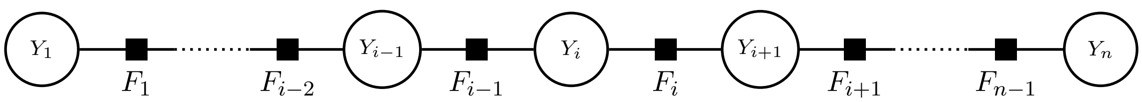
Weekly Exercise 9

Dr. Csaba Domokos
Technische Universität München, Computer Vision Group
July 3rd, 2017

Probabilistic Inference (8 points)

Exercise 1 (**Inference on chains**, 2 points)**.** Consider the following factor graph, which is a chain:



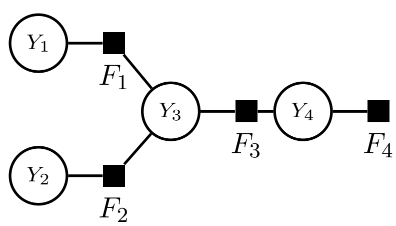
The joint distribution can be written in the form

$$p(\mathbf{y}) = \frac{1}{Z}\psi_{F_1}(y_1, y_2)\psi_{F_2}(y_2, y_3) \cdot \ldots \cdot \psi_{F_{n-1}}(y_{n-1}, y_n) \ ,$$

where $Z = \sum_{\mathbf{y}} \prod_{i=1}^{n-1} \psi_{F_i}(y_i, y_{i+1})$ denotes the partition function. Show that the *marginal distribution* $p(y_i)$ decomposes into the product of two factors:

$$p(y_i) = \frac{1}{Z} r_{F_{i-1} \to Y_i}(y_i) r_{F_i \to Y_i}(y_i) \ .$$

Exercise 2 (**Sum-product and max-sum**, 6 points)**.** Consider the following factor graph,



The potential functions are defined for $k \in \{1, 2, 3\}$

$$\psi_{F_k}(y_i, y_j) = \exp\left(-(|y_i - y_j| + (c_k - y_i)^2)\right) \ , \text{ where } c_1 = 0, c_2 = 1, c_3 = 1 \ , \text{ and}$$
$$\psi_{F_4}(y_4) = \exp(-(2 - y_4)^2) \ .$$

Assume Y_4 as the root node and $\mathbf{y} \in \mathcal{L}^4 = \{0, 1, 2\}^4$.

a) Perform the *sum-product algorithm* in order to achieve *probabilistic inference* of the model expressed by the graph above. Show the intermediate steps in details.

b) Perform the *max-sum algorithm* to achieve *MAP inference* of the model expressed by the graph above. Show the intermediate steps in details.