Convex Optimization for Computer Vision
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Exercises: E. Laude
Summer Semester 2016

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Weekly Exercises 3

Room: 02.09.023
Friday, 6.5.2016, 09:00-11:00
Submission deadline: Wednesday, 4.5.2016, 14:00, Room 02.09.023

Theory: Strong convexity, Lipschitz continuity and subgradient descent (12 Points)

Exercise 1 (4 Points)**.** Prove the following theorem: If $E \in \mathcal{S}^{1,1}_{m,L}(\mathbb{R}^n)$, then for any $u, v \in \mathbb{R}^n$ we have

$$\langle \nabla E(u) - \nabla E(v), u - v \rangle \geq \frac{mL}{m+L} \|u - v\|^2 + \frac{1}{m+L} \|\nabla E(u) - \nabla E(v)\|^2$$

Exercise 2 (4 Points)**.** Compute the subdifferentials of the following convex functions:

1. $f : \mathbb{R}^n \to \mathbb{R}$ with $f(x) = \|x\|_1$.
2. $f : \mathbb{R}^n \to \mathbb{R}$ with $f(x) = \|x\|_2$.
3. $f : \mathbb{R}^{n \times m} \to \mathbb{R}$ with $f(X) = \|X\|_{2,1} := \sum_{i=1}^m \|x^i\|_2$, where $x^i \in \mathbb{R}^n$ is the i-th column of X.

Exercise 3 (4 Points)**.** Let $f \in \mathbb{R}^n$. Show that the ℓ_1-norm proximity operator of f defined as the solution u of the convex optimization problem

$$\arg\min_{u \in \mathbb{R}^n} \frac{1}{2\lambda} \|u - f\|^2 + \|u\|_1,$$

is given as

$$u \in \mathbb{R}^n, \quad u_i := \begin{cases} f_i + \lambda & \text{if } f_i < -\lambda \\ 0 & \text{if } f_i \in [-\lambda, \lambda] \\ f_i - \lambda & \text{if } f_i > \lambda. \end{cases}$$

Hint: Note that the above optimization problem is decoupled in the sense that one can look for the individual entries u_i of the optimal u separately.

Programming: TV-ℓ_1-denoising (12 Points)

Exercise 4 (12 Points)**.** Denoise the noisy input image f, given in the file `fish_saltpepper.png` by minimizing the following robust denoising energy:

$$E(u) = \frac{\lambda}{2} \|u - f\|_1 + \|Du\|_{2,1}$$

with subgradient descent, where D is a finite difference color gradient operator, and the $\ell_{2,1}$-norm is defined as on exercise sheet 0.