# Analysis of 3D Shapes (IN2238)

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Summer Semester 2017



**We are always looking for master students!**


3D Reconstruction


Optical Flow

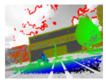

Shape Analysis


Robot Vision


RGB-D Vision


Image Segmentation


Convex Relaxation


Visual SLAM

Please talk to the appointed contact person directly.





# 1. Introduction



# Schedule



The lecture **Analysis of Three-Dimensional Shapes** will be organized as following:

- **Tuesday Lecture:** 10-11 and 11-12 in Room 02.09.023
- **Wednesday Tutorial:** 14-16 in Room 02.09.023
- **Thursday Lecture:** 14:00–14:45 and 15:00–15:45 in Room 02.09.023

The tutorial combines theoretical and programming assignments:

- **Assignment Distribution:** Tuesday 11:00-11:15 in Room 02.09.023
- **Theoretical Assignment Due:** Tuesday 23:59 per email
- **Assignment Presentation:** Wednesday 14-16 in Room 02.09.023





## May 2017

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|
| 1 | 2 Lecture 1 | 3 | 4 Lecture 2 | 5 | 6 | 7 |
| 8 | 9 Lecture 3 | 10 Matlab | 11 Lecture 4 | 12 | 13 | 14 |
| 15 | 16 Lecture 5 | 17 Tutorial 1 | 18 Lecture 6 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 Lecture 7 | 31 Tutorial 2 | | | | |





## June 2017

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|
| | | | 1 Lecture 8 | 2 | 3 | 4 |
| 5 | 6 | 7 Tutorial 3 | 8 Lecture 9 | 9 | 10 | 11 |
| 12 | 13 Lecture 10 | 14 Tutorial 4 | 15 | 16 | 17 | 18 |
| 19 | 20 Lecture 11 | 21 Tutorial 5 | 22 Lecture 12 | 23 | 24 | 25 |
| 26 | 27 Lecture 13 | 28 Tutorial 6 | 29 Lecture 14 | 30 | | |





## July 2017

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|
| | | | | | 1 | 2 |
| 3 | 4 Lecture 15 | 5 Tutorial 7 | 6 Lecture 16 | 7 | 8 | 9 |
| 10 | 11 Lecture 17 | 12 Tutorial 8 | 13 Lecture 18 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| 24 | 25 | 26 | 27 | 28 | 29 | 30 |

## Exams

Requirements for being admitted to the exam:

- **Registration:** Students need to be registered prior to the exam: May, $29^{\text{th}}$ – June, $30^{\text{th}}$ via TUM online.
- **Exam:** In the week of August, $14^{\text{th}}$ – $18^{\text{th}}$.

Participation at the tutorial:

- **Not mandatory, but highly recommended:** Theoretical assignments will help to understand the topics of the lecture. Programming assignments will help to apply the theory to practical problems.
- **Bonus:** Active students who solve 60% of the assignments earn a bonus.
- **Exam:** If one receives a mark between 1.3 and 4.0 in the exam, the mark will be improved by 0.3 resp. 0.4. Marks of 1.0 or 5.0 cannot be improved.





## Bonus

To achieve the bonus, the following requirements have to be fulfilled:

**Theory**

- 60% of all theoretical assignments have to be solved. (PDF-Submissions (using LaTeX) happen only online via email)
- At least one theoretical exercise has to be presented in front of the class.

**Programming**

- 60% of all programming assignments have to be presented during the tutorial.

To promote team work, we advocate to form groups of **two** or **three** students in order to solve and submit the assignments.





## Team

**Lecturers** **Teaching Assistant**

Dr. Frank R. Schmidt

Matthias Vestner

Zorah Lähner

Please do not hesitate to contact us in order to set up an appointment:

- `f.schmidt@in.tum.de`
- `matthias.vestner@in.tum.de`
- `laehner@in.tum.de`





## Course Material



## Course Page

On the internal site of the course page you have access to extra course material:
`https://vision.in.tum.de/teaching/ss2017/shape_2238`

