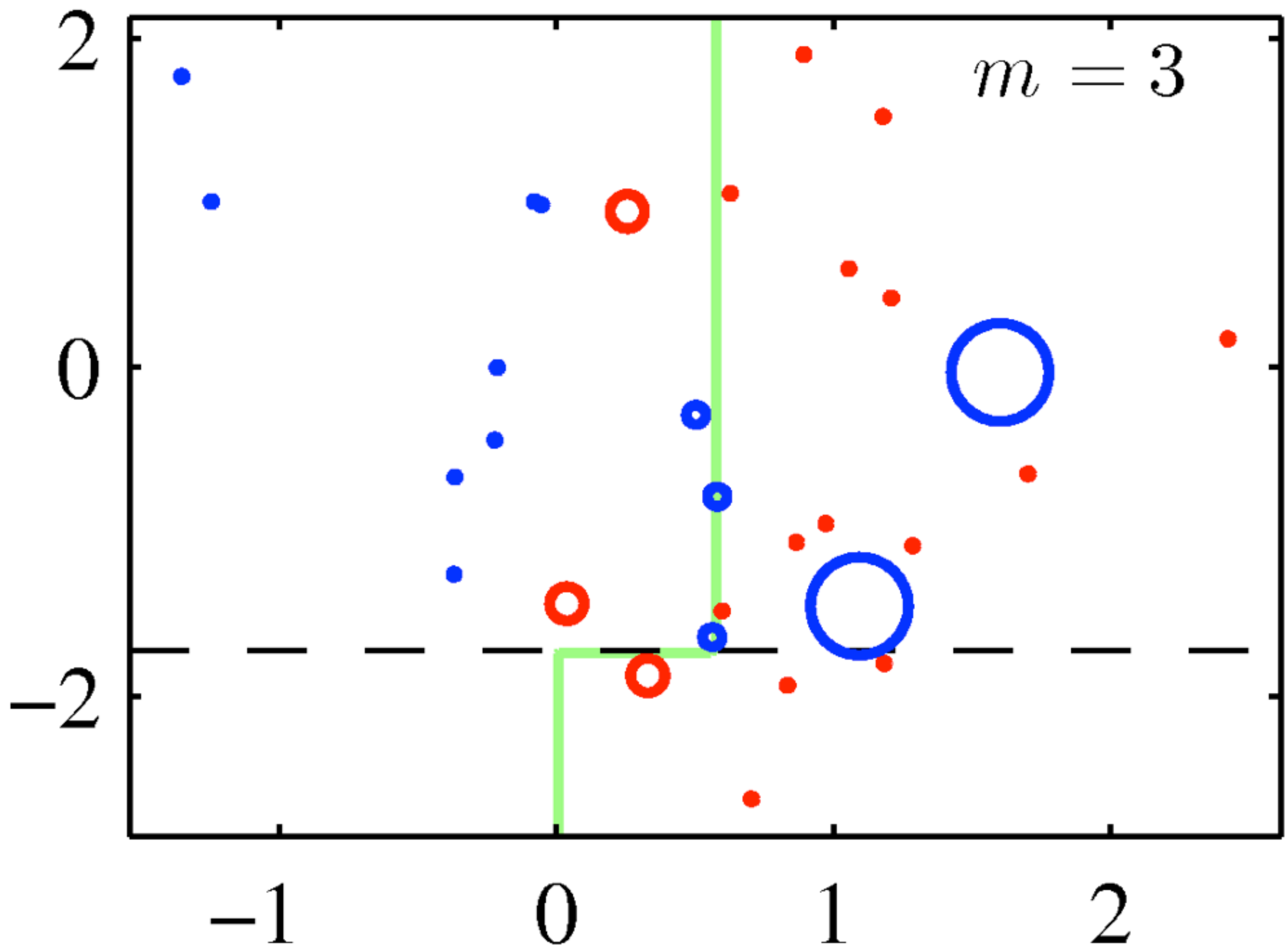
Classification Example



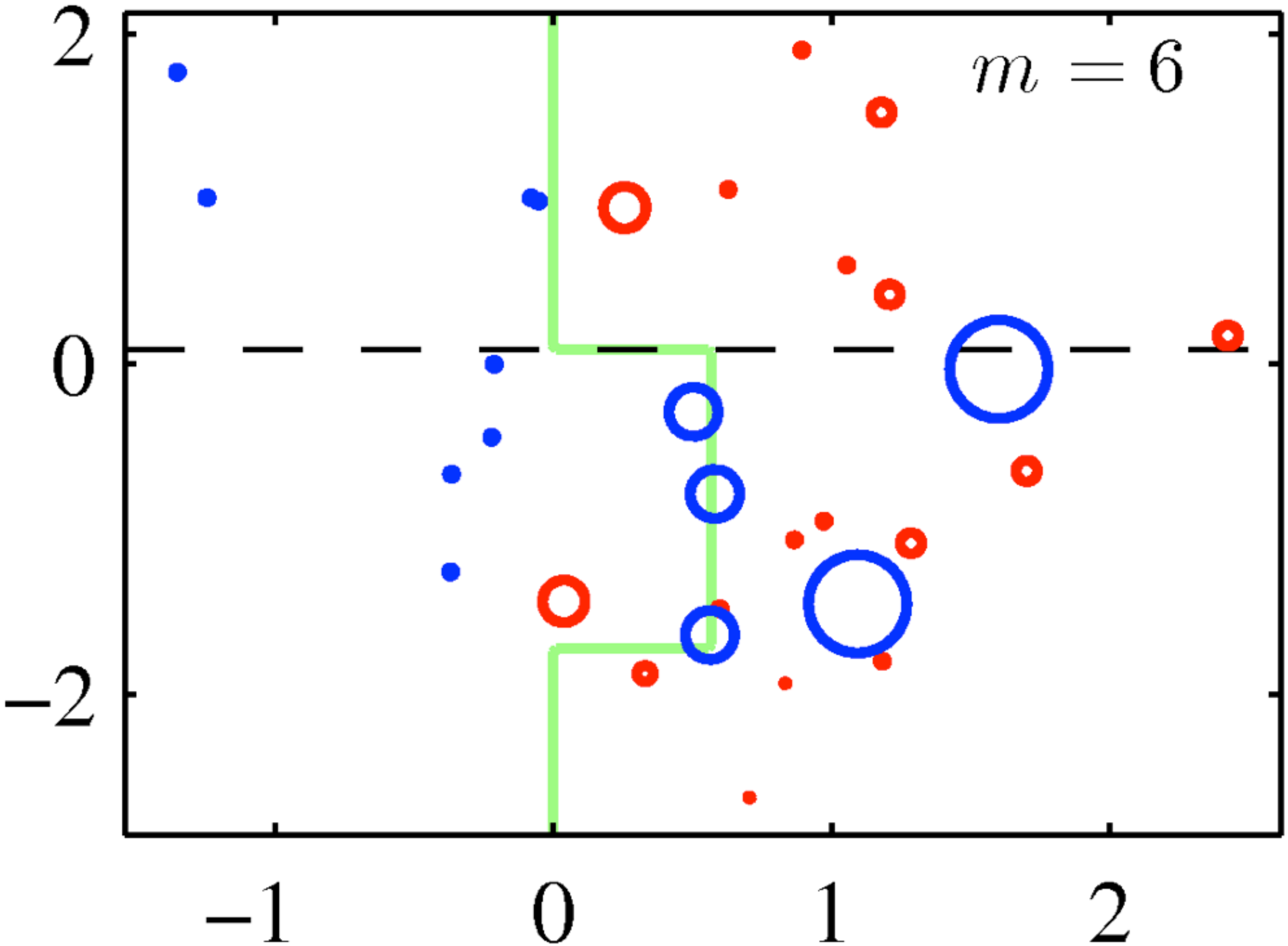
Classification Example



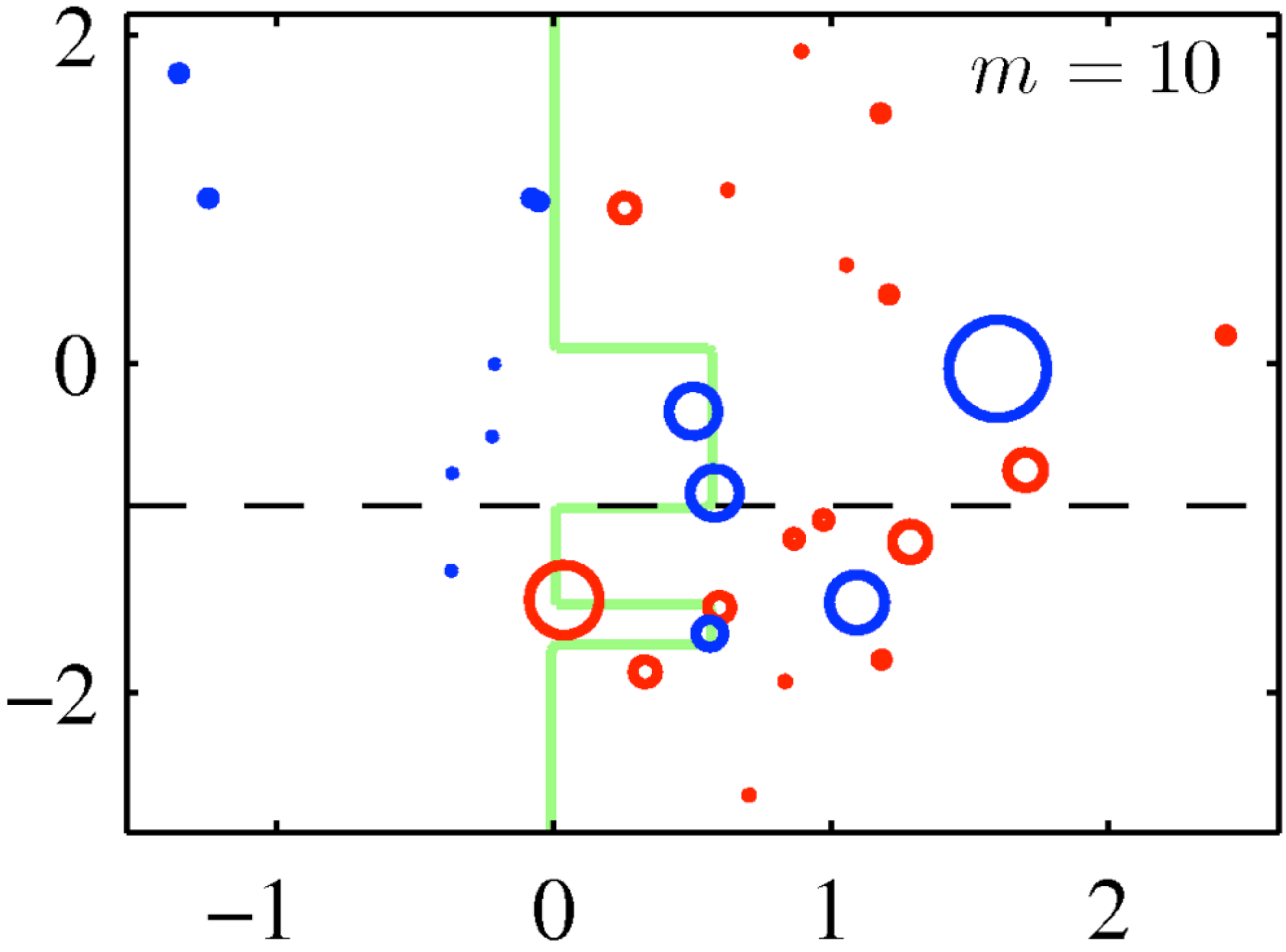
