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Machine Learning for Computer Vision
Summer term 2017

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Topic: Laplace Approximation, K-Means, EM

Exercise 1: Laplace Approximation

In Gaussian Process classification, we cannot integrate exactly over the parameters $\mathbf{w}$.

a) Why is this the case? Why is this a problem?
Name 3 approaches one can use to tackle this problem?

b) One simple way to circumvent this problem is Laplace approximation. Consider the case of a single continuous random variable z with (non-Gaussian) distribution $p(z) = \frac{1}{Z} f(z)$, where $f(z)$ is the unnormalized density. What is the solution given by Laplace approximation?

Exercise 2: Expectation-Maximization for GMM

In the standard EM algorithm, we first define the responsibilities γ as

$$\gamma_{nk} = p(z_{nk} = 1 | x_n) = \frac{\pi_k \mathcal{N}(x_n | \mu_k, \Sigma_k)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(x_n | \mu_j, \Sigma_j)} \quad , z_{nk} \in \{0, 1\}, \sum_{k=1}^{K} z_{nk} = 1$$

a) Find the optimal means, covariances and mixing coefficients that *maximize the data likelihood*. How can you interpret the results?

b) (Optional) Define the complete-data-log-likelihood. What is the difference to the standard log-likelihood?

Exercise 3: K-Means Compression and EM for GMM (Programming)

Implement following exercises in your preferred programming language. Download the *clustering.zip* found on the website and extract its contents.

a) Implement K-Means and use it to compress the *mona-lisa.jpg* image, by using only $2, 3, \ldots, 10$ colors.

b) Implement the Expectation-Maximization algorithm (EM) for Gaussian Mixture Models (GMM) to cluster the *old faithful* dataset.