**Password** `Sh4p3`

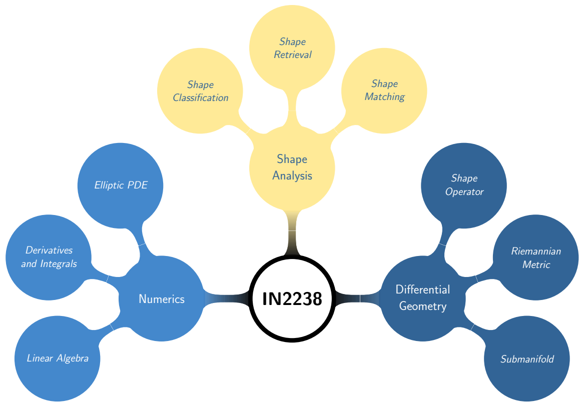
- Slides for each lecture (Available prior to the lecture)
- Assignment Sheets (Available after the Tuesday lecture)
- Solution Sheets (Available after the Wednesday tutorial)

**The course page will also be used for extra announcements.**





## Topics of the Lecture








## Shape



## What is a Shape?

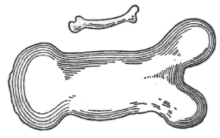
Cave Painting

Paper Cutting

- A **shape** describes the geometry of an object without relying on a specific location or orientation of this object, *i.e.*, *An egg remains an egg, no matter where we put it on a table.*
- A shape should reflect the human perception of objects: *The whole is more than the sum of its parts.*

# What is a Shape?





Source: Galileo, 1638

**Definition 1** (Kendall, 1977). **Shape** is all the *geometrical information* that remains when *location, scale and rotational effects* are filtered out.

Two 3D-objects $O_1, O_2 \subset \mathbb{R}^3$ are **of the same shape** if we find a rotation $R \in \mathrm{SO}(3)$, a translation $T \in \mathbb{R}^3$ and a scaling factor $\sigma \in \mathbb{R}_+$ such that $O_1$ becomes $O_2$ after scaling, rotation and translation.



# Shape Transformation



We can transform a 3D-object $O_1$ by applying a function $\Phi\colon \mathbb{R}^3 \to \mathbb{R}^3$ to this object, *i.e.*,

$$\Phi(O_1) := \{\Phi(x) | x \in O_1\}$$

We write

$$\begin{aligned} O_1 + T :=& \Phi(O_1) & \Phi(x) =& x + T & \text{(Translation)} \\ \sigma \cdot O_1 :=& \Phi(O_1) & \Phi(x) =& \sigma \cdot x & \text{(Scaling)} \\ R \cdot O_1 :=& \Phi(O_1) & \Phi(x) =& R \cdot x & \text{(Rotation)} \end{aligned}$$

This means that $O_2$ is of the same shape as $O_1$ if the following is satisfied

$$O_2 = \sigma \cdot (R \cdot O_1 + T)$$



# Shape as Equivalence Class of Objects



Note that the above mentioned relation between objects defines an **equivalence relation**:

$$O_1 \sim O_2 \quad :\Leftrightarrow \quad \left[\exists (R, T, \sigma) \in \mathrm{SO}(3) \times \mathbb{R}^3 \times \mathbb{R}_+ : O_2 = \sigma \cdot (R \cdot O_1 + T)\right]$$

One can easily show that the following holds:

$$\begin{aligned} & O_1 \sim O_1 & & & \text{(Reflexivity)} \\ & O_1 \sim O_2 & \Leftrightarrow \quad & O_2 \sim O_1 & \text{(Symmetry)} \\ & O_1 \sim O_2, O_2 \sim O_3 & \Rightarrow \quad & O_1 \sim O_3 & \text{(Transitivity)} \end{aligned}$$

Every equivalence relation defines an equivalence class of equivalent objects. The equivalence class $[O] := \{O' \subset \mathbb{R}^3 | O \sim O'\}$ of an object $O \subset \mathbb{R}^3$ is the **shape** of this object.



# Restricted and Extended Shape Notions



In some applications, we like to differentiate between objects of different sizes. Analogously, we can define the **Shape-and-Scale equivalence relation**:

$$O_1 \sim O_2 \quad :\Leftrightarrow \quad \left[\exists (R, T) \in \mathrm{SO}(3) \times \mathbb{R}^3 : O_2 = R \cdot O_1 + T\right]$$

The equivalence class $[O] := \{O' \subset \mathbb{R}^3 | O \sim O'\}$ of an object $O \subset \mathbb{R}^3$ is the **shape-and-scale** of this object.

Some authors call also the shape-and-scale of an object their shape.

