# Chapter Overview
## Multiscale Methods

*Nonlinear Multiscale Methods for Image and Signal Analysis*
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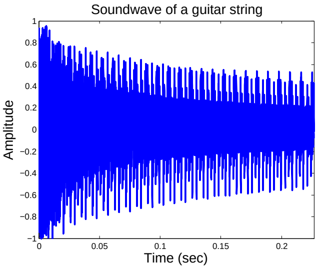
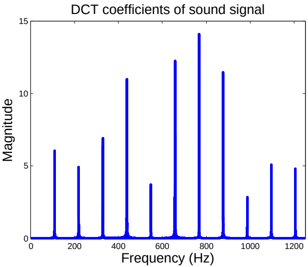
# Motivation

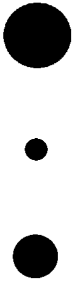
Let's look at the sound signal ...

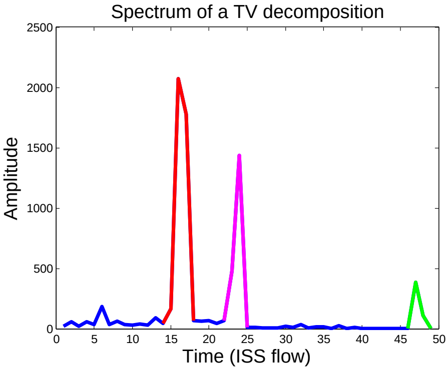


# Motivation

Spectral representation of images with sharp edges?





# Convex Analysis

**Background in convex analysis:**

- Convexity
- Domain
- Proper
- Existence of $\min_u E(u)$
- Subdifferential + Optimality condition
- Subdifferentiability
- Sum + chain rule

# Convex Analysis

## TV Minimization:

- Regularizing with $\sum_{i,j} \sqrt{(u_{i,j} - u_{i+1,j})^2 + (u_{i,j} - u_{i,j+1})^2}$
- Anisotropic case: $\|Ku\|_1$.
- Minimization of $\frac{1}{2}\|u - f\|_2^2 + \alpha\|Ku\|_1$ is difficult.
- Idea: $\|Ku\|_1 = \sup_{p,\ \|p\|_\infty \leq 1} \langle Ku, p \rangle$
- Derivation of gradient projection algorithm

# Convex Analysis

## Duality

- Convex conjugate
- Fenchel-Young Inequality
- Biconjugate
- Subgradient of convex conjugate
- Primal, dual, and saddle point formulation

# Nonlinear singular vectors

## Generalized singular vectors

- Classical case: $J(u) = \frac{1}{2}\|Ku\|_2^2$ leads to $\lambda v_\lambda \in \partial J(v_\lambda)$.
- Generalization to convex functionals via $\lambda v_\lambda \in \partial J(v_\lambda)$.
- 1-homogeneous functions
  - Non-negative
  - Triangle inequality
  - Domain and kernel are linear subspaces
  - Characterization of subdifferential
- Existence of singular vectors via ground states
- Failure of the Rayleigh principle
- Exact recovery of (noisy) singular vectors

# Nonlinear singular vectors

## Multiscale decompositions with variational methods

- How does $u(t) = \arg\min_u \frac{1}{2}\|u - f\|_2^2 + tJ(u)$ behave?
- Explicit solution for $f$ being a singular vector
- $\partial_{tt} u(t)$ has peaks
- $\phi(t) = t\partial_{tt} u(t)$ is wavelength representation
- Finite time extinction of variational method
- Recovery of $f$ by integrating $\phi$
- Spectral filtering via $\int_0^\infty w(t)\ \phi(t)\ dt$

# Nonlinear singular vectors

## Multiscale decompositions with gradient flows

- How does $\partial_t u(t) = -p(t)$, $p(t) \in \partial J(u(t))$, $u(0) = f$ behave?
- Explicit solution for $f$ being a singular vector
- Proceed as in the variational method case

# Nonlinear singular vectors

## Multiscale decompositions with inverse scale space flows

- Systematic error / loss of constrast of variational methods
- Bregman iteration, inverse scale space flow
- How does $\partial_t p(t) = f - u(t)$, $p(t) \in \partial J(u(t))$, $p(0) = 0$ behave?
- Explicit solution for $f$ being a singular vector
- $\partial_t u(t)$ has peaks and can define a spectral representation
- Behavior is inverse to the previous two cases
- Define frequency decomposition
- Transform to switch between frequency and wavelength