Chapter 0
Organization and Introduction

Nonlinear Multiscale Methods for Image and Signal Analysis
SS 2015

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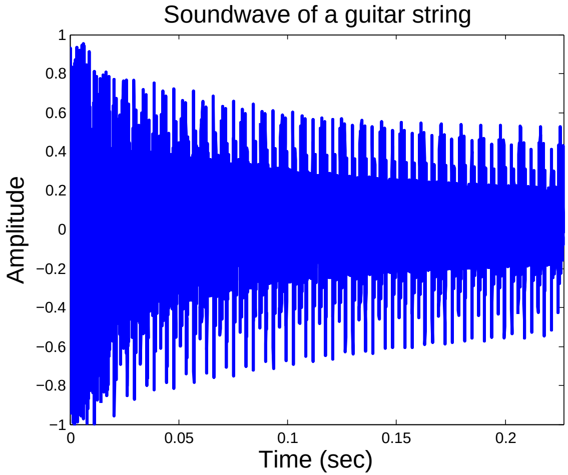
Motivation

What do you hear?

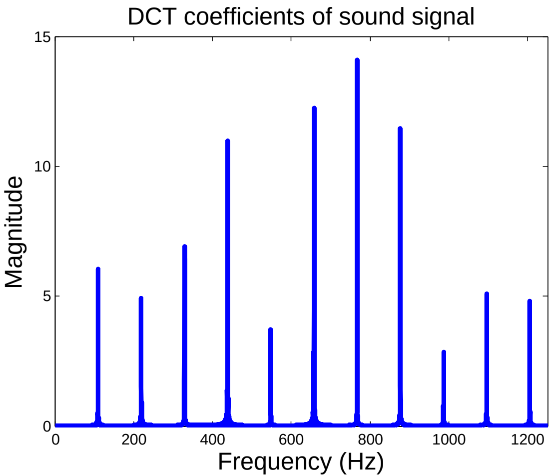
Motivation

Let's look at the sound signal ...



Motivation

Much better: DCT coefficients!



Motivation

What do you see?

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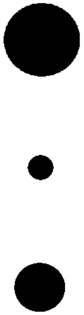

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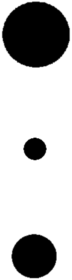

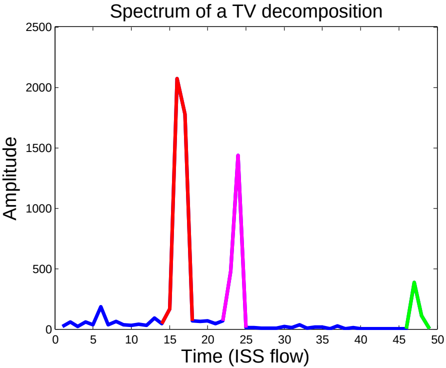

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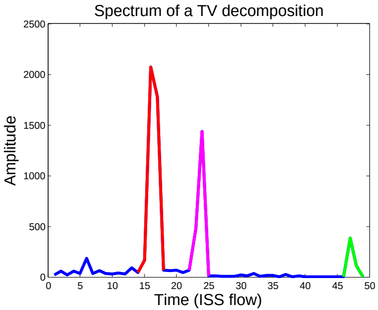

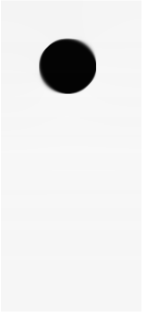

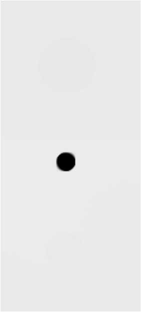

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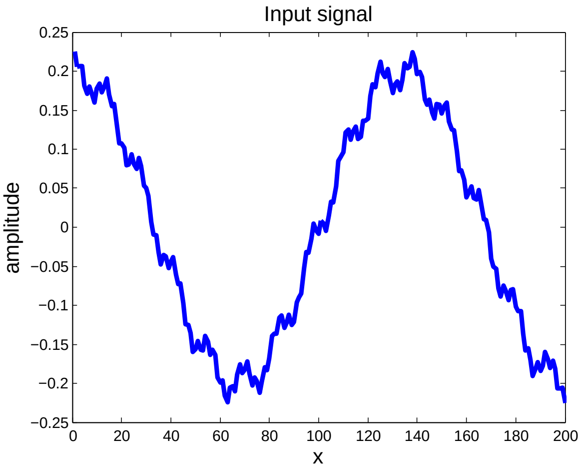
What could spectral representations be good for?