In some applications, we also like to extend the shape equivalence relation in order to be invariant with respect to different "poses", *i.e.*:
*A horse is a horse, no matter whether it is standing or galopping.*

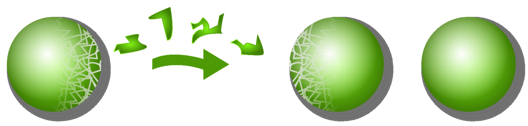




# Object

# Banach-Tarski Paradox





Source: Wikimedia.org

**Theorem 1** (Banach and Tarski, 1924). *The 3D ball $B = \{x \in \mathbb{R}^3 | \langle x, x \rangle \leqslant 1\}$ can be partitioned into finite many sets $A_i$,* i.e., $B = \bigsqcup_{i=1}^k A_i$, *which can be rotated to form two copies of $B$.*

The proof of this paradox uses a heavy machinery based on set theory. In particular the so called *Axiom of Choice* is used to create the sets $A_i$.

Interestingly, we cannot measure the volume of any of the sets $A_i$. To avoid any similar problem, we are only interested in objects $O \subset \mathbb{R}^n$ that are open.



# Open Sets



**Definition 2** (Open Set). A set $O \subset \mathbb{R}^n$ is called **open** iff

$$x \in O \quad \Rightarrow \quad \exists \varepsilon > 0 : B_\varepsilon(x) \subset O,$$

where $B_\varepsilon(x) := \{y \in \mathbb{R}^n | \, \|x - y\| < \varepsilon\}$ is a ball of radius $\varepsilon$ centered at $x \in \mathbb{R}^n$.

**Definition 3** (Relatively Open Set). Given $X \subset \mathbb{R}^n$, we call $O \subset X$ **relatively open with respect to $X$** iff there exists an open set $\hat{O} \subset \mathbb{R}^n$ such that

$$O = \hat{O} \cap X$$

The set of all open sets is called its topology $\mathcal{T}(X)$.

**Definition 4** (Neighborhood). Given a subset $X \subset \mathbb{R}^n$ and $x \in X$, we call $O(x) \in \mathcal{T}(X)$ an **open neighborhood** of $x$ if $x \in O$.



# Boundary

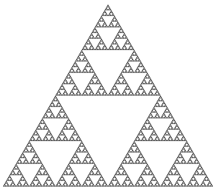


**Definition 5** (Boundary). Given a set $X \subset \mathbb{R}^n$, we call

$$\partial X := \{x \in \mathbb{R}^n | \forall O(x) \in \mathcal{T}(\mathbb{R}^n) : O \cap X \neq \varnothing \wedge O \backslash X \neq \varnothing\}$$

its **boundary**.

- The boundary of $\mathbb{R}^n$ is $\partial \mathbb{R}^n = \varnothing$.
- The boundary of the closed interval $[0, 1]$ is $\partial[0, 1] = \{0, 1\}$.
- The boundary of $[0, 1[$ is $\partial[0, 1[ = \{0, 1\}$.
- The boundary of $]0, 1[$ is $\partial]0, 1[ = \{0, 1\}$.
- The boundary of the ball $B^n = \{x \in \mathbb{R}^n | \langle x, x \rangle \leqslant 1\}$ is $\partial B = \mathbb{S}^{n-1} = \{x \in \mathbb{R}^n | \langle x, x \rangle = 1\}$.



Source: Wikimedia.org

To define what a shape is, we first need to define an **object**.

**Definition 6** (Informal Definition)**.** An **object** of dimension $d$ is an open subset $X \subset \mathbb{R}^d$ such that $\dim \partial X = d - 1$.

Note that we exclude sets whose boundaries are **fractals**. Instead, we want to have a boundary with the well-defined dimension $d - 1$.





### Shapes


- Galileo, *Discorsi e dimostrazioni matematiche, informo a due nuove scienze attenti alla mecanica i movimenti locali*, 1638, appresso gli Elsivirii; Opere VIII. (2)
- Kendall, *The diffusion of shape*, 1977, Advances in Applied Probabilities (9), 428–430.
- Dryden and Mardia, *Statistical Shape Analysis*, 1998, Wiley, 376 pages.


### Set Theoretical Results


- Banach and Tarski, *Sur la décomposition des ensembles de points en parties respectivement congruentes*, 1924, Fundamenta Mathematica (6), 244–277.
- Carathéodory, *Vorlesungen über reelle Funktionen*, 1918, B. G. Teubner, 704 pages.