Chapter 0
Organization and Overview

Convex Optimization for Machine Learning & Computer Vision
SS 2018

Tao Wu
Emanuel Laude
Zhenzhang Ye

Computer Vision Group
Department of Informatics
TU Munich

Organization

Course Overview

Tao Wu
Emanuel Laude
Zhenzhang Ye

Organization

Course Overview

Organization

Whether this lecture fits you?

Prerequisites

- Background in mathematical analysis and linear algebra
- Numerical implementation in Matlab or Python
- Interest in mathematical theory (know why!)

Tao Wu
Emanuel Laude
Zhenzhang Ye

Whether this lecture fits you?

Prerequisites

- Background in mathematical analysis and linear algebra
- Numerical implementation in Matlab or Python
- Interest in mathematical theory (know why!)

Nice plus (but not necessary)

- Experience in machine learning or computer vision
  e.g., CV I & II, ML for CV, Probab. Graphical Models in CV.
- Knowledge in continuous optimization
  e.g., Nonlinear Optimization.
- Knowledge in functional analysis
  Further understanding of the theory in full generality.

Course overview

Lectures

1. Theory of convex analysis
2. Design and analysis of optimization algorithms
3. Selected topic: Stochastic optimization

Applications are mostly covered in exercise session...

Tao Wu
Emanuel Laude
Zhenzhang Ye

Exercise session

Organizers: Emanuel Laude and Zhenzhang Ye

- Exercise sheets covering the content of the lecture will be passed out every Wednesday.
- Exercises contain theoretical as well as programming questions.
- If solutions have obviously been copied, both groups will get 0 points.
- You may work on the exercises in groups of two.
- You are encouraged to present your solution on board at exercise class.
- To get a 0.3 grade bonus, you need to complete 75% of the total exercise points.

Tao Wu
Emanuel Laude
Zhenzhang Ye

Contact us

Miscellaneous info

- Tao's office: 02.09.061
- Emanuel's office: 02.09.039
- Zhenzhang's office: 02.09.037
- Office hours: Please write an email.
- Lecture: Starts at quarter past. Short break in between.
- Course website:
  `https://vision.in.tum.de/teaching/ss2018/in2330`

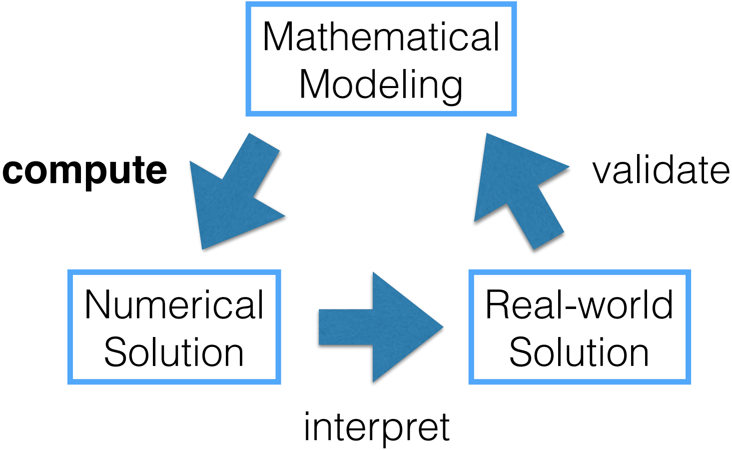
Tao Wu
Emanuel Laude
Zhenzhang Ye

Organization

Course Overview

First Glimpse of the Course

Driving cycle



Tao Wu
Emanuel Laude
Zhenzhang Ye

An example from computer vision / machine learning

- Image segmentation / multi-labeling:

image

segmentation ($L = 4$)

An example from computer vision / machine learning

- Image segmentation / multi-labeling:

image

segmentation ($L = 4$)

- Variational method for finding label function $u : \Omega \to \Delta^L$

$$\text{minimize} \sum_{j \in \Omega} \Big(\delta\{u_j \in \Delta^L\} + \langle u_j, f_j \rangle \Big) + \alpha \sum_{l=1}^{L} \sum_i \omega_i \cdot (\nabla u^l)_i,$$

where

- Pointwise constraint: Δ^L is the unit $(L-1)$-simplex.
- Unary term: $f : \Omega \to \mathbb{R}^L$ is a pre-computed vector.
- Pairwise term: $\sum_i \omega_i \cdot (\nabla u^l)_i$ is the weighted total-variation.

Prototypical workflow

- The variational model

$$\text{minimize} \sum_{j\in\Omega}\Big(\delta\{u_j \in \Delta^L\} + \langle u_j, f_j\rangle\Big) + \alpha\sum_{l=1}^{L}\sum_{i}\omega_i\cdot(\nabla u^l)_i,$$

  is a special case of **convex optimization**

$$\text{minimize } J(u) + \delta\{u \in C\},$$

  with **convex objective** J and **convex constraint** C.

- This course is about **theory** and **practice** for solving convex optimization (arising from computer vision and machine learning).

Prototypical workflow

- Put into canonical form:

$$\min_u F(Ku) + G(u),$$

where F, G are convex functions, K is a linear operator.

Prototypical workflow

- Put into canonical form:

$$\min_u F(Ku) + G(u),$$

where F, G are convex functions, K is a linear operator.

- Reformulate the problem (by introducing *dual variable* p):

$$\max_p \min_u \langle Ku, p\rangle - F^*(p) + G(u),$$

where F^* is the *convex conjugate* of F.

Tao Wu
Emanuel Laude
Zhenzhang Ye

Prototypical workflow

- Put into canonical form:

$$\min_u F(Ku) + G(u),$$

where F, G are convex functions, K is a linear operator.

- Reformulate the problem (by introducing *dual variable* p):

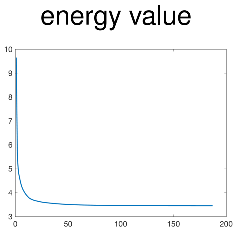
$$\max_p \min_u \langle Ku, p\rangle - F^*(p) + G(u),$$

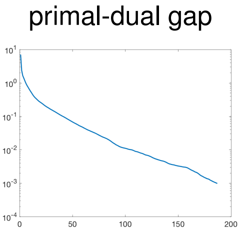
where F^* is the *convex conjugate* of F.

- Apply an iterative scheme (e.g. PDHG):

$$u^{k+1} = \arg\min_u \left\langle u, K^\top p^k \right\rangle + G(u) + \frac{s}{2}\|u - u^k\|^2,$$
$$p^{k+1} = \arg\min_p -\left\langle K(2u^{k+1} - u^k), p \right\rangle + F^*(p) + \frac{t}{2}\|p - p^k\|^2.$$

Questions of our concern





- Does a minimizer always exist?
- How to characterize a minimizer?
- How to derive an optimization algorithm?
- How to analyze/guarantee the convergence?
- How to accelerate the convergence?
- Efficient implementation, etc.

Ready to start?

Tao Wu
Emanuel Laude
Zhenzhang Ye