Weekly Exercise 2

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Probability theory (10 Points)

Exercise 1 (Bayes' rule, 1 point**).** Let A, B, C be *events*. Assuming $P(B \mid C) \neq 0$, prove that

$$P(A \mid B \cap C) = \frac{P(B \mid A \cap C) \cdot P(A \mid C)}{P(B \mid C)} \,.$$

Exercise 2 (Bayes' rule, 5 points**).** Siegfried the ornithologist does a study on the green-speckled swallow. Since he has a huge collection of bird photographs he wants to find all images depicting a green-speckled swallow. Due to it's distinctive features it is an easy task for Eduard, Siegfried's friend and computer vision scientist, to program a green-speckled swallow detector that marks all images containing such a bird. Unfortunately the detector does not work perfectly. If the image contains a green-speckled swallow the detector marks it correctly with a chance of 99.5%. If the image does not contain a green-speckled swallow the detector marks it correctly with a chance of 99.3%. The bird is also very rare: If we randomly draw an image from the collection, there is only a chance of 0.001% that the image contains a green-speckled swallow.

a) Do a formal modeling of the experiment. How does the discrete probability space look like?

b) What is the probability that a green-speckled swallow is on a given image, if the detector gives a positive answer?

c) What is the probability that a green-speckled swallow is on a given image, if the detector gives a negative answer?

Exercise 3 (Independence, 2 points**).** Let A and B be events. Show that

$$P(A \mid B) = P(A) \quad \Leftrightarrow \quad P(A \cap B) = P(A) \cdot P(B) \,.$$

Exercise 4 (Conditional independence, 2 points**).** Let A, B and C be events. Show that the two definitions of the conditional independence are equivalent, that is,

$$P(A \mid C) = P(A \mid B \cap C) \quad \Leftrightarrow \quad P(A \cap B \mid C) = P(A \mid C) \cdot P(B \mid C) \,.$$