Suggested Homework

Nonlinear Multiscale Methods for Image and Signal Analysis

Exercise 1. Let $q \in \mathbb{R}^{n\times m}$. Convince yourself that the projection $\hat{p}$ of q onto the $\ell^{2,\infty}$ ball of radius one,

$$B_{\ell^{2,\infty}} = \left\{ p \in \mathbb{R}^{n\times m} \mid \sqrt{\sum_j (p_{ij})^2} \leq 1,\ \forall i \right\},$$

i.e.

$$\hat{p} = \arg\min_{p \in B_{\ell^{2,\infty}}} \|p - q\|^2,$$

is given by

$$\hat{p}_{ij} = \frac{q_{ij}}{\max\left(\sqrt{\sum_j (q_{ij})^2}, 1\right)}.$$

Exercise 2. Implement the gradient projection algorithm for isotropic as well as for anisotropic TV image denoising. How do the results differ for large regularization parameters?