Classification Example



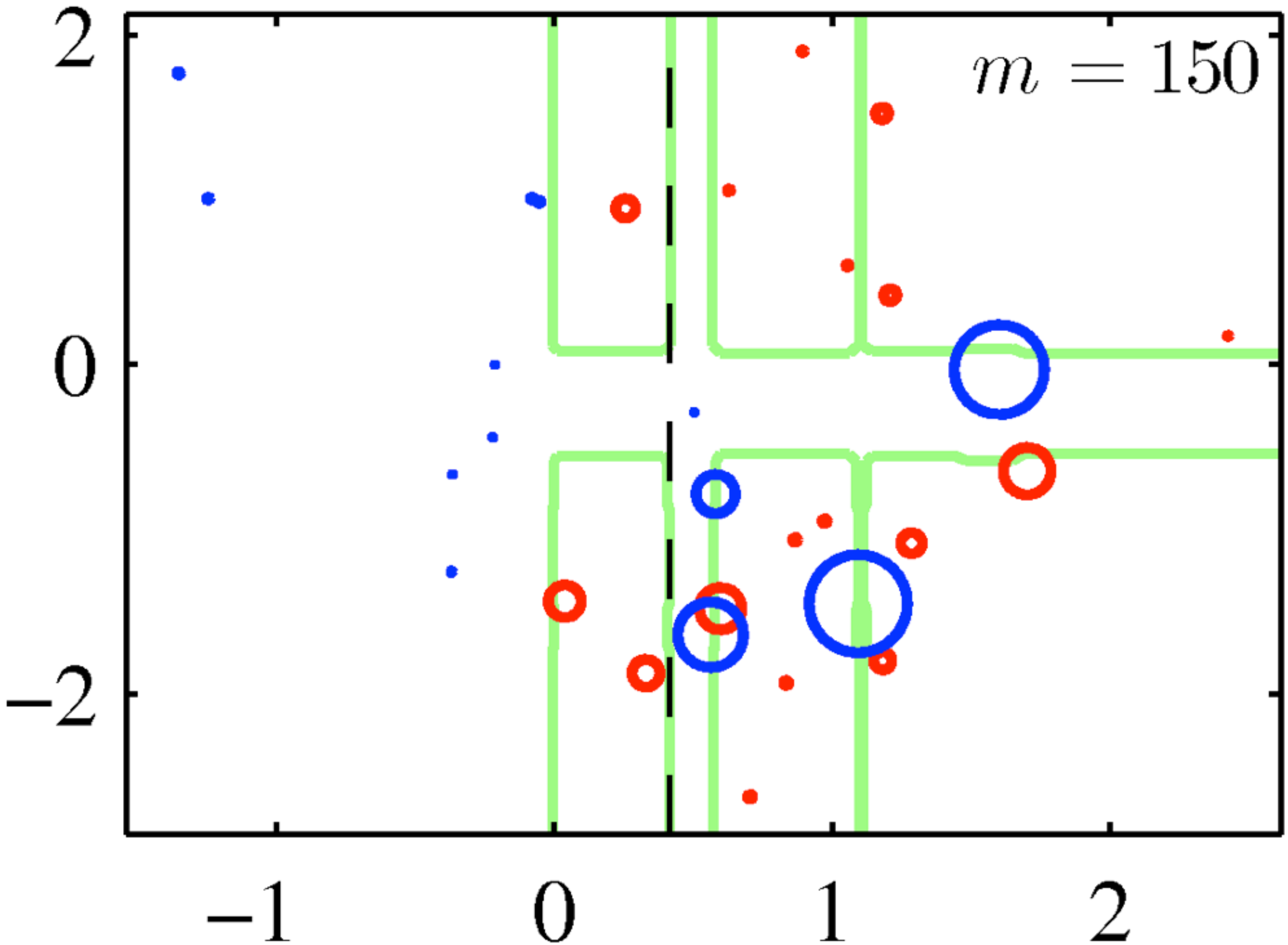
Classification Example



Classification Example

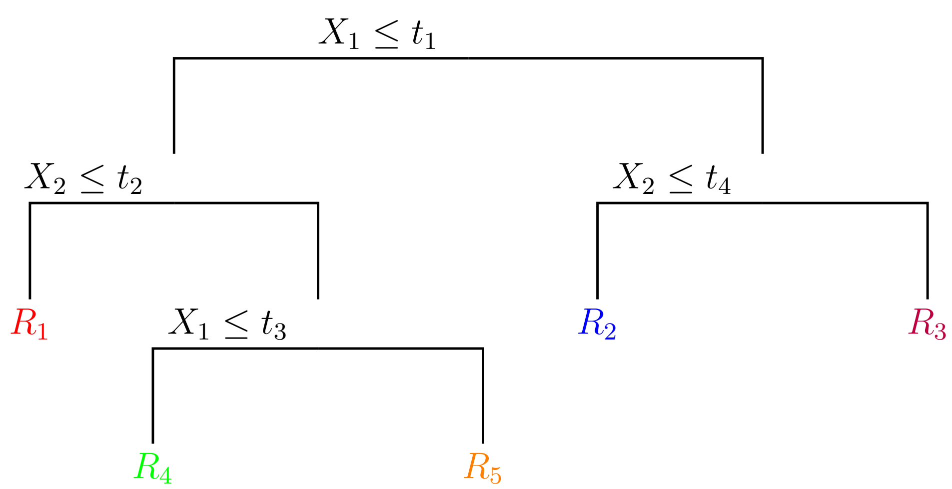


