GPU Programming in Computer Vision: Day 3

Date: Wednesday, 13. September 2017

Exercise 9: Isotropic Diffusion (8P)

Implement the nonlinear isotropic diffusion

$$\partial_t u = \operatorname{div}\Big(\widehat{g}\big(|\nabla u|\big)\nabla u\Big),$$

for different choices of $\widehat{g}$. This means updating u in the following way:

$$u^{n+1} = u^n + \tau \operatorname{div}(\widehat{g}\big(|\nabla u^n|\big)\nabla u^n\Big).$$

As a initial condition, pick u^0 as the input image. For simplicity, we consider $\widehat{g}(s) = 1$ at first.

1. Use forward differences to compute the derivatives $v_1 := \partial_x^+ u$ and $v_2 := \partial_y^+ u$. Reuse your code from exercise 4.
2. Compute the diffusivity g from v_1, v_2. Use a "`__host__ __device__`" function for $\widehat{g}$. *Hint:* Note that g is *scalar*, there is only one value g, which is shared for all channels.
3. Multiply v_1, v_2 by g, and store the result again in v_1, v_2. If you want, you can combine steps 2 and 3 into a single kernel. Note that then you don't need an array for g, because you can compute g locally in the kernel.
4. Use backward differences to compute the divergence: $d := \operatorname{div}\binom{v_1}{v_2} = \partial_x^- v_1 + \partial_y^- v_2$. Reuse your code from exercise 4. y
5. Compute the update step for u, update all of the n_c channels in a single kernel. You can implement this as a separate kernel, or as part of the div-kernel from step 3.
6. Compute N iterations of the diffusion and visualize the end result. Experiment with different time steps τ and different numbers of iterations N. A necessary condition for convergence is $\tau < 0.25/\widehat{g}(0)$, i.e. $\tau < 0.25$ for $\widehat{g}(s) = 1$. What happens is τ is chosen too big? What happens for very large N?
7. Compare the result to Gaussian convolution $G_\sigma * u$ with $\sigma = \sqrt{2\tau N}$. What do you observe?
8. Now try using a different diffusivity function:
   - $\widehat{g}(s) = \frac{1}{\max(\varepsilon, s)}$,
   - $\widehat{g}(s) = \exp(-s^2/\varepsilon)/\varepsilon$.

   How do the results change in each case?

Exercise 10: Anisotropic Diffusion (5P)

Implement the *anisotropic* linear diffusion

$$\partial_t u = \operatorname{div}(G\nabla u),$$

by iterating the explicit forward Euler scheme

$$u^{n+1} = u^n + \tau \operatorname{div}(G\nabla u^n).$$

Here, $G(x,y) \in \mathbb{R}^{2\times 2}$ is a 2×2 diffusion tensor which is different at every pixel (x,y) in the domain and is calculated once from the input image. Its aims is to make the diffusion process preserve image edges.

1. Compute the diffusion tensor G from the input image:

   (a) First calculate the structure tensor of the input image. Postsmooth the structure tensor with a gaussian kernel with a different standard deviation parameter ρ. Reuse parts of your code from exercise 7.

   (b) Now calculate the Eigenvalues $\lambda_1 \geq \lambda_2$ and corresponding eigenvectors e_1,e_2 of the structure tensor G in each point. For that, extend the `__device__` function from exercise 8 to also calculate and return the eigenvectors.

   (c) Set the diffusion tensor to $G = \mu_1 e_1 e_1^T + \mu_2 e_2 e_2^T$ with

   $$\mu_1 = \alpha,$$

   $$\mu_2 = \begin{cases} \alpha & \lambda_1 = \lambda_2, \\ \alpha + (1-\alpha)\exp\left(-\frac{C}{(\lambda_1-\lambda_2)^2}\right) & \text{else,} \end{cases}$$

   where $C > 0$ and $\alpha \in (0,1)$ are parameters.

2. Implement the diffusion as outlined in the previous exercise, except that the diffusivity is now matrix valued and constant. Perform a 2×2 matrix-vector multiplication on the gradient instead of the scalar multiplication in exercise 10.

3. Try out different choices of C, α and iteratons numbers N. Remember to chose $\tau < 0.25$ small enough. Try it out on the image `van-gogh.png` and pick $C = 5 \cdot 10^{-6}$, $\alpha = 0.01$, $\sigma = 0.5$ and $\rho = 3$ as a start. *Hint: for debugging purposes it might be useful to visualize the diffusion tensor!*