Analysis of Three-Dimensional Shapes
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Weekly Exercises 1

Room: 02.09.023

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Mathematics: Calculus recap and Manifolds

Recap the definition of *partial derivative* if you are not familiar with it anymore. Quick introduction of notation: For a differentiable function $f : \mathbb{R}^n \to \mathbb{R}^m$ the partial derivative of the $j-th$ component of f by the i-th variable can be written as

1. $\partial_i f^j$ with $i \in \{1, ..., n\}, j \in \{1, ..., m\}$
2. $\frac{\partial f^j}{\partial x_i}$ describing the same thing but assuming that the variable are given names as is normally case (e.g. $(x, y, z) \mapsto (x, y + z)$)

The notation is a matter of taste but some are less confusion depending on the situation.

The *differential* is the best linear approximation of a function. For a function $f : \mathbb{R}^n \to \mathbb{R}^m$ it can be represented by its Jacobi matrix:

$$Df = \begin{pmatrix} \partial_1 f^1 & \dots & \partial_n f^1 \\ \vdots & & \vdots \\ \partial_1 f^m & \dots & \partial_n f^m \end{pmatrix}$$

or (if taking partial derivatives is not trivial)

$$Df(x)[h] \doteq f(x + h) - f(x)$$

In this case the equality holds only for linear terms in h.

Exercise 1 (2 points). 1. Let f be

$$f : \mathbb{R}^2 \to \mathbb{R}$$
$$(x, y) \mapsto \begin{cases} 0 & \text{if } x = y = 0 \\ \frac{xy}{x^2+y^2} & \text{otherwise} \end{cases}$$

Calculate the partial derivatives $\partial_1 f$ and $\partial_2 f$. What happens at $\partial_1 f(0, 0)$?

2. Consider $g:\ \mathbb{R}^2 \to \mathbb{R}$ and $f:\ \mathbb{R}^2 \to \mathbb{R}$ differentiable with

$$g(x_1, x_2) = f(x_1^2, x_1 + x_2)$$

Calculate $\frac{\partial g}{\partial x_1}$ (in relation to f).

Exercise 2 (2 points).

1. Calculate the differential of

$$\begin{aligned} f_1 : \mathbb{R}^3 &\to \mathbb{R}^2 \\ (x, y, z) &\mapsto (x(1-y), xyz) \end{aligned}$$

2. Calculate the differential of

$$\begin{aligned} f_2 : \mathbb{R}^2 &\to \mathbb{R}^3 \\ (u, v) &\mapsto (u^2 + v^2, u - v, 4v^4) \end{aligned}$$

Exercise 3 (3 points). Consider the vector spaces $U = \mathbb{R}^3$ and $V = \mathbb{R}^2$ which can be equipped with the canonical basis C_3, C_2 or the following ones:

$$X_1 = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right) \qquad X_2 = \left(\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix} \right)$$

$$Y_1 = \left(\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix} \right) \qquad Y_2 = \left(\begin{pmatrix} 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right)$$

Let $L : U \to V$ be a linear mapping that can be represented by $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ in the canonical basis.

1. Write down $\mathcal{M}_{Y_1}^{X_1}(L)$.
2. Write down $\mathcal{M}_{Y_2}^{X_2}(L)$.
3. You have $a \in U$ written in the basis X_1 with the coefficients $(1, 1, 1)$. What is the result of applying L to a written in the canonical basis?

Tip: no need to calculate the matrix inverses by hand.