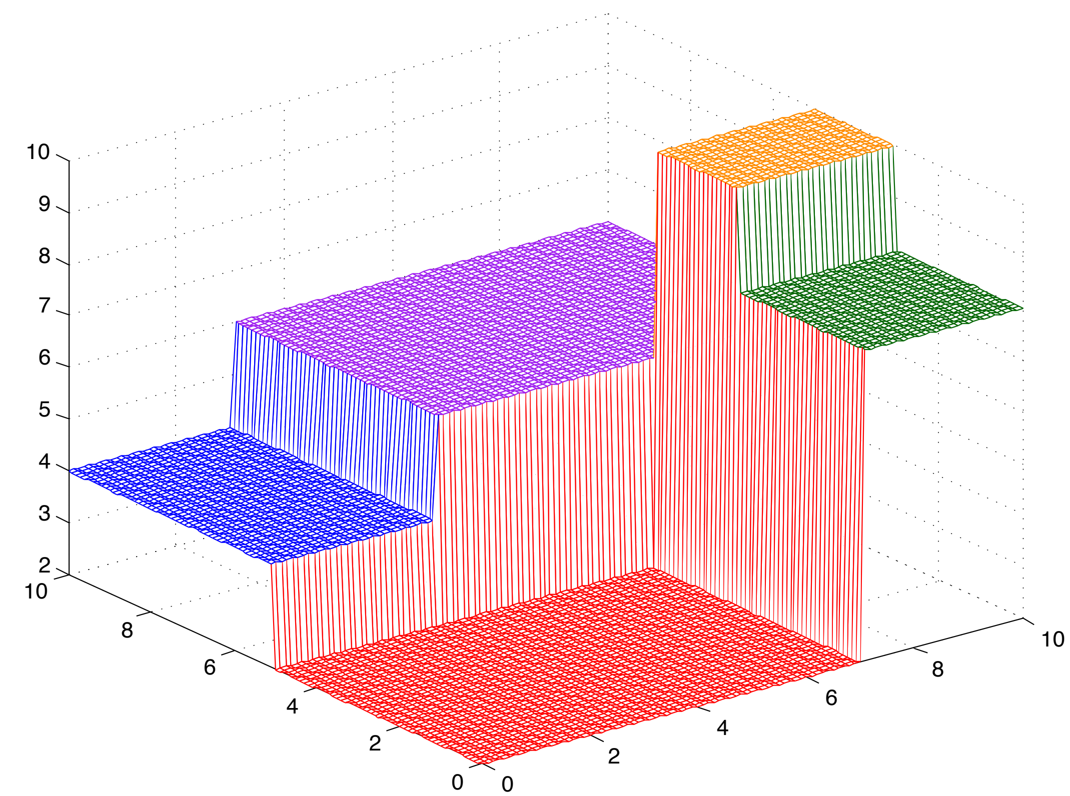
Classification Example



Decision Trees

- A more general version of decision stumps are decision **trees:**

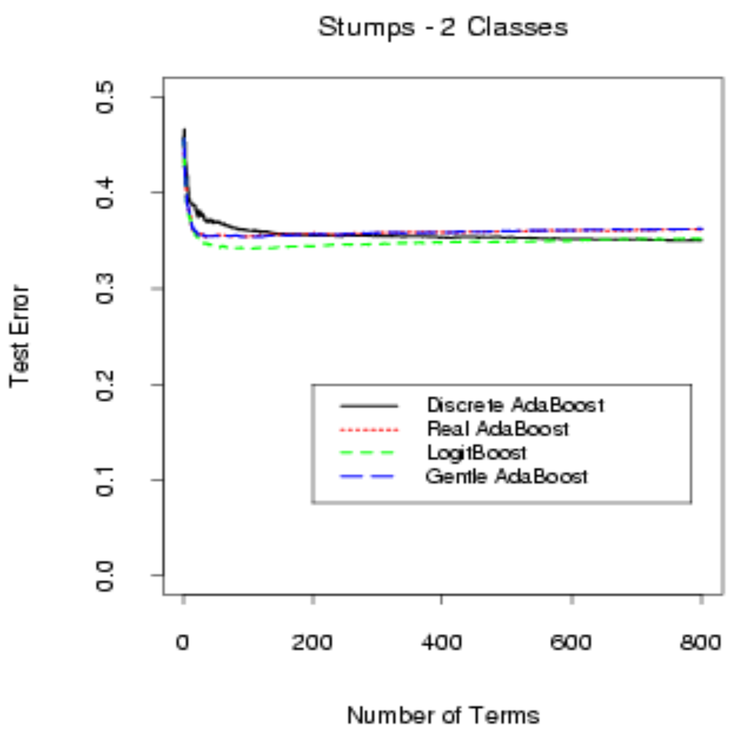


