Multiple View Geometry: Exercise Sheet 1

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Part I: Theory

The following exercises have to be **solved at home**. You will present your answer during the tutorials.

1. Show for each of the following sets (1) whether they are linearly independent, (2) whether they span $\mathbb{R}^3$ and (3) whether they form a basis of $\mathbb{R}^3$:

   - $B_1 = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

   - $B_2 = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

   - $B_3 = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

2. Which of the following sets forms a group (with matrix-multiplication)? Prove or disprove!

   - $G_1 := \left\{ A \in \mathbb{R}^{n \times n} \mid det(A) \neq 0 \wedge A^T = A \right\}$
   - $G_2 := \left\{ A \in \mathbb{R}^{n \times n} \mid det(A) = -1 \right\}$
   - $G_3 := \left\{ A \in \mathbb{R}^{n \times n} \mid det(A) > 0 \right\}$

3. Prove or disprove: There exist non-zero vectors $v_1, \ldots, v_4 \in \mathbb{R}^3 \setminus \mathbf{0}$, which are pairwise orthogonal (i.e., $\forall i, j \colon \langle v_i, v_j \rangle = 0$).

Part II: Practical Exercises

1. Basic image processing

   (a) Download `ex1.zip`

   (b) Load the image `lena.png`.

   (c) Determine the size of the image and show the image.

   (d) Convert the image to gray scale and determine the maximum and the minimum value of the image.

   (e) Apply a gaussian smoothing filter (e.g. using the Matlab-functions `imfilter, fspecial`) and save the output image

   (f) Show 1) the original image, 2) the gray scale image and 3) the filtered image in one figure.

   (g) Compare the gray scale image and the filtered image for different values of the smoothing.

2. Basic operations

   (a) Let $A = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 8 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 5 \\ 15 \end{pmatrix}$. Solve $Ax = b$ for x.

   (b) Define a matrix B equal to A.

   (c) Change the second element in the first row of A to 4.

   (d) Compute the following:
   $c = 0;$
   `for` $i \in \{-4, 0, 4\}$
   $\qquad c = c + i * A^\top * b$
   `end`
   print c

   (e) Compare a) `A .* B` and b) `A' * B` and explain the difference.

3. Write a function `approxequal`(x, y, ϵ) comparing two vectors x and y if they are almost equal, i.e.:
   $$\forall i: \quad ||x_i - y_i|| \le \epsilon.$$
   The output should be logical 1 or 0.

   If the input consists of two matrices, your function should compare the columns of the matrices if they are almost equal. In this case, the output should be a vector with logical values 1 or 0.

4. Write a function `addprimes`(s, e) returning the sum of all prime numbers between s and e.

   Use the Matlab-function `isprime`.