Weekly Exercise 7

Dr. Csaba Domokos
Technische Universität München, Computer Vision Group
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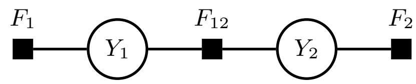
Fast Primal-Dual Schema (6 Points)

Exercise 1 (**Primal-dual LP**, 4 Points)**.** Let us consider the following *factor graph* model with $\mathcal{Y}_1 = \{1, 2, 3\}$ and $\mathcal{Y}_2 = \{1, 2\}$



where the factor energies are defined as follows:

y_1	$E_1(y_1)$
1	5
2	2
3	7

y_2	$E_2(y_2)$
1	1
2	2

$E_{12}(y_1, y_2)$	1	2
1	0	1
2	1	0
3	4	1

Define the **primal** and the **dual** *linear programs*

$$\min_{\mathbf{x}} \ \langle \mathbf{c}, \mathbf{x} \rangle \qquad \max_{\mathbf{y}} \ \langle \mathbf{b}, \mathbf{y} \rangle$$
$$\mathbf{A}\mathbf{x} = \mathbf{b} \qquad \mathbf{A}^T\mathbf{y} \leq \mathbf{c}$$
$$\mathbf{x} \geq \mathbf{0}$$

for the *multi-labeling problem* corresponding to the factor graph above:

$$E(\mathbf{y}) = E_1(y_1) + E_2(y_2) + E_{12}(y_1, y_2) \ .$$

Exercise 2 (**Complementary slackness,** 2 Points)**.** Let $(\mathbf{x}, \mathbf{y})$ be a pair of *integral primal* and *dual feasible* solutions to the linear programming relaxation

$$\min_{\mathbf{x}} \ \langle \mathbf{c}, \mathbf{x} \rangle \qquad \max_{\mathbf{y}} \ \langle \mathbf{b}, \mathbf{y} \rangle$$
$$\mathbf{A}\mathbf{x} = \mathbf{b} \qquad \mathbf{A}^T\mathbf{y} \leq \mathbf{c}$$
$$\mathbf{x} \geq \mathbf{0}$$

corresponding to the multi-labeling problem. Show that if $(\mathbf{x}, \mathbf{y})$ satisfies the *relaxed primal complementary slackness conditions*, that is

$$\forall x_j > 0 \quad \Rightarrow \quad \sum_{i=1} a_{ij} y_i \geq \frac{c_j}{\varepsilon_j} \ ,$$

then $\mathbf{x}$ is an ε-approximation to the *optimal integral solution* x^* with $\varepsilon = \max_j \varepsilon_j$.

Programming (6 Points)

Exercise 3 (Semantic image segmentation with α-expansion, 6 Points**).** Consider the energy function

$$E(\mathbf{y};\mathbf{x}) = \sum_{i\in\mathcal{V}} E_i(y_i;x_i) + w \sum_{i,j\in\mathcal{E}} E_{ij}(y_i,y_j;x_i,x_j) \quad , \tag{1}$$

for the multi-labeling problem, i.e. $\mathbf{y} \in \mathcal{L}^{\mathcal{V}}$, where $\mathcal{L}$ stands for the label set. $\mathcal{V}$ is the set of pixels and $\mathcal{E}$ consists of all four-neighboring pixels. Implement the α-expansion algorithm to solve the **semantic image segmentation** for the images shown in figure 1. Try to choose different w for Equation (1) and compare the segmentation results.

Figure 1: The test images for semantic image segmentation.

These test images have been obtained from the MSRC image understanding dataset[1], which contains 21 classes, i.e. $\mathcal{L} = \{1, 2, \ldots, 21\}$. The meaning of the classes are given in the `21class.txt` file. Use it to check whether your results are reasonable.

To define the **unary energy functions** E_i, use the `*.c_unary` files provided in the supplementary material (`supp_07`). Each test image has its own unary file, specified by the same filename. From each unary file, you can read out a $K \times H \times W$ array of float numbers. The H and W are the image height and width, and $K = 21$ is the number of classes. This array contains the 21-class probability distribution for each pixel. You may find the `multilabel_demo.cpp` in the supplementary material, which demonstrates how to load a unary file and read out the corresponding probability values. The unary energy functions E_i for all $i \in \mathcal{V}$ are then defined as the *negative log-likelihood*.

The **pairwise energy functions** E_{ij} are defined by the *contrast sensitive Potts-model*

$$E_{ij}(y_i,y_j;x_i,x_j) = \exp(-\lambda\|x_i - x_j\|^2)[\![y_i \neq y_j]\!] \quad ,$$

where x_i is the intensity vector for pixel i, and you may choose $\lambda = 0.5$.

[1] `https://www.microsoft.com/en-us/research/project/image-understanding/`