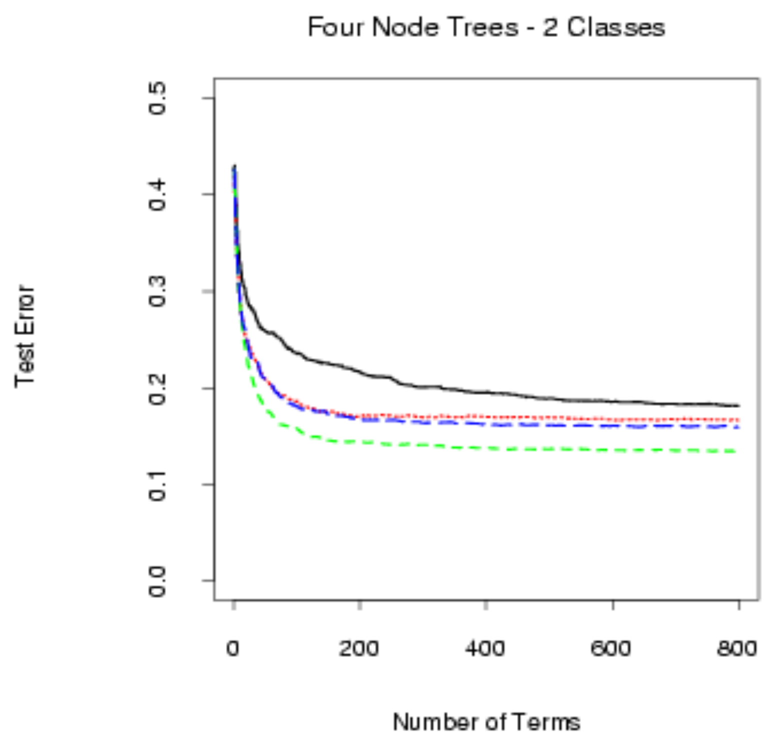


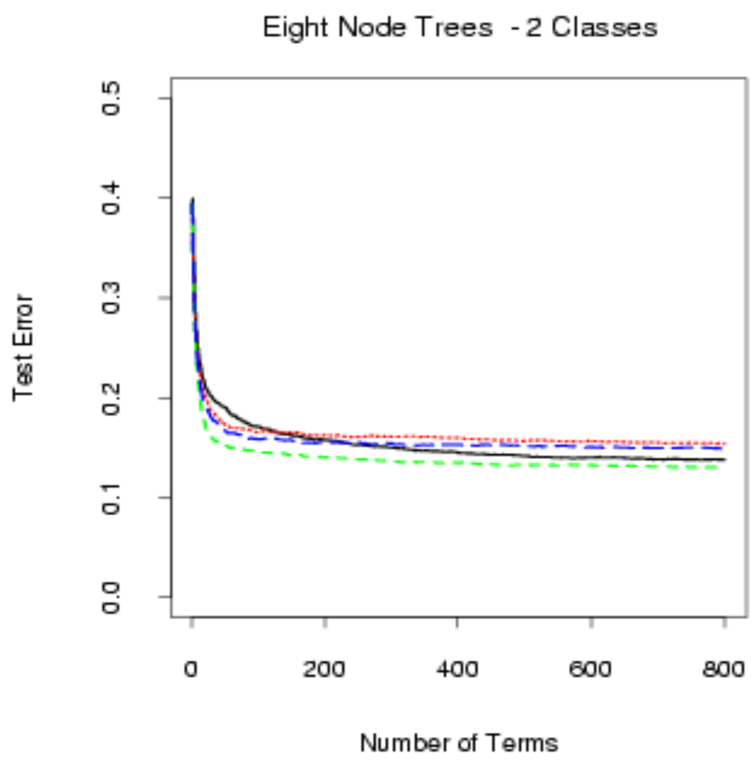
- At every node, a decision is made
- Dan be used for classification and for regression (Classification And Regression Trees CART)

Back to Boosting

- AdaBoost has been shown to perform very well, especially when using decision trees as weak classifiers

