

Weekly Exercises 6

Room: 02.09.023

Wed, 08.06.2016, 14:00-16:00

Submission deadline: Tue, 07.06.2016, 23:59 to laehner@in.tum.de

Mathematics

Exercise 1 (1 point). Show that the principle curvature can be calculated if the mean and Gauss curvature are given.

Exercise 2 (3 points). Show that the gaussian curvature of the torus in 3D (see Exercise sheet 2) integrates to zero. Tip: the parametrization and differential of the torus can be found in the solutions, choose arbitrary values for a, r to simplify the calculations. ($a = 2, r = 1$ for example) First, try to find the Gauss Map and calculate its differential. It suffices to write the Gauss Map as a function $N : U \rightarrow \mathbb{S}^2$. Then write the differential in an basis of the tangent space (the columns of Dx for example).

Programming

If you want to try out other shapes than the cat, take a look at http://tosca.cs.technion.ac.il/book/resources_data.html under the link TOSCA high-resolution.

- Exercise 3** (2 points).
1. Use your code from last week (or download the solution from the homepage) to calculate the gradient on a triangle mesh. The gradient is defined triangle-wise, so you have to implement a function `gradient.m` that takes a struct containing vertices and triangles and a function $f \in \mathbb{R}^n$ defined vertex-wise and returns the gradient of f as a $\mathbb{R}^{m \times 3}$ matrix (one vector for each triangle). Remember there is a closed formula for the inverse of a 2×2 matrix.
 2. Compare the gradient from the previous step with the obvious solution of just calculating the gradient on the reference triangle and transferring that vector onto the manifold $Dx_i \nabla \tilde{f}$. You can use `quiver3` to plot 3D-vector fields, additionally to the vectors it needs their origins as an input. You can use the mass center of each triangle $(1/3(a+b+c))$ if (a, b, c) is the triangle. The command `hold on` will keep previous plots that you made and plot your new command over the old one, so you can plot the mesh and two sets of vector fields in the same plot (the vector fields will also be assigned different colors automatically).