



Multiple View Geometry: Solution Exercise Sheet 1

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Part I: Theory

1. To summarize:

	B_1	B_2	B_3
(1) Are linearly independent	yes	yes	no
(2) Span \mathbb{R}^3	yes	no	yes
(3) Form a basis of \mathbb{R}^3	yes	no	no

More details:

B_1 : Can be shown by building a matrix and calculating the determinant: $\det \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \neq 0$.

As the determinant is not zero, we know that the vectors are linearly independent. Three linear independent vectors in \mathbb{R}^3 span \mathbb{R}^3 . Furthermore, three spanning vectors build a minimal set, hence, they also form a basis of \mathbb{R}^3 .

B_2 : To span \mathbb{R}^3 , there are at least three vectors needed.

B_3 : In \mathbb{R}^3 , there cannot be more than three independent vectors.

2. To summarize:

	G_1	G_2	G_3
Form a group	no	no	yes

More details:

G_1 : Closure not given!

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 5 \end{pmatrix}}_{\in G_1} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{\in G_1} = \underbrace{\begin{pmatrix} 1 & 4 & 9 \\ 2 & 0 & 12 \\ 3 & 8 & 15 \end{pmatrix}}_{\notin G_1}$$

G_2 : Neutral element not included, as $\det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 \neq -1$

G_3 : Yes, as can easily be shown using $\det(A^{-1}) = \frac{1}{\det(A)}$.

3. **To summarize:** No.

More details (proof): Assuming the existence of four pairwise orthogonal, non-zero vectors $v_1, \dots, v_4 \in \mathbb{R}^3$, we obtain a contradiction:

We know that, in \mathbb{R}^3 , there are at most 3 linearly independent vectors. Hence, we know that $\exists a_i :$
 $\sum_{i=1}^4 a_i v_i = 0$, with at least one $a_i \neq 0$. Without loss of generality, we can assume that $a_1 = 1$, giving

$$v_1 = a_2 v_2 + a_3 v_3 + a_4 v_4$$

As the vectors are pairwise orthogonal, we can derive

$$\begin{aligned} ||v_1||^2 &= \langle v_1, v_1 \rangle \\ &= \langle v_1, a_2 v_2 + a_3 v_3 + a_4 v_4 \rangle \\ &= \langle v_1, v_2 \rangle a_2 + \langle v_1, v_3 \rangle a_3 + \langle v_1, v_4 \rangle a_4 = 0, \end{aligned}$$

which contradicts $v_1 \neq \mathbf{0}$.

Part I: Matlab

1. Basic image processing

- (a) -
- (b) `I = imread('lena.png');`
- (c) `[r,c,ch] = size(I)`
`imshow(I)`
- (d) `J = rgb2gray(I);`
`min_val = min(min(J))`
`max_val = max(max(J))`
- (e) `h = fspecial('gaussian');`
`J2 = im2double(J);`
`K = imfilter(J2,h);`
`imwrite(K,'smoothed.png','PNG');`
- (f) `figure`
`subplot(1,3,1), imshow(I), title('original')`
`subplot(1,3,2), imshow(J2), title('gray scale')`
`subplot(1,3,3), imshow(K), title('smoothed')`
- (g) `h = fspecial('gaussian', [9 2], 1);`
`K = imfilter(J2,h);`
`figure, subplot(1,2,1), imshow(J2), subplot(1,2,2), imshow(K)`

2. Basic operations

- (a) `x = A \ b`
- (b) `B = A`
- (c) `A(1,2) = 4`
- (d) `c = 0;`
`for i = -4:4:4`
`c = c + i*A'*b`
`end`
`fprintf('c = %0.1f',c)`
- (e) a) element-wise multiplication of matrix-elements
b) matrix multiplication ($A^T B$)

3. There is a number of possible solution which do not require a loop, such as:

- `out = all(all(abs(x-y) <= eps))`
- `out = sum(sum(abs(x-y) > eps)) == 0`
- `out = max(abs(x-y)) <= eps`
- `out = max((x-y) .* (x-y)) <= eps*eps`

4. Again there are several possible commands, such as:

- `A = s:e;`
`out = sum(isprime(A) .* A);`
- `A = s:e;`
`out = sum(A(isprime(A)));`