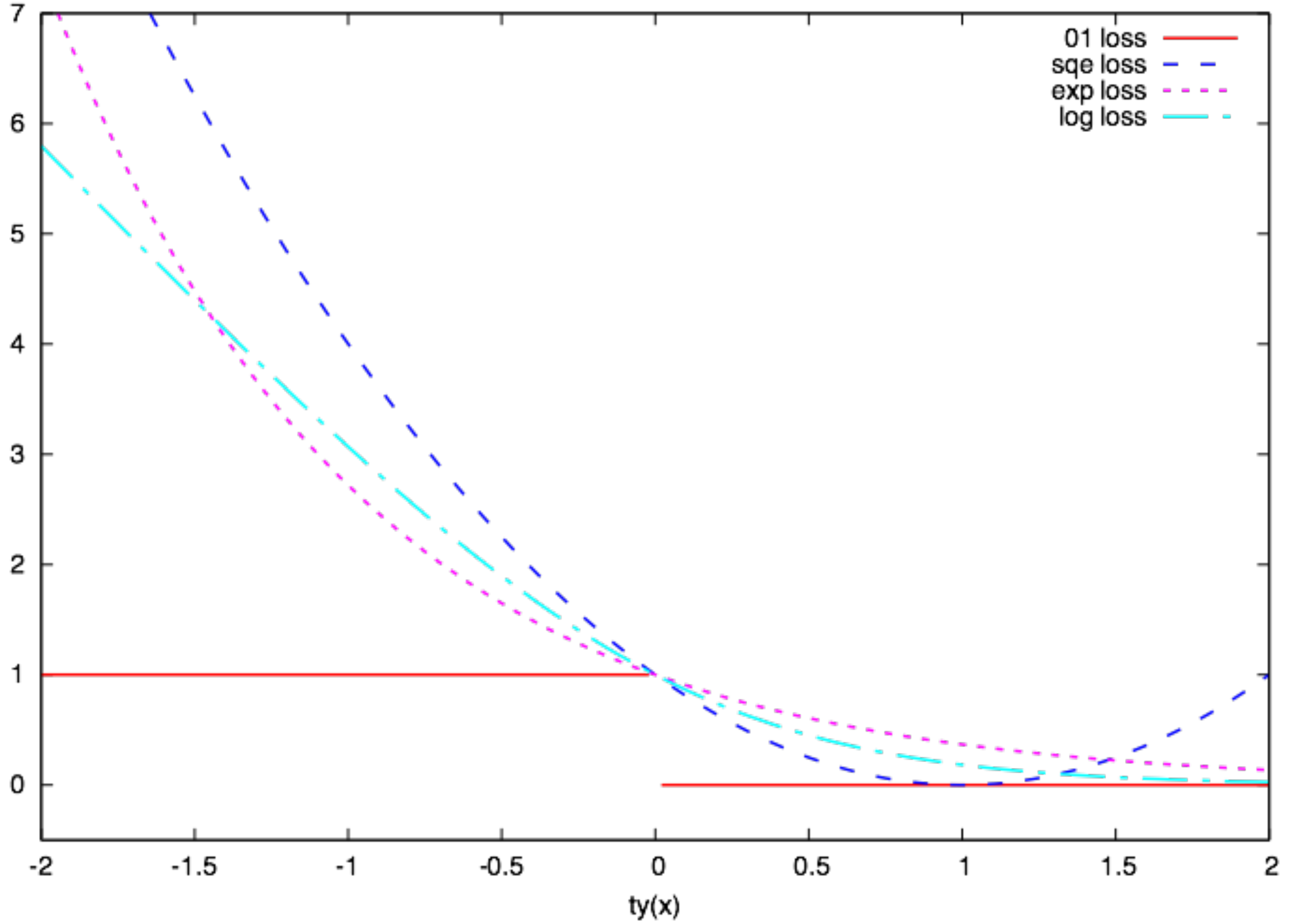






- However: the exponential loss weighs misclassified examples very high!

Using the Log-Loss



- The log-loss is defined as:

$$L(t, y(\mathbf{x})) = \log_2(1 + \exp(-2ty(\mathbf{x}))$$

- It penalizes misclassifications only **linearly**

The LogitBoost Algorithm

1. For $i = 1, \ldots, N$: $v_i \leftarrow 1/N \quad \pi_i \leftarrow 1/2$
2. For $m = 1, \ldots, M$

   Compute the working response $z_i = \dfrac{t_i - \pi_i}{\pi_i(1-\pi_i)}$

   Compute the weights $v_i = \pi_i(1-\pi_i)$

   Find ϕ_m that minimizes

   $$\sum_{i=1}^{N} v_i (z_i - \phi(\mathbf{x}_i))^2$$

   Update $y(\mathbf{x}) \leftarrow y(\mathbf{x}) + \frac{1}{2}\phi_m(\mathbf{x})$ and $\pi_i \leftarrow \dfrac{1}{1+\exp(-2y(\mathbf{x}_i))}$
3. Use the resulting classifier:

$$y(\mathbf{x}) = \operatorname{sgn} \sum_{m=1}^{M} \phi_m(\mathbf{x})$$

Weighted Least-Squares Regression

- Instead of a weak classifier, LogitBoost uses "weighted least-squares regression"
- This is very similar to standard least-squares regression:

$$E(\mathbf{w}) = \frac{1}{2}\sum_{i=1}^{N} v_i(\mathbf{w}^T\phi(\mathbf{x}_i) - t_i)^2$$

- This results in a matrix $\hat{\Phi} = V^{1/2}\Phi$ where

$$V^{1/2} = \mathrm{diag}(\sqrt{v_1}, \ldots, \sqrt{v_N})$$

- The solution is

$$\mathbf{w} = (\hat{\Phi}^T\hat{\Phi})^{-1}\hat{\Phi}^T\mathbf{t}$$

GentleBoost

Gentle AdaBoost

1. Start with weights $w_i = 1/N$, $i = 1, 2, \ldots, N$, $F(x) = 0$.
2. Repeat for $m = 1, 2, \ldots, M$:
   (a) Fit the regression function $f_m(x)$ by weighted least-squares of y_i to x_i with weights w_i.
   (b) Update $F(x) \leftarrow F(x) + f_m(x)$
   (c) Update $w_i \leftarrow w_i e^{-y_i f_m(x_i)}$ and renormalize.
3. Output the classifier $\text{sign}[F(x)] = \text{sign}[\sum_{m=1}^{M} f_m(x)]$

Algorithm 4: *A modified version of the Real AdaBoost algorithm, using Newton stepping rather than exact optimization at each step*

- Numerically more stable than LogitBoost
- Tends to perform better than AdaBoost and LogitBoost

