Weekly Exercise 9

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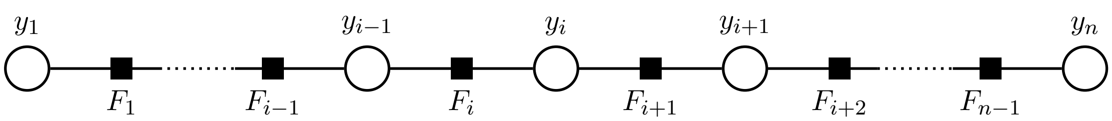
Probabilistic Inference (8 Points)

Exercise 1 (**Inference on chains**, 2 Points)**.** Consider the following factor graph, which is a chain:



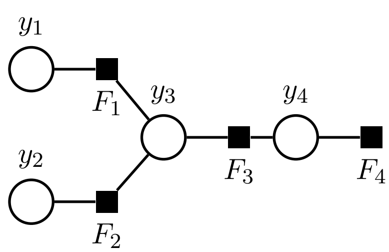
The joint distribution can be written in the form

$$p(\mathbf{y}) = \frac{1}{Z} F_1(y_1, y_2) F_2(y_2, y_3) \cdot \ldots \cdot F_{n-1}(y_{n-1}, y_n),$$

where $Z = \sum_{\mathbf{y}} \prod_{i=1}^{n-1} F_i(y_i, y_{i+1})$ denotes the partition function. Show that the marginal distribution $p(y_i)$ decomposes into the product of two factors:

$$p(y_i) = \frac{1}{Z} r_{F_i \to y_i}(y_i) r_{F_{i+1} \to y_i}(y_i).$$

Exercise 2 (**Sum-product and max-sum**, 6 Points)**.** Consider the following factor graph,



The factors are defined through the following:

$$\begin{aligned} &F_x(y_i, y_j) = \exp\left(-(|y_i - y_j| + (c_x - y_i)^2)\right), x \in \{1, 2, 3\}, \\ &c_1 = 0, c_2 = 1, c_3 = 1, c_4 = 2, \\ &F_4(y_4) = \exp(-(c_4 - y_4)^2). \end{aligned} \tag{1}$$

Assume y_4 as the root node and $\mathbf{y} \in \{0, 1, 2\}^4$.

a) Perform the sum-product algorithm in order to achieve probabilistic inference of the model expressed by the graph above. Show the intermediate steps in details.

b) Perform the max-sum algorithm to achieve MAP inference of the model expressed by the graph above. Show the intermediate steps in details.