# Suggested Homework

## Nonlinear Multiscale Methods for Image and Signal Analysis

**Exercise 1.** Convince yourself that

- every norm is convex.
- if $E : \mathbb{R}^n \to \mathbb{R}$ is convex, then $(E \circ A)$ for $A \in \mathbb{R}^{n\times m}$ is convex.

**Exercise 2.** Prove the following (1d) statement from the lecture. If $E : \mathbb{R} \to \mathbb{R} \cup \{\infty\}$ is convex, then $E$ is locally Lipschitz on $\mathrm{int}(\mathrm{dom}(E))$.

*Hint:* First show that for $x, x_1, x_2 \in \mathrm{int}(\mathrm{dom}(E))$ with $x_1 < x < x_2$ it holds that

$$\frac{E(x) - E(x_1)}{x - x_1} \leq \frac{E(x_2) - E(x_1)}{x_2 - x_1} \leq \frac{E(x_2) - E(x)}{x_2 - x}.$$

Then, for a given $x \in \mathrm{int}(\mathrm{dom}(E))$ pick $a < x_1 < x < x_2 < b$ in $\mathrm{int}(\mathrm{dom}(E))$ and prove that $E$ is Lipschitz on $]x_1, x_2[$.

**Exercise 3.** Find an example of a convex function $E : \mathbb{R} \to \mathbb{R}\cup\{\infty\}$ which is not continuous on $\mathrm{dom}(E)$.