Application of AdaBoost: Face Detection

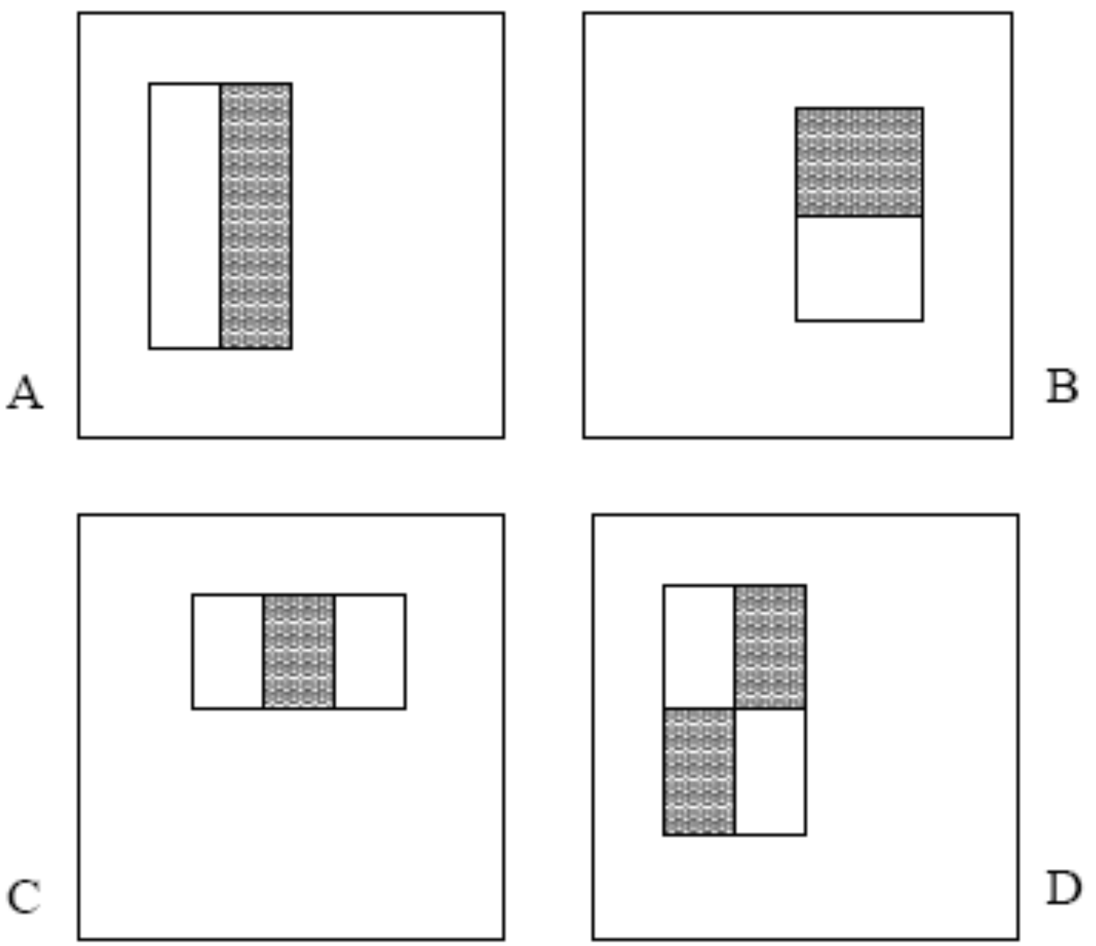
- The biggest impact of AdaBoost was made in face detection
- Idea: extract features (“Haar-like features”) and train AdaBoost, use a cascade of classifiers
- Features can be computed very efficiently
- Weak classifiers can be decision stumps or decision trees
- As inference in AdaBoost is fast, the face detector can run in **real-time**!

Haar-like Features

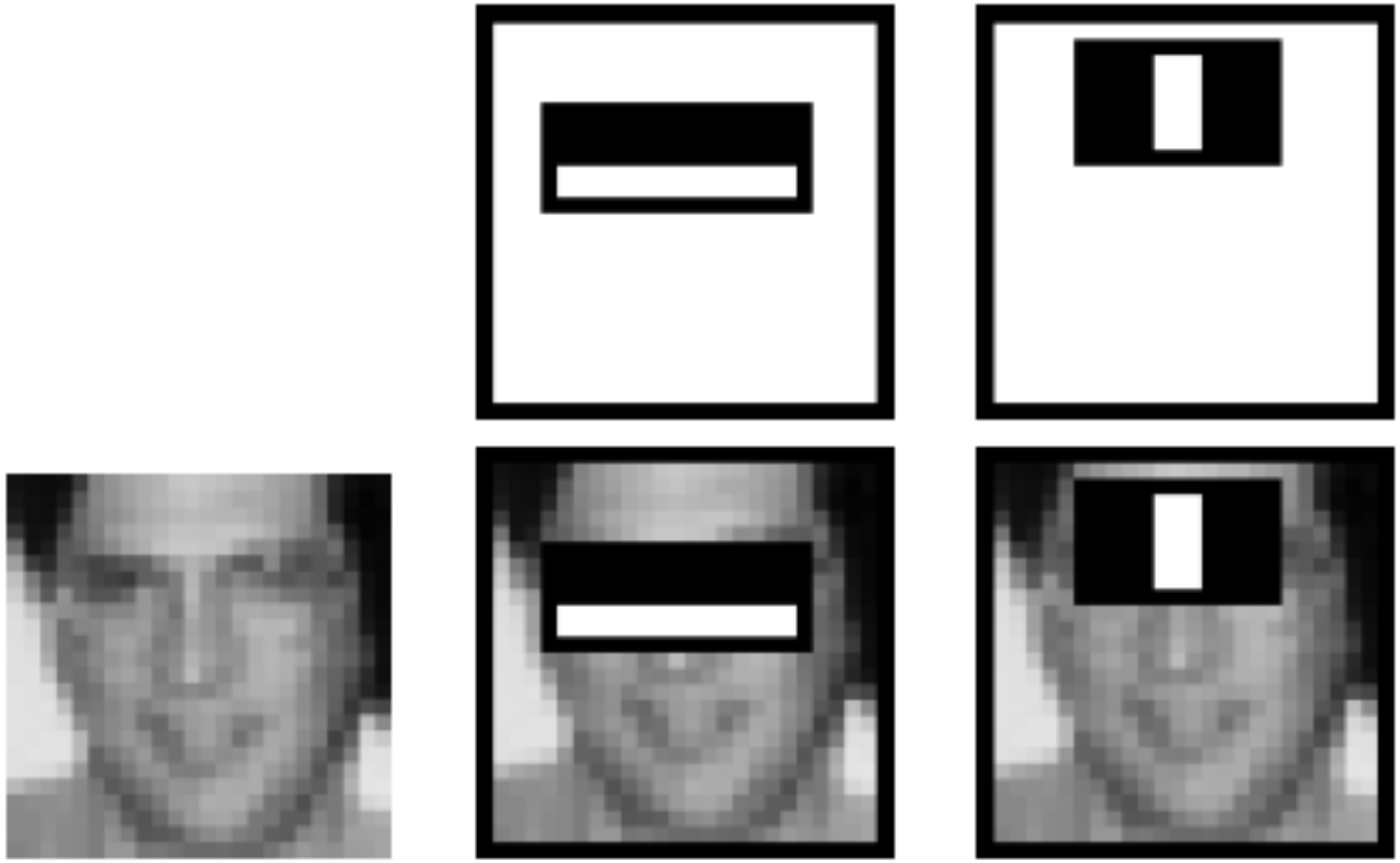
- Defined as difference of rectangular integral area:
  - The sum of the pixels which lie within the white rectangles are subtracted from the sum of pixels in the grey rectangles.

