Suggested Homework

Nonlinear Multiscale Methods for Image and Signal Analysis

Exercise 1. Let $J \in \Gamma_0(\mathbb{R}^n)$ be 1-homogeneous. Prove that J^* is the indicator function of $\partial J(0)$.

Exercise 2. Let $J \in \Gamma_0(\mathbb{R}^n)$ be 1-homogeneous with $\ker(J) = \{0\}$. Show that

$$|p|_* := \sup_{u, J(u) \leq 1} \langle p, u \rangle$$

defines a norm on $\mathbb{R}^n$. Furthermore, show that $\partial J(0) = \{p \in \mathbb{R}^n \mid |p|_* \leq 1\}$.

Exercise 3. Implement a function *isSubgradient = isSubgradient(u,K,q)* that verifies if $\|q\|_\infty \leq 1$ and $\langle K^T q, u \rangle = \|Ku\|_1$. Convince yourself that this implies $K^T q \in \partial J(u)$ for $J(u) = \|Ku\|_1$.

Advanced: Let K be a 1d finite difference matrix such that $J(u) = \|Ku\|_1$ is the 1d total variation. Can you construct a q such that *isSubgradient*$(q, K, K^T q)$ is true, i.e. $K^T q \in \partial J(K^T q)$?

Hint: q is piecewise linear.