Suggested Homework

Nonlinear Multiscale Methods for Image and Signal Analysis

Exercise 1. Let $J \in \Gamma_0(\mathbb{R}^n)$ be one-homogeneous. Show formally that for any (sufficiently regular) path $u(t)$ it holds that

$$\langle \partial_t p(t), u(t) \rangle = 0, \quad \text{with } p(t) \in \partial J(u(t)).$$

Hint: Consider the equality $\partial_t J(u(t)) = \partial_t(\langle u(t), p(t) \rangle)$. Use the chain rule for the first, and the product rule for the second term.

Conclude that the solution to

$$\partial_t u(t) = -p(t), \qquad p(t) \in \partial J(u(t)),\ u(0) = f$$

with $\phi(t) = t\partial_{tt} u(t)$ satisfies

$$\langle \phi(t), u(t) \rangle = 0.$$

In this sense, at each time the wavelength representation $\phi(t)$ is orthogonal to the remaining signal $u(t)$.

Exercise 2. Implement the nonlinear spectral decomposition of input data f based on both, the variational method as well as the gradient flow, for the case of total variational regularization. Compute such decompositions on an image of your choice. Experiment with interesting filtering approaches.