$$\left(\iint_{White} I(x,y)dxdy\right) - \left(\iint_{Grey} I(x,y)dxdy\right)$$

- One feature defined as:
  - Feature type: A,B,C or D
  - Feature position and size

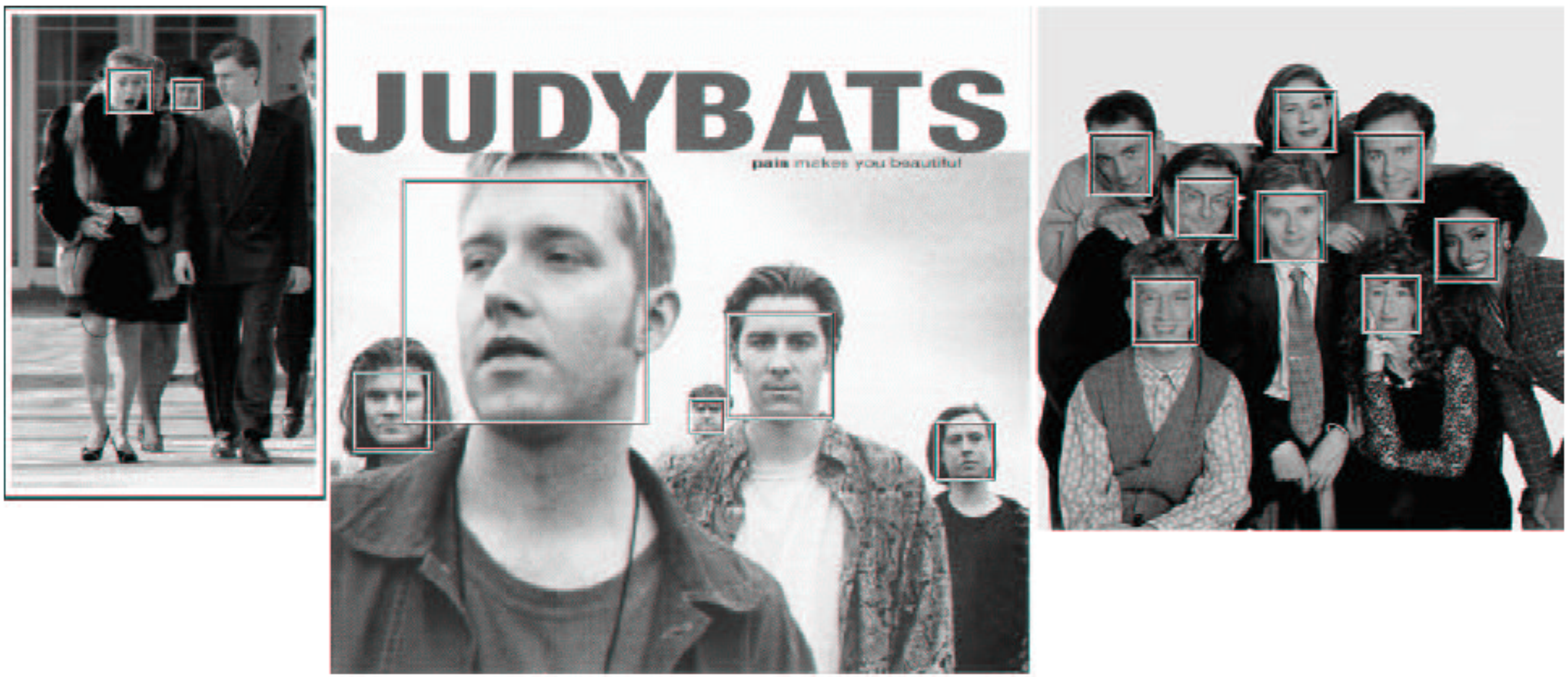


Two First Classifiers Selected by AdaBoost



A classifier with only this two features can be trained to recognise 100% of the faces, with 40% of false positives

Results (1)



Results (2)






Summary

- Boosting is a method to use a weak classifier and turn it into a strong one (arbitrarily small training error!)
- AdaBoost minimizes the exponential loss
- To be more robust against outliers, we can use LogitBoost or GentleBoost
- Weak learners can be decision stumps or decision trees
- Face detection can be solved with Boosting