Weekly Exercises 2

Room: 02.09.023

Wed, 04.05.2016, 14:00-16:00

Submission deadline: Tue, 03.05.2016, 23:59 to laehner@in.tum.de

Mathematics: Tangent spaces and Curvature

Exercise 1 (2 points).

1. Consider the following two coordinate maps of the sphere in $\mathbb{R}^3$.

$$\begin{aligned} x_1 &: \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\} \to \mathbb{R}^3 \\ &(u,v) \mapsto (u, v, \sqrt{1 - u^2 - v^2}) \qquad (1) \\ x_2 &:]-10, 10[\times]-10, 10[\to \mathbb{R}^3 \\ &(u,v) \mapsto \frac{1}{u^2+v^2+1}(2u, 2v, u^2+v^2-1) \qquad (2) \end{aligned}$$

   Calculate two vectors in $\mathbb{R}^3$ spanning the tangent space at $p = (2/3, 2/3, 1/3)$. Which vectors a, b in the domain of Dx_1, Dx_2 are mapped to the tangent vectors you gave before?

2. Proof that the tangent space $T_pM \subset \mathbb{R}^n$ of a submanifold $M \subset \mathbb{R}^n$ of dimension $m \leq n$ is a linear subspace and does not depend on the choice of the choosen coordinate map (x_i, U_i). (Lemma 3 from the Manifold chapter)

Exercise 2 (1 point). Find a coordinate map $\mathbf{x} : \mathbb{R}^2 \to \mathbb{R}^3$ of the torus

$$T = \left\{ (x,y,z) \in \mathbb{R}^3 | \left(\sqrt{x^2+y^2} - a\right)^2 + z^2 = r^2 \right\}$$

with $a > r > 0$. (It can of course not cover the complete manifold.)

Exercise 3 (1 point). The circle with radius r can be represented by the implicit formulation

$$\begin{aligned} \varphi : &\mathbb{R}^2 \to \mathbb{R} \\ &x \mapsto \sqrt{\langle x, x \rangle} \\ C_r &= \varphi^{-1}(r) \end{aligned}$$

Calculate the curvature on each point of C_r.

Exercise 4 (2 points). Show that the push-forward is a linear mapping.

Programming: Curvature

Download the supplementary material from the homepage. It contains some black-white silhouette images from the MPEG7 dataset (http://www.dabi.temple.edu/~shape/MPEG7/dataset.html) and a file to extract contour information from these images.

Exercise 5 (2 points)**.** Read out the image files `bat-9.gif`, `device7-1.gif`, `turtle-1.gif` into matrices (use `imread`, it reads positive integers. Changing the type to double will help). Include an image for each sub-exercise in your solution sheet.

1. Calculate the curvature on the contour. It can be seen as the level set function somewhere between 0 and 1 so the formula $\kappa(p) = \text{div}\left(\frac{\nabla F(\cdot)}{||\nabla F(\cdot)||}\right)(p)$ from the lecture can be used. There are `imgradient` and `imgradientxy` in Matlab or implement your own finite element gradient as an exercise (it's quite easy). The divergence can be calculated with `divergence`.

2. The result from the last exercise was pretty ugly. The reason is that the function we considered was not smooth but went zig-zag along the edges of the pixels. Use a gaussian filter on the image before calculating the curvature. (See `imgaussfilt`) Use the `extract_contour.m` from the supplementary material to get a binary mask for the contour with different thickness. Play around with different σ for the filter and thicknesses of the mask.

Exercise 6 (1 point)**.** In most applications we want to find out which shapes are similar to each other. Create a descriptor for each shape in the supplementary material and create a histogram of the different curvatures on the contour (don't forget to normalize because normally the contours will not have the same amount of points). There is a Matlab function `histogram` if you are not familiar with histograms.
Compare the descriptor of `device7-1` to all other descriptors (for example with the Euclidean distance) and sort the remaining shapes in order of similarity to device7-1.