Convex Optimization for Machine Learning and Computer Vision

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Weekly Exercises 8

Room: 02.09.023
Monday, 03.07.2017, 12:15-14:00
Submission deadline: Wednesday, 28.06.2017, Room 02.09.023

Primal-Dual Methods (6 Points)

Exercise 1 (2 Points)**.** Consider the optimization problem

$$\min_{x \in \mathbb{R}^n} \; g(x) + \sum_{i=1}^{k} f_i(K_i x), \tag{1}$$

with $g : \mathbb{R}^n \to \overline{\mathbb{R}}$, $f_i : \mathbb{R}^{m_i} \to \overline{\mathbb{R}}$ closed, proper, convex and $K_i : \mathbb{R}^n \to \mathbb{R}^{m_i}$ linear. Assume that g and all f_i are *simple* in the sense that their proximal mapping

$$\text{prox}_{\tau f_i}(y) := \text{argmin}_{x \in \mathbb{R}^{m_i}} \; f_i(x) + \frac{1}{2\tau} \|x - y\|^2,$$

can be efficiently computed. Explain how (1) can be solved with PDHG and write down the explicit update equations.
Hint: Stack the individual K_i into a single matrix K.

Exercise 2 (4 Points)**.** Recall the energy from the cartooning programming exercise on the second exercise sheet:

$$E(u) = \langle u, f \rangle + \sum_{j=1}^{n} \delta\{u(:,j) \in \Delta^k\} + \alpha \sum_{i=1}^{k} \|Du(i,:)\|_1. \tag{2}$$

Write down the explicit update equations for solving (2) with a) PDHG and b) ADMM. Make sure to apply the algorithms in such a way that each subproblem has a simple, closed form solution (such as orthogonal projection onto the simplex, solving a linear system or proximity operator of the ℓ_1-norm).

Programming (5 Bonus)

Exercise 3. Implement one of the methods you derived in exercise 2 using MATLAB. For PDHG, choose the step sizes s and t such that they satisfy $st > \|K\|_{\text{spec}}^2$.