## Machine Learning for Robotics and Computer Vision Summer term 2016

Homework Assignment 2

Topic 1: Regression May 2, 2016

### Exercise 1: Bayesian Update

Consider a linear regression model with basis functions  $\phi(x)$  as presented in the lecture. We assume a Gaussian prior distribution for the weights:

$$p(\mathbf{w}) = \mathcal{N}(\mathbf{w}|m_0, S_0)$$

Suppose we have already observed N data points, so the posterior distribution is

$$p(\mathbf{w}|\mathbf{t}) = \mathcal{N}(\mathbf{w}|m_N, S_N)$$

with

$$m_N = S_N(S_0^{-1}m_0 + \sigma^{-2}\Phi^T \mathbf{t})$$
 and  $S_N = S_0^{-1} + \sigma^{-2}\Phi^T\Phi$ .

Now, we observe a new data point  $(x_{N+1}, t_{N+1})$ . What is the new posterior?

### Exercise 2: Quadrocopter (Programming)

We are testing a tracking program. We evaluate it with the help of a quadrocopter. The quadrocopter sends estimates of its velocity and the tracking program estimates its global position with respect to the quadrocopter's initial position (before flying).

a) The tracker yields these tracked position estimates at a frequency of 1Hz:

$$\mathcal{T} = \left\{ \begin{pmatrix} 2\\0\\1 \end{pmatrix} \begin{pmatrix} 1.08\\1.68\\2.38 \end{pmatrix} \begin{pmatrix} -0.83\\1.82\\2.49 \end{pmatrix} \begin{pmatrix} -1.97\\0.28\\2.15 \end{pmatrix} \begin{pmatrix} -1.31\\-1.51\\2.59 \end{pmatrix} \begin{pmatrix} 0.57\\-1.91\\4.32 \end{pmatrix} \right\}$$

Plot these data with your tool of choice (e.g. Matlab).

- b) Assuming the quadrocopter flies with constant speed, which speed does it have? What is the residual error of the estimation?
- c) Now assume that the quadrocopter flies with constant acceleration. What is the residual error now? Is the error higher or lower? Why?
- d) According to our last model, what is the quadrocopter's most likely position in the next second?

Hint for b) and c): Use the Polynomial Regression method introduced in the lecture.

# Topic 2: Probabilistic Graphical Models

### Exercise 3: Reading a graphical model

We have the following graphical model:

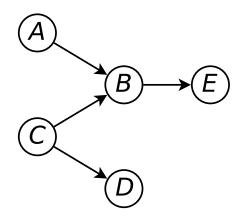
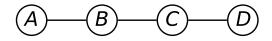


Abbildung 1: Graphical model.

- a) Write the joint probability distribution corresponding to the graphical model depicted in Fig. 1.
- b) What are the conditional independence assumptions of this model?
- c) Which of the following assertions are true, and why?
  - B is d-separated from D by C,
  - A is d-separated from C by E,
  - A is d-separated from C by D,
  - E is d-separated from D by B,
  - E is d-separated from D by A.

## Exercise 4: Markov Chain

We have the following Markov Chain:



- a) Write the joint probability distribution associated to this Markov Chain.
- b) Each variable can take value 0 or 1, and we want to express that it is 9 times more probable that neighboring variables have equal values than they have different value. Give the potential functions of this Markov Chain.
- c) Compute the probability distributions p(A) and p(C).
- d) Now, we observe that D is 1, recompute the distributions over A and C:  $p(A \mid [D = 1])$  and  $p(C \mid [D = 1])$ .
- e) Compute p(C | [A = 0], [D = 1]).

The next exercise class will take place on May 13th, 2016.

For downloads of slides and of homework assignments and for further information on the course see

https://vision.in.tum.de/teaching/ss2016/mlcv16