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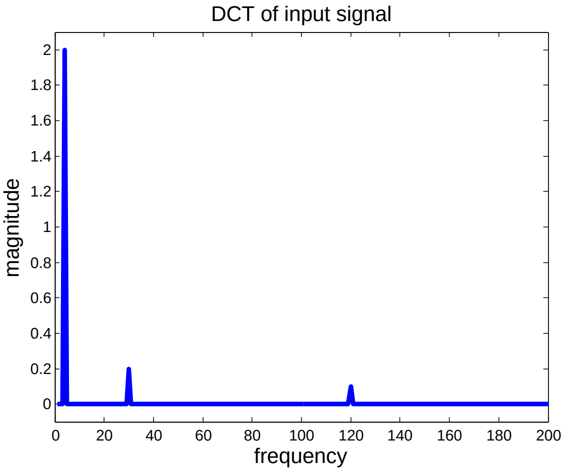

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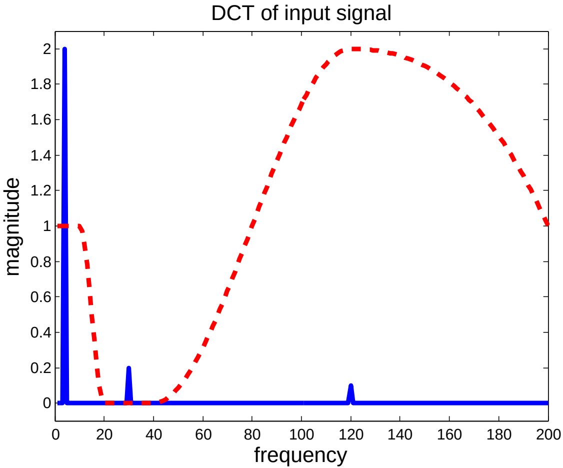

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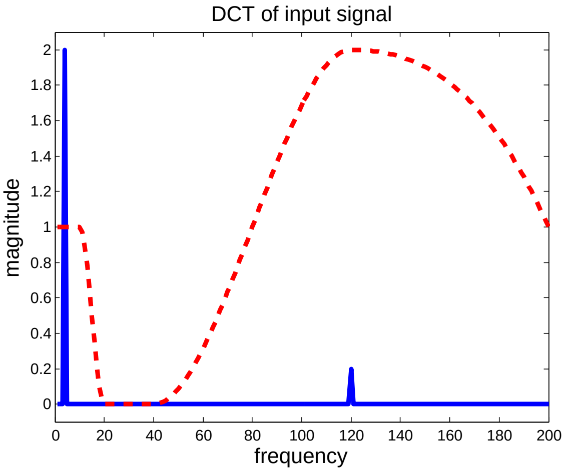

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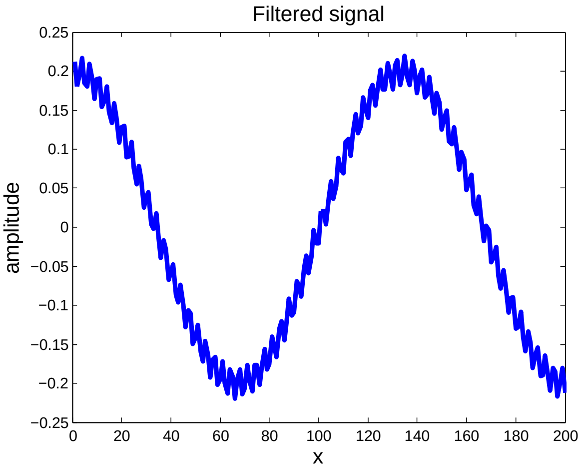

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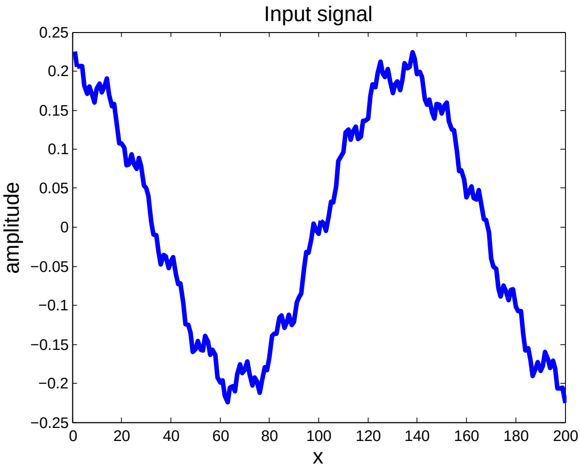

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Motivation

GUI

Motivation

How does this work?

Variational Approach

$$u(t) = \arg\min_u \frac{1}{2}\|u - f\|_2^2 + t\,J(u)$$

$$\phi(t) = t\,\partial_{tt} u(t), \qquad \psi(t) = \phi(1/t)\frac{1}{t^2}$$

Forward Flow

$$\partial_t u(t) = -p(t), \qquad p(t) \in \partial J(u(t))$$

$$\phi(t) = t\,\partial_{tt} u(t), \qquad \psi(t) = \phi(1/t)\frac{1}{t^2}$$

Inverse Scale Space Flow

$$\partial_t p(t) = f - v(t), \qquad p(t) \in \partial J(v(t))$$

$$\psi(t) = \partial_t v(t), \qquad \phi(t) = \psi(1/t)\frac{1}{t^2}$$

Filtering

$$u_{\text{filtered}} = \int_0^\infty \omega(t)\psi(t)\,dt$$

Organizational Stuff

Requirements, or “is this something for me?”

Necessary

- Interest in mathematical theory
- Image processing and convex analysis
- Numerics (Matlab)

Nice to know

- Optimization
- Partial differential equations

Requirements, or “is this something for me?”

For those who heard *Variational Methods for Computer Vision*

- It will become more theoretical and more mathematically challenging.
- We will prove theorems on the board.
- You’ll have an advantage in terms of possible applications and their numerical implementation.

Requirements, or "is this something for me?"

For those who heard *Variational Methods for Computer Vision*

- It will become more theoretical and more mathematically challenging.
- We will prove theorems on the board.
- You'll have an advantage in terms of possible applications and their numerical implementation.

For those who heard *Ill-posed Problems*

- We will need very little functional analysis – everything will be in $\mathbb{R}^n$.
- You'll have an advantage in terms of the theoretical mathematical concepts.
- I highly recommend learning how to implement the discussed approaches. It is useful and fun!

Formalities

Exercises

- There are no exercises for this lecture.
- Occasionally: theory or programming problems.
- Solution one week later in the lecture.
- The more we discuss in the lecture, the more interesting the course will be! Please don't be shy to say something!
- We'll use OnlineTed to make the lecture more interactive.

Formalities

Examination

- Depending on the number of attendees, the final exam will be either oral or written.
- ECTS credits: 4

Formalities

Miscellaneous

- My office: 02.09.061
- Office hours: Tuesday 4–5pm
- Lecture: Starts at 2:15pm. Short break in between.
- Course website: `https://vision.in.tum.de/teaching/ss2015/multiscale_methods`

Overview

Introduction

Step 1: Make sure we are all on the same page!

Basics of convex analysis:

- Convex extended real valued functions in $\mathbb{R}^n$
- Minimization problems (existence, optimality condition)
- Duality, Saddle point problems

Goal: Everyone knows all necessary tools to follow the lecture!

Introduction

Step 2: Make sure everyone can try out what we are doing!

An optimization method for non-smooth convex minimization

- Praxis oriented – focus on a Matlab implementation.
- Idea rather than convergence analysis.

Goal: Everyone can try out him-/herself what we are doing!

Multiscale decomposition

Step 3: Analyzing multiscale methods!

3.1 Classical theory: How does linear filtering work?

- Transform signal to different representation.
- Filter coefficients.
- Transform back.

$\rightarrow$ Analyze behavior via eigendecomposition!

Multiscale decomposition

Step 3: Analyzing multiscale methods!

3.2 A nonlinear singular vector analysis

- Is there any analogy to singular vectors for nonlinear regularization methods?
- What properties do nonlinear singular vectors have?

Multiscale decomposition

Step 3: Analyzing multiscale methods!

3.3 Nonlinear variational methods

- Define a spectral decomposition for one-homogeneous regularizations.

Multiscale decomposition

Step 3: Analyzing multiscale methods!

3.4 Nonlinear scale space flows

- Analyze behavior of scale space flows.
- Define a spectral decomposition for one-homogeneous regularizations.

Multiscale decomposition

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3.5 Nonlinear inverse scale space flows

- Analyze behavior of inverse scale space flows
- Define a spectral decomposition for one-homogeneous regularizations.

Multiscale decomposition

Step 3: Analyzing multiscale methods!

3.6 Properties of spectral decomposition methods

- Under which conditions do the spectral decomposition approaches yield a discrete spectrum?
- Under which conditions are the three spectral decomposition approaches equivalent?
- Under which conditions do we obtain a nonlinear eigenvalue decomposition?