Multiple View Geometry: Solution Exercise Sheet 4

Prof. Dr. Daniel Cremers, Julia Diebold, Robert Maier, TU Munich

http://vision.in.tum.de/teaching/ss2015/mvg2015

Part I: Theory

Image Formation

1. Let $\mathbf{p} = (x\ y\ z)$ be a point on the smaller object and $\mathbf{p}' = (x'\ y'\ z')$ a point on the larger object. Since $\mathbf{p}'$ is twice as far away, we have $z' = 2z$, and twice as big we have $x' = 2x$ and $y' = 2y$. Hence, it follows that $\mathbf{p}$ and $\mathbf{p}'$ lie on the same projection ray.

$$\pi(\mathbf{p}') = \pi \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} = \begin{pmatrix} 2x/2z \\ 2y/2z \end{pmatrix} = \begin{pmatrix} x/z \\ y/z \end{pmatrix} = \pi \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \pi(\mathbf{p})$$

2. Let $\mathbf{p}_h := \begin{pmatrix} \mathbf{p}_h \\ 1 \end{pmatrix} = (0\ \ 0\ \ 4\ \ 1)^\top$. Hence:

   $\tilde{\mathbf{p}}_\mathbf{1} = \pi(\mathbf{P}_1 \cdot \mathbf{p}_h) = \pi(-3\ \ 0\ \ 4)^\top = (-0.75\ \ 0)^\top$

   $\tilde{\mathbf{p}}_\mathbf{2} = \pi(\mathbf{P}_2 \cdot \mathbf{p}_h) = \pi(1\ \ 0\ \ 4)^\top = (0.25\ \ 0)^\top$

Radial Distortion

1. No, as it can only model points for which the viewing ray intersects the image plane.
2. $f(r) = 1 + a_1 r^2 + a_2 r^4$ is much easier to invert than $f(r) = 1 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4$. An inverse model is *needed to get 3D points* from image points and depth.
   (Inverting f is required for calculating the viewing ray corresponding to an image point (un-projection), this is an important property for some algorithms, like tracking algorithms.)