Convex Optimization for Machine Learning and Computer Vision

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Weekly Exercises 4

Room: 02.09.023

Monday, 29.05.2017, 12:15-14:00

Submission deadline: Wednesday, 24.05.2017, Room 02.09.023

Convex Duality (8 Points + 4 Bonus)

Exercise 1 (4 Points). Compute the convex conjugates of the following functions:

1. $f_1 : \mathbb{R} \to \mathbb{R} \cup \{\infty\}$ where $f_1(x) = \sqrt{1 + x^2}$.
2. $f_2 : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ where $f_2(x) = \log\left(\sum_{i=1}^n e^{x_i}\right)$.

Don't forget to specify the domains $\text{dom}(f_1^*), \text{dom}(f_2^*)$.

Exercise 2 (4 Points). Compute the convex envelope f^{**} of the functions

1. $f : \mathbb{R} \to \overline{\mathbb{R}}$, $f(x) = \begin{cases} 0 & \text{if } \ x = 0, \\ \lambda & \text{if } \ x \neq 0, |x| \leq 1, \\ \infty & \text{otherwise.} \end{cases}$
2. $f : \mathbb{R}^{n \times n} \to \overline{\mathbb{R}}$, $f(X) = \text{rank}(X) + \delta\{\|X\|_{\text{spec}} \leq 1\}$.

by taking the convex conjugate twice.

Definition. The convex hull of an arbitrary set $C \subset \mathbb{R}^n$ is defined as

$$\text{conv}(C) = \left\{ \sum_{i=1}^{p} \lambda_i x_i : x_i \in C, \lambda_i \geq 0, \sum_{i=1}^{p} \lambda_i = 1, p \geq 0 \right\}. \tag{1}$$

Exercise 3 (4 Points). Show that for a set $C \neq \emptyset$ in $\mathbb{R}^n$, every point of $\text{conv}(C)$ can be expressed as a convex combination of $n+1$ points of C (not necessarily different).

Image Cartooning

Finish the programming exercise from the second exercise sheet.