# Multiple View Geometry: Exercise Sheet 7

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## Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. **Epipoles**

   (a) Why do the following relations between the epipoles and the fundamental matrix hold?
   $Fe_1 = 0 \quad \text{and} \quad e_2^\top F = 0 \; .$

   (b) Denote by $[R, T]$ the relative coordinate transformation between two camera views. Then the associated essential matrix is denoted by $F = \hat{T}R$. Show that the following holds for the epipoles $e_1$ and $e_2$:
   $$e_2 \sim T \;\text{ and }\; e_1 \sim R^\top e_2$$
   where $\sim$ means equivalence in the sense of homogeneous coordinates.

2. **Coinages of Points and Lines**

   Suppose $p_1, p_2$ are two points on the line $L$. Let $x_1, x_2$ be the images of the points $p_1, p_2$, respectively, and let $l$ be the coimage of the line $L$.

   Furthermore suppose $L_1, L_2$ are two lines intersecting in the point $p$. Let $x$ be the image of the point $p$ and let $l_1, l_2$ be the coimages of the lines $L_1, L_2$, respectively.

   Draw a picture and convince yourself of the following relationships:

   (a) Show that
   $$l \sim \hat{x_1} x_2, \qquad x \sim \hat{l_1} l_2,$$

   (b) Show that for some $r, s, u, v \in \mathbb{R}^3$,
   $$l_1 \sim \hat{x} u, \qquad l_2 \sim \hat{x} v, \qquad x_1 \sim \hat{l} r, \qquad x_2 \sim \hat{l} s$$

   where $\sim$ means equivalence in the sense of homogeneous coordinates.

3. **Rank Constraints**

   Let $x_1$ and $x_2$ be two image points with projection matrices $\Pi_1$, $\Pi_2$. Show that the rank constraint
   $$\text{rank} \begin{pmatrix} \hat{x_1} \Pi_1 \\ \hat{x_2} \Pi_2 \end{pmatrix} \leqq 3$$
   ensures that $x_1$ and $x_2$ are images (/ projections) of the same three-dimensional point $X$.

4. **Projection and Essential Matrix**

   Suppose two projection matrices $\Pi = [R, T]$ and $\Pi' = [R', T'] \in \mathbb{R}^{3\times 4}$ are related by a common transformation $H$ of the form

   $$H = \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} \in \mathbb{R}^{4\times 4} \quad \text{where} \quad v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

   That is, $[R, T]H \sim [R', T']$ are equal up to scale.

   Show that $\Pi$ and $\Pi'$ give the same essential matrices ($E = \hat{T}R$ and $E' = \hat{T}'R'$) up to a scale factor.

# Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

### Epipolar lines

1. Download the package `ex7.zip` from the website. Extract the images `batinria0.pgm` and `batinria1.pgm`. Their corresponding camera calibration matrices can be found in the file `calibration.txt`.
2. Show the two images with matlab and select a point in the first image. You can use the command `[x,y]=ginput(n)` to retrieve the image coordinates of a mouse click.
3. Think about where the corresponding epipolar line $l_2$ in the second image could be.
4. Now compute the epipolar line $l_2 = Fx_1$ in the second image corresponding to the point $x_1$ in the first image. To this end you will need to compute the fundamental matrix $F$ between the two images.
   (Use the calibration data from the file `calibration.txt`.)
5. Test your program for different points $x_1$. What do you observe?
6. *Bonus: Determine the best matching point on the epipolar line via normalized cross correlation.*