

## Weekly Exercise 10

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### Mean Field Approximation

(4 Points)

**Exercise 1 (Naive Mean Field, 4 Points).** Assume a graphical model  $G = (\mathcal{V}, \mathcal{E})$  and consider a factorized distribution in the following form:

$$q(y) = \prod_{i \in \mathcal{V}} q_i(y_i). \quad (1)$$

a) Show that the marginal distribution of a factor  $F$  is given by:

$$\mu_{F, y_F}(q) = q_{N(F)}(y_F) = \prod_{i \in N(F)} q_i(y_i).$$

b) Show that the entropy decomposes as:

$$H(q) = \sum_{i \in \mathcal{V}} H_i(q_i),$$

where

$$H_i(q) = - \sum_{y_i \in \mathcal{Y}_i} q_i(y_i) \log q_i(y_i).$$

### Programming

(10 Points)

**Exercise 2 (Multi-class image segmentation with fully connected CRF, 10 points).** Let us consider the multi-class labeling problem for image segmentation. Assuming the label set  $\mathcal{L}$ , we define the following energy function for  $\mathbf{y} \in \mathcal{L}^{\mathcal{V}}$ :

$$E(\mathbf{y}) = \sum_{i \in \mathcal{V}} E_i(y_i) + \sum_{(i,j) \in \mathcal{E}} E_{ij}(y_i, y_j), \quad (2)$$

on a fully connected CRF model, i.e.  $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} \mid i < j\}$ .

The goal of the exercise is to implement the naive mean field approximation algorithm in order to obtain multi-class segmentation results for test images shown in Figure 1. As you have seen in the previous exercises, these images come from a subset of the MSRC image understanding dataset, which contains 21 classes, i.e.  $\mathcal{L} = \{1, 2, \dots, 21\}$ .

To define the unary energy functions  $E_i$  for  $i \in \mathcal{V}$ , use the provided `*.c_unary` files. Each test image has its own unary file, specified by the same filename. In each unary

file, you can read out a  $K \times H \times W$  array of float numbers. The  $H$  and  $W$  are the image height and width, and  $K = 21$  is the number of possible classes. This array contains the 21-class probability distribution for each pixel. We provide the `multilabel_demo.cpp` to demonstrate how to load the unary file and read out the corresponding probability values. The unary energy functions  $E_i$  for all  $i \in \mathcal{V}$  are then defined as

$$E_i(y_i = l) = -\log(p_l) \ .$$

To define the pairwise energy  $E_{ij}$  for  $(i, j) \in \mathcal{E}$ , use the contrast sensitive Potts model:

$$E_{ij}(y_i, y_j) = \mathbb{1}[y_i \neq y_j] \left( w_1 \exp \left( -\frac{|p_i - p_j|^2}{2\theta_\alpha^2} - \frac{|I_i - I_j|^2}{2\theta_\beta^2} \right) + w_2 \exp \left( -\frac{|p_i - p_j|^2}{2\theta_\gamma^2} \right) \right),$$

where the parameter are chosen as

$$w_1 = 10, w_2 = 3, \theta_\alpha = 80, \theta_\beta = 13, \theta_\gamma = 3 \ .$$



Figure 1: Test images for multi